

Jan Carlsson

# A FDTD Program for Computing Responses on Branched Multi-Conductor Transmission Lines

## Abstract

This document gives a description of a finite difference time domain (FDTD) program that is intended for the prediction of voltages and currents on branched multi-conductor transmission lines. The excitation of the lines can either be an incident electromagnetic field or voltage sources placed at the ends of the lines. It is even possible to compute the responses when both types of excitations exist simultaneously.

The program can be run under Windows 95 or Windows NT 4.0 or later.

**Key words:** Crosstalk, FDTD, Finite difference time domain method, Multi-conductor transmission lines, Transmission line network.

**SP Sveriges Provnings- och  
Forskningsinstitut**  
SP Rapport 1998:16  
ISBN 91-7848-722-6  
ISSN 0284-5172  
Borås 1998

**SP Swedish National Testing and  
Research Institute**  
SP Report 1998:16

Postal address:  
Box 857, S-501 15 BORÅS,  
Sweden  
Telephone +46 33 16 50 00  
Telex 36252 Testing S  
Telefax +46 33 13 55 02

# Contents

<i>Abstract</i> .....	2
<i>Contents</i> .....	3
<i>Summary</i> .....	5
<b>1 Introduction</b> .....	7
<b>2 Disposition</b> .....	8
<b>3 Program overview</b> .....	9
<b>3.1 Intended use</b> .....	9
<b>3.2 Installation</b> .....	9
<b>3.3 Description</b> .....	9
<b>3.4 A quick look</b> .....	9
<b>4 Program use</b> .....	11
<b>4.1 Defining transmission line properties</b> .....	11
4.1.1 Line discretisation .....	12
4.1.2 Connection of transmission lines .....	12
<b>4.2 Defining excitations and terminations</b> .....	14
4.2.1 Termination resistances and voltage sources .....	14
4.2.2 Excitation by incident plane wave .....	14
4.2.3 Available waveforms .....	17
<b>4.4 Time discretisation</b> .....	19
<b>4.5 Simulation</b> .....	19
<b>4.6 Presentation of computed results</b> .....	20
4.6.1 Voltages and currents as a function of position and time .....	21
4.6.2 End-responses .....	21
<b>5 Line characteristics, <math>L</math> and <math>C</math> matrices</b> .....	23
<b>5.1 Definition</b> .....	23
<b>5.2 Simple formulas</b> .....	24
5.2.1 Two-wire line .....	24
5.2.2 Coaxial cable .....	25
5.2.3 Single wire over ground plane .....	25
5.2.4 $(n+1)$ wires .....	26
5.2.5 $n$ wires over ground plane .....	26
<b>6 Sample simulations, validation</b> .....	28
<b>6.1 Pulse propagation on a simple transmission line</b> .....	28
6.1.1 Line characteristics .....	28
6.1.2 Propagation on matched line .....	28
6.1.3 Propagation on shorted line .....	29
6.1.4 Propagation on open line .....	29
<b>6.2 Crosstalk between parallel conductors</b> .....	29
6.2.1 Line characteristics .....	29
6.2.2 End-response in frequency domain .....	30
<b>6.3 Propagation on a transmission line network</b> .....	31
6.3.1 Line characteristics .....	31

6.3.2 Computed response .....	31
<b>6.4 End-responses due to an incident plane wave .....</b>	<b>33</b>
6.4.1 Line characteristics .....	33
6.4.2 Exciting plane wave .....	33
6.4.3 Computed response .....	33
<b>7 Theory .....</b>	<b>35</b>
7.1 Multi-conductor transmission line equations .....	35
7.2 Solution by using FDTD .....	36
7.3 Extension to include junctions .....	39
<b>8 References .....</b>	<b>41</b>
<b>Appendix File formats .....</b>	<b>42</b>
A1 Project file .....	42
A2 L, C and R files .....	42
A3 End-response files .....	43
A4 Normalisation file .....	44
A5 Field excitation files .....	44

## Summary

The FDTD program described in this document is intended to be used for predicting crosstalk between conductors in multi-conductor transmission lines as well as induced currents and voltages in terminations due to incident fields. The program can handle several transmission lines that are connected to each other. Computation is done in the time domain but an internal FFT routine can be used for obtaining results also in the frequency domain.

A transmission line is described for the program by setting several properties for the line. Among the properties that must be defined are the length of the line, the number of conductors, inductance and capacitance matrices, termination impedances and discretisation parameters. Since the only properties describing line characteristics are the inductance and capacitance matrices lines are assumed to be lossless.

The transmission lines can have up to nine conductors and are terminated in resistors connected in a way that can be described by resistance matrices. Excitation of the lines can either be done by placing voltage sources in the ends of the lines or by defining an incident electromagnetic field. Since the field excitation is read from files, that for the case of incident plane waves can be generated by the program, fields computed by another program can also be used for exciting the lines. This means that the field excitation is not only restricted to incident plane waves but any field can be used.

During a simulation the voltages and currents on any of the conductors in a transmission line can be shown. Thus, e.g. a pulse travelling back and forth the line can be seen in real time as a function of the position along the line. Also the voltages and currents at terminations can be saved in files and be further analysed after a simulation. Termination responses saved in files can be plotted by the program and can also be transformed to the frequency domain through the internal FFT routine.



# 1 Introduction

In the context of EMC it is of great importance to be able to compute the crosstalk between individual conductors in a cable or between conductors on a printed circuit board. The crosstalk can either be computed using so called full-wave methods or it can be computed using some simplified formulation. The full-wave methods are based on a rigorous formulation using Maxwells equations which then are solved by a numerical method such as FDTD, MoM, FEM etc. The advantage of these methods are that all effects are taken into account, the only approximation that is introduced is the one due to the numerical method being used. However, the disadvantages by using full-wave methods are that the analysis is complicated and often very time consuming.

One simplified formulation that often is used for computing crosstalk is to use the transmission line equations for multi-conductors. This is a simplified formulation since a TEM mode of propagation along the transmission line is assumed. Radiation effects are also not taken into account. However, it turns out that the simplification is not severe and for many practical cases the two approaches agrees very well, see e.g. [1]. The advantages by using the transmission line equations are that the equations are relatively simple and can be solved in an efficient way.

The computer code that is described in this document is based upon the transmission line equations for multi-conductors. The equations are solved by using a finite difference time domain method, [2]-[4]. The formulation in [2]-[4] is extended so that several transmission lines can be connected forming a transmission line network.

The results obtained by the BMTL program have been tested against previously published results and against computations done with other programs.

## **2 Disposition**

A brief overview of the BMTL program and a description of the capabilities are given in chapter 3. The next following chapters describe how to use the program for analysing transmission line networks.

Before a simulation can be started the properties of the transmission lines and how they are connected have to be described for the program. These topics are described in chapter 4.1.

The transmission line network can be excited by voltage sources and/or incident fields. For both excitation types several different waveforms can be chosen from. These, together with how the lines can be terminated in resistance networks are described in chapter 4.2.

The last sub-chapters of chapter 4 deal with the simulation and how to visualise computed results.

Chapter 5 defines the parameters that are needed for describing the characteristics of transmission lines. Simple formulas for a few common types of transmission lines are also given in this chapter.

In order to show how the program can be used a few examples are given in chapter 6. Some of these examples also serve as a validation of results obtained by the program.

The theory behind the program is covered in chapter 7 and the format for files used and created by the program are described in the appendix.

## 3 Program overview

### 3.1 Intended use

The BMTL program is in the first place intended to be used in EMC applications where the interest is in determining the crosstalk between individual conductors in a multi-conductor transmission line and also the induced voltages and currents at the ends of a transmission line due to an incident electromagnetic field.

Since the transmission lines are described by their per unit-length inductance and capacitance matrices a transmission line can just as well represent e.g. a cable or the parallel conductors on a printed circuit board.

### 3.2 Installation

The program can be installed on a PC running Windows 95 or Windows 4.0 or later. The installation is done by executing the "Setup.EXE" on the installation diskette. During the installation the following necessary files will be installed on the computer:

- BMTL.EXE, program file
- BMTL.HLP, help file

In addition two folders will also be created in the program folder, the folder "Examples" containing example projects and the folder "Parameters" containing per unit-length parameters for a few simple transmission lines.

### 3.3 Description

The BMTL program is a finite difference time domain program that solves the transmission line equations for multi-conductor transmission lines. Transmission lines can be connected to each other forming a transmission line network. Transmission lines are terminated by resistors that are connected in a way that can be described by a resistance matrix. The transmission line network can be excited by voltage sources placed at the ends of the transmission lines. Several different waveforms can be chosen from and different forms can be used simultaneously. A transmission line can also be excited by an incident electromagnetic field. This field can either be generated internally by the program (plane wave) or read from a file generated by another program or by measurements. An internal FFT routine makes it possible to transform computed responses in the time domain to the frequency domain.

### 3.4 A quick look

In using the BMTL program the *Transmission Line Properties* window plays a central role, Fig. 3:1. Through this window most of the parameters that need to be defined before a computation can be performed are set.

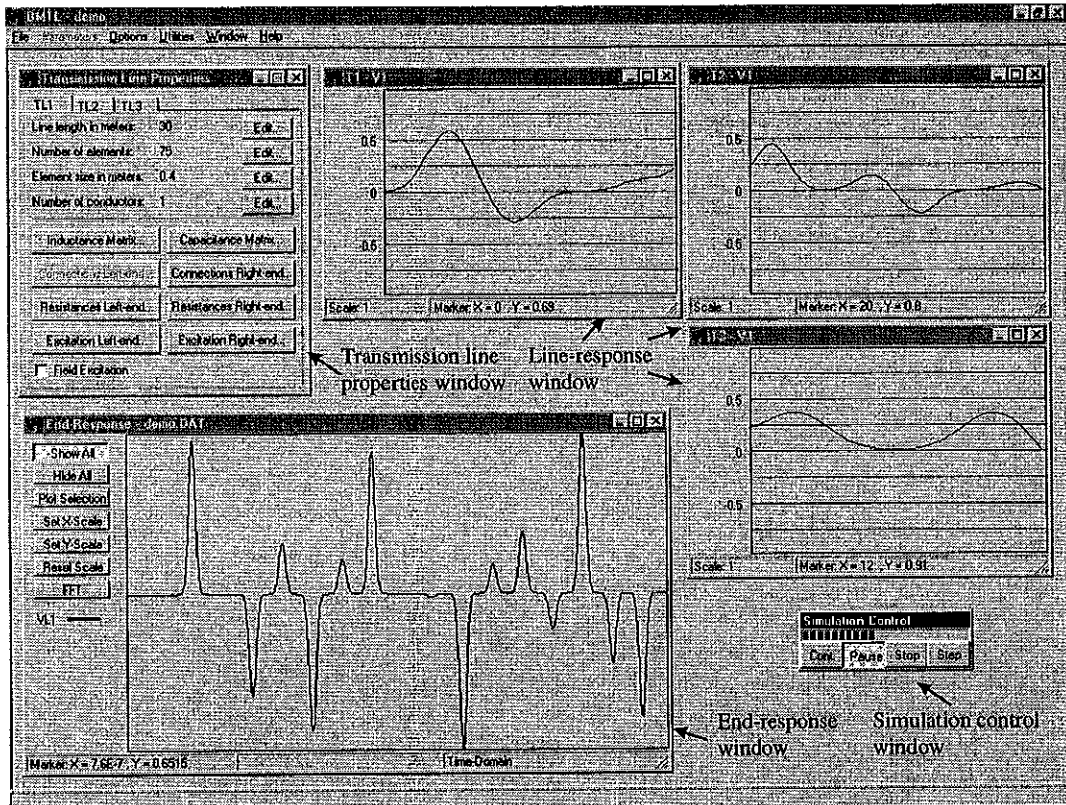


Fig. 3:1. The user interface.

When all parameters are set through the *Transmission Line Properties* window a simulation can be started by pressing the *Start* button in the *Simulation Control* window, Fig. 3:1. It is also possible to halt a simulation any time by pressing the *Stop* or the *Pause* button in the *Simulation Control* window. When a simulation is paused the simulation can be stepped one time step at a time by pressing the *Step* button. This makes it possible to in detail study what happens e.g. when a pulse is reflected at the terminations. During a simulation voltages and currents along individual conductors in a transmission line can be viewed in *Line-response* windows, Fig. 3:1. In these windows e.g. a pulse travelling forth and back the line can be seen in real time. Selected voltages and currents at the ends of a transmission line can be saved in files during a simulation. These responses can after a simulation be viewed in *End-response* windows, Fig. 3:1. End-responses can also be transformed to the frequency domain in these windows.

Connections of transmission lines are defined in a dialog box but can also be viewed in a separate window in order to make it easy to check the connections, Fig. 3:2.

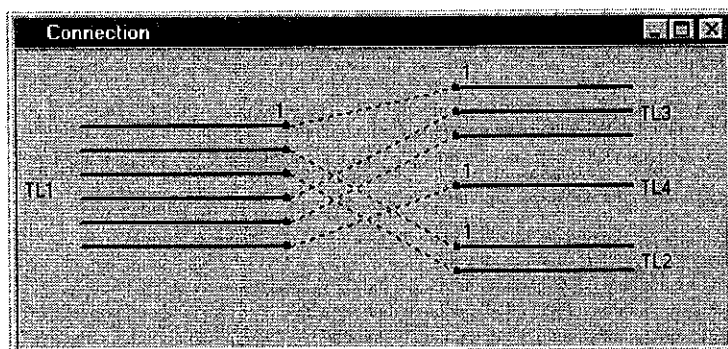


Fig. 3:2. Connection window showing the connection of four transmission lines.

## 4 Program use

This chapter explains the terminology used in the program and how to set the different parameters that are needed before a simulation can be performed. Also how to visualise and further process computed results are explained.

The first parameter that should be set before any other parameter is set is the number of transmission lines. This can be done through the main menu under *Parameters*. When the number of transmission lines is changed all other parameters are restored to default values and the change is directly reflected in the *Transmission Line Properties* window. The maximum number that can be set is 99 (might not be possible due to available memory in the computer).

### 4.1 Defining transmission line properties

In order to describe a particular transmission line for the program a number of properties, or parameters, have to be set. All necessary properties for a transmission line are set through the *Transmission Line Properties* window, Fig. 4:1. If several transmission lines are defined a particular transmission line is set to be the active by clicking on the page tab with the corresponding name.

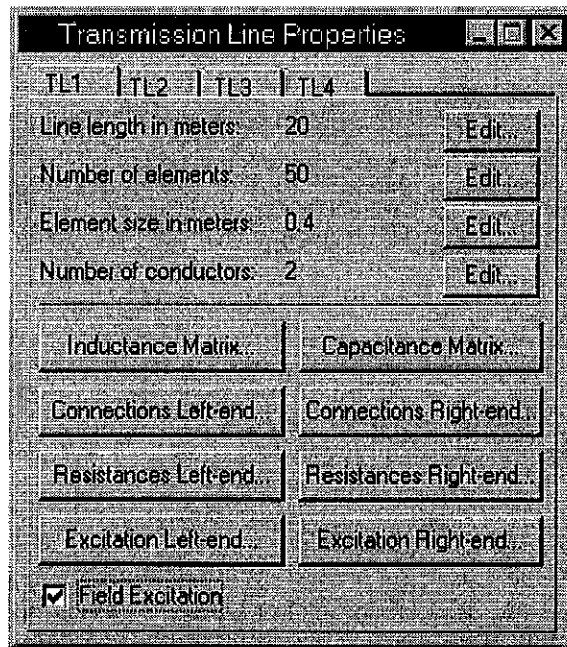


Fig. 4:1. Transmission Line Properties window. Four transmission lines defined of which number 1 is the active (page tab TL1 is in front of the others).

Properties are edited by clicking on either one of the *Edit* buttons or on any of the larger buttons (*Inductance Matrix...* etc.). By clicking on any of these buttons a dialog box where the parameters can be edited (or read from file) will appear.

It should be noted that the number of conductors is defined as the number of the conductors in the transmission line with the reference conductor not counted. So, e.g. a coaxial line will have only one conductor with this definition.

### 4.1.1 Line discretisation

Perhaps the most obvious property for a transmission line is the length of the line. The length is connected to the line discretisation in the following manner:  $Length = \Delta z N_z$ , where  $\Delta z$  is the length of the elements and  $N_z$  is the number of elements the line is divided into. When changing the element length,  $\Delta z$ , the number of elements,  $N_z$ , are changed accordingly so that the length of the line doesn't change, and vice versa.

As is discussed in the theory chapter it is important to divide the line into a sufficiently large number of elements, i.e. the elements should not be too long in terms of the wavelength at the highest frequency of interest. As a rule of thumb the length should not

be longer than one tenth of the wavelength, i.e.  $\Delta z \leq \frac{v_{\min}}{10f_{\max}}$ . In this relation  $v_{\min}$  is the

lowest mode velocity, in m/s, on the transmission line. Since the lowest mode velocity is most often not known it can be computed (together with the other mode velocities) by choosing *Compute Mode Velocities* under *Utilities* in the main menu. In order to use this utility the line parameters *Inductance Matrix* and *Capacitance Matrix* have to be set first.

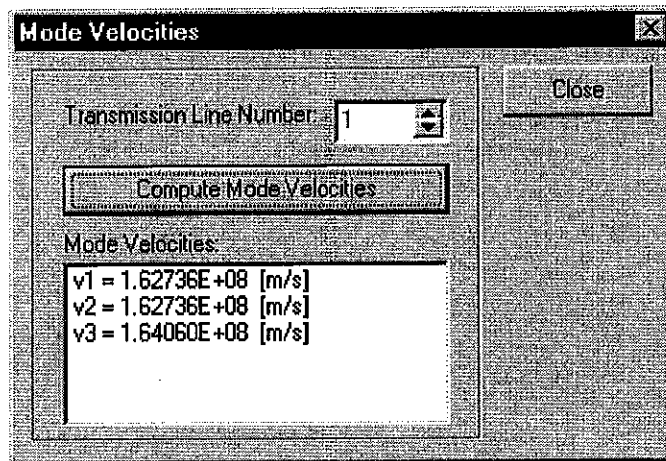


Fig. 4:2. Example of computed mode velocities by using the *Compute Mode Velocities* under *Utilities* in the main menu.

### 4.1.2 Connection of transmission lines

If several transmission lines are defined they can be connected together in the ends forming a transmission line network. Connections are defined by pressing the *Connections Right-end* button in the *Transmission Line Properties* window. By clicking this button a dialog box showing a connection matrix will be shown, Fig. 4:3. In the matrix, numbers for connected transmission lines and conductors should be input. It should be noted that connections can only be edited for the right end of a transmission line. Connections in the left end are updated accordingly and can be inspected by clicking on the button *Connections Left-end* in the *Transmission Line Properties* window.

	TL no.	Cond no.	TL no.	Cond no.	TL no.	Cond no.
Cond no 1	2	1	0	0	0	0
Cond no 2	2	2	0	0	0	0
Cond no 3	3	1	4	1	0	0

Note:  
Right end of TL  
can only be  
connected to  
left end of TL  
with higher  
number.

Fig. 4.3. Connection matrix window showing connections in right end for TL1 (corresponding network shown in Fig. 4:4-5).

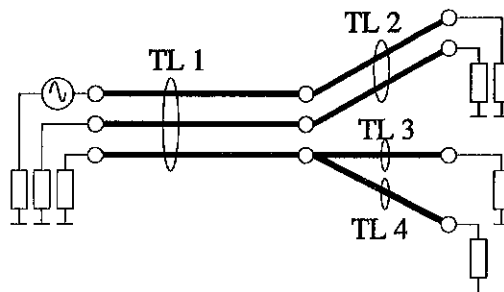


Fig. 4.4. Transmission line network corresponding to the connection matrix in Fig. 4:3.

Connections in the right end of a transmission line can be visualised in the program by choosing *Show Connections* under *Options* in the main menu, Fig. 4:5. If several transmission lines have connections in the right end, first a dialog box where a particular transmission line can be selected will be shown.

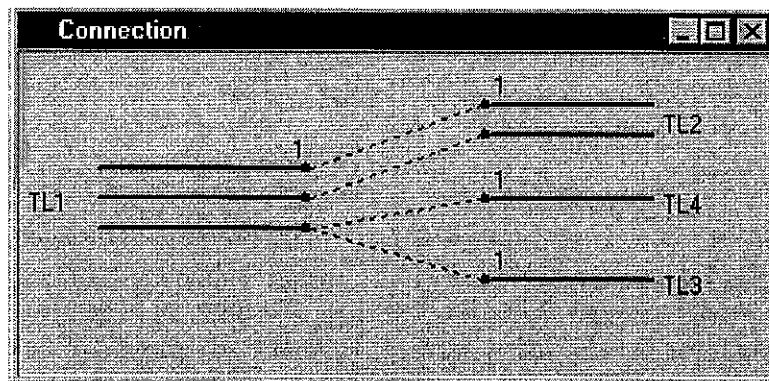


Fig. 4.5. Connection window illustrating the connection matrix in Fig. 4:3.

## 4.2 Defining excitations and terminations

A transmission line network is excited by voltage sources in the ends of transmission lines and/or incident fields. Several voltage sources and incident fields can be used simultaneously and also have different waveforms.

### 4.2.1 Termination resistances and voltage sources

The termination networks at the ends of a transmission line consists of resistors and voltage sources connected in a way that can be described by the following relations:

$$z = 0: \mathbf{V}(0,t) = \mathbf{V}_s(t) - \mathbf{R}_s \mathbf{I}(0,t)$$

$$z = L: \mathbf{V}(L,t) = \mathbf{V}_L(t) + \mathbf{R}_L \mathbf{I}(L,t)$$

In these relations the voltage and current vectors have the dimension  $n \times 1$  and the resistance matrices have the dimensions  $n \times n$ , where  $n$  is the number of conductors in the transmission line (reference conductor not counted). In the program *Left end* is the same as  $z = 0$  and *Right end* is the same as  $z = L$  in the relations above.

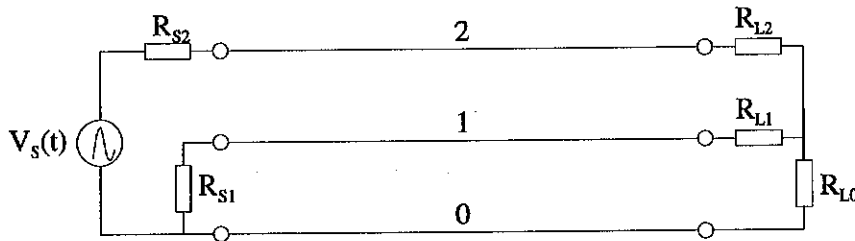


Fig. 4:6. Example of termination networks.

For the example in Fig. 4:6 the source vectors and resistance matrices are:

$$\mathbf{V}_s(t) = \begin{bmatrix} 0 \\ V_s(t) \end{bmatrix}, \quad \mathbf{R}_s = \begin{bmatrix} R_{s1} & 0 \\ 0 & R_{s2} \end{bmatrix}, \quad \mathbf{V}_L(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{R}_L = \begin{bmatrix} R_{L1} + R_{L0} & R_{L0} \\ R_{L0} & R_{L2} + R_{L0} \end{bmatrix}$$

Voltage vectors and resistance matrices are defined by pressing the *Excitation...* and *Resistances...* buttons, respectively, in the *Transmission Line Properties* window. For voltage sources several different waveforms can be chosen from, see 4.2.3.

### 4.2.2 Excitation by incident plane wave

Transmission lines can be excited not only by lumped voltage sources in the ends but also by incident fields illuminating the whole line. The excitation due to impinging electromagnetic fields are read from files with a format that is described in appendix. The incident field can in principal be any type of field, i.e. the field is not only restricted to an incident plane wave. However, the only type of incident field that can be generated by the program is the field due to an incident plane wave, other types of fields have to be generated by some other program (or by measurements).

It is important to remember that the program is based upon the transmission line equations whereby, by definition, there exist no common mode current on the transmission line (i.e. all currents, including current in reference conductor, at any cross

section along the line sums up to zero). When a transmission line is exposed to a field there might be a common mode current on the line which therefore cannot be predicted with the transmission line model that the program is based upon. However, if the cross section of the line is small in terms of the wavelength (which it must be for the transmission line equations to be valid) the sum of all currents, i.e. common mode as well as differential mode, at the ends of the transmission line must be zero. This means that the common mode current not predicted by the program must be zero at the ends of the transmission line and is therefore of no importance in computing the responses at the terminations. *So, the only thing that we can compute with the program when a transmission line is exposed to an incident field is the responses at the ends.*

Before the field response of a transmission line (or several connected lines) can be computed the necessary field excitation files have to be generated. For the case of an incident plane wave this can be done through *Generate Field Excitation Files* under *Utilities* in the main menu, Fig. 4:7.

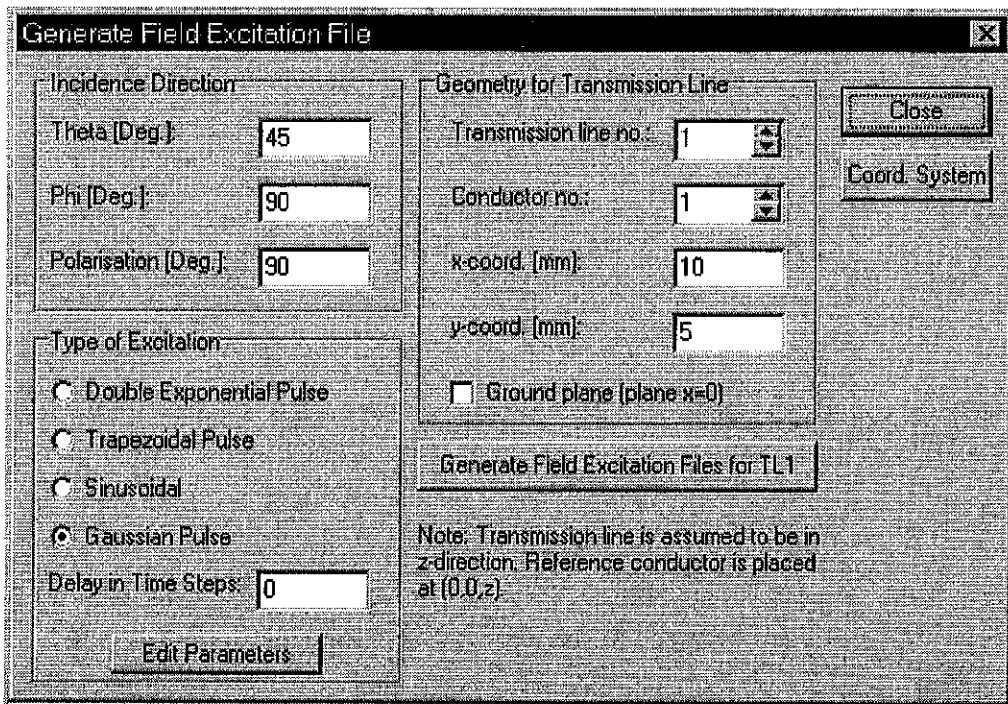


Fig. 4:7. Generate field excitation files dialog box.

It should be noted that depending on the direction of incidence it might be necessary to delay the excitation pulse so that the pulse is arriving at the transmission line at the right time. If no delay is used the pulse is arriving at the coordinate origo at time equals zero. The definition of the different angles is shown in Fig. 4:8 and the coordinates for defining the cross section of the transmission line in Fig. 4:9.

The field strength for the incident plane wave is 1 V/m.

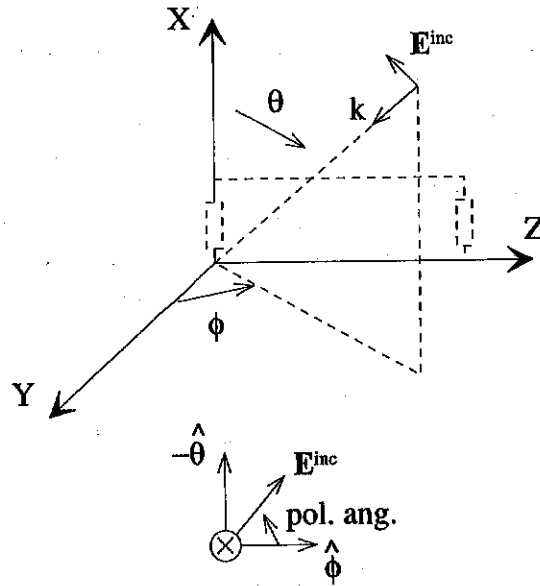


Fig. 4:8. Definition of incidence direction.

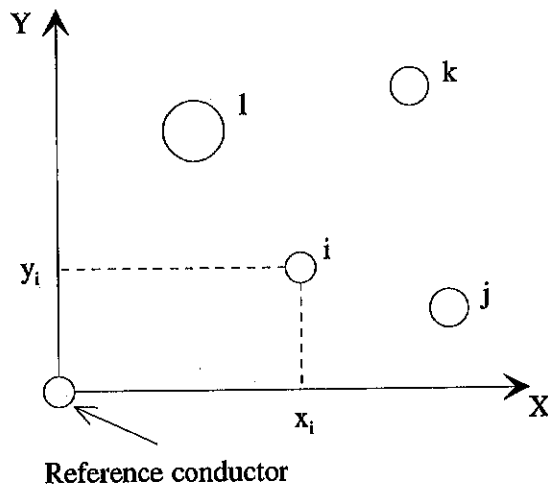


Fig. 4:9. Definition of coordinates for transmission line cross section.

### 4.2.3 Available waveforms

Several different waveshapes for excitation voltage sources and incident plane waves can be chosen from. Different forms for different sources can be used simultaneously for a simulation.

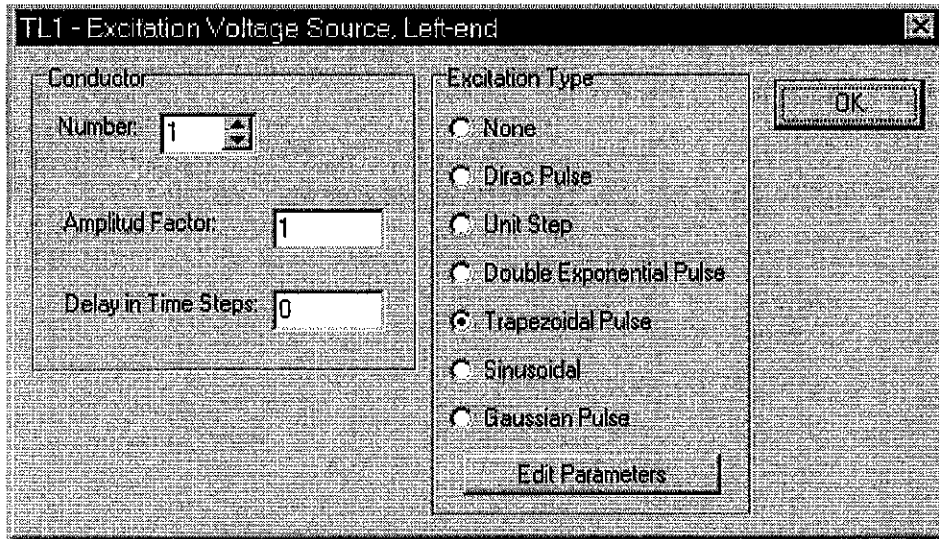


Fig. 4:10. Dialog box for definition of waveforms for excitation voltage sources.

Referring to Fig. 4:10, the conductor number determines where the particular voltage source is placed (the position in the excitation vector), the amplitud factor determines the amplitude for the waveform and the delay determines when the wave starts to develop. By choosing the excitation type *None* in the dialog box above the particular conductor is not excited, it is only terminated by the defined resistance matrix.

The different waveforms are defined in the following.

#### 4.2.3.1 Dirac pulse

A short pulse that starts at the time given by the time delay and lasts for only one time step. The amplitude is given by the amplitude factor. This pulse has a very high frequency contents and should only be used when the criteria for the so called "magic time step" is fulfilled, see 4.4.

#### 4.2.3.2 Unit Step

Starts from zero amplitude and rises to the amplitude given by the amplitude factor in the time given by the rise time. The amplitude starts to rise after the time given by the delay. Can e.g. be used for determining the step response.

#### 4.2.3.3 Double exponential pulse

The pulse form is given by the expression:

$$V(t) = \begin{cases} e^{-\alpha(n-T_d)\Delta t} - e^{-\beta(n-T_d)\Delta t}, & (n-T_d)\Delta t > 0 \\ 0, & (n-T_d)\Delta t < 0 \end{cases}$$

where:  $T_d$  is the time delay in time steps.

The double exponential pulse is often used for simulating an EMP, whereby the

constants are given by, [5]: 
$$\begin{cases} \alpha = 4.0 \cdot 10^6 \\ \beta = 4.76 \cdot 10^8 \end{cases}$$

#### 4.2.3.4 Trapezoidal pulse

The trapezoidal pulse can e.g. be used for simulating a digital signal. The pulse is described by the rise-time, fall-time and the total width of the pulse, Fig. 4:11.

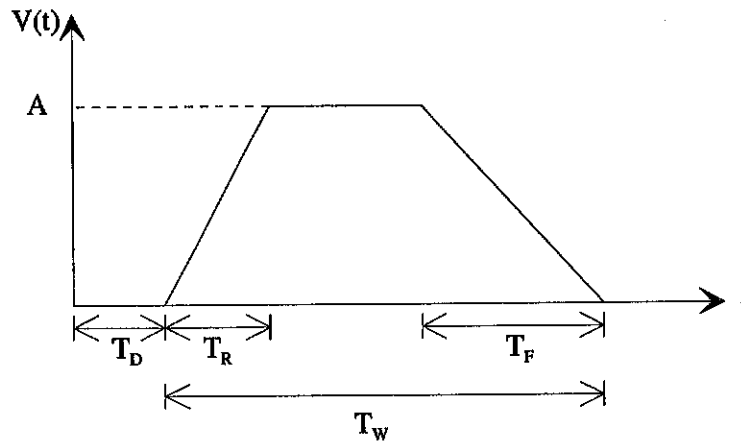


Fig. 4:11. Definition of trapezoidal pulse.

#### 4.2.3.5 Sinusoidal

This is the only periodic waveform available.

$$V(t) = \begin{cases} \sin[2\pi f(n - T_d)\Delta t], & (n - T_d)\Delta t > 0 \\ 0, & (n - T_d)\Delta t < 0 \end{cases}$$

where  $T_d$  is the time delay in time steps.

#### 4.2.3.6 Gaussian pulse

The Gaussian pulse is given by the expression:

$$V(t) = \begin{cases} e^{-\alpha[(n - T_d)\Delta t - \beta\Delta t]^2}, & (n - T_d)\Delta t > 0 \\ 0, & (n - T_d)\Delta t < 0 \end{cases}$$

where  $T_d$  is the time delay in time steps.

Essentially the parameter  $\alpha$  determines the width of the pulse and  $\beta$  when it starts. In the program the only parameter that is set is  $\beta$  and  $\alpha$  is set according to the relation:

$\alpha = \left( \frac{4}{\beta \Delta t} \right)^2$ . The reason for this adaptation is to adjust the pulse width so that the frequency content in the pulse is not too high taking the time step into account. The Gaussian pulse is well suited to use when frequency responses through the internal FFT routine are wanted.

## 4.4 Time discretisation

In order for the simulation to be stable the time discretisation must be chosen according to the Courant criteria, i.e.  $\Delta t \leq \frac{\Delta z}{v_{\max}}$ . This criteria simply requires the time step not to be longer than the propagation time over each element. For the case of equality in the above relation one often refers to the "magic time step". When setting the time discretisation (*Time Discretisation* under *Parameters*) it is possible to compute the magic time step by pressing the button *Compute Optimum Time Step*. Since the program first have to compute the mode velocities to be able to do this, the inductance and capacitance matrices have to be defined.

It can be shown that for transmission lines were the magic time step can be chosen (i.e. when all mode velocities are the same) the exact solution to the transmission line equations are obtained. For these cases we can even use the Dirac pulse as excitation, which has a very high frequency contents, and get exact results.

## 4.5 Simulation

When all parameters have been set a simulation can be performed. Simulations are controlled through the *Simulation Control* window, Fig. 4:12.

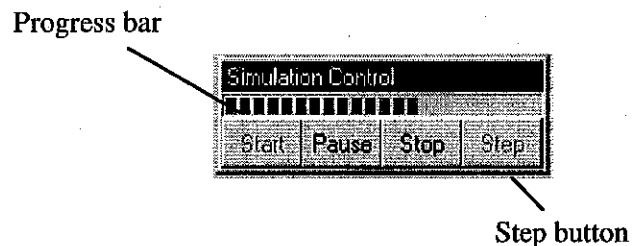


Fig. 4:12. The simulation control window.

A simulation is started by pressing the *Start* button whereby the time discretisation and other parameters are checked. If the parameters are OK a simulation is started and the simulation progress can be seen in the *Progress bar*. Any time during a simulation the simulation can either be paused or stopped by pressing the corresponding button. When a simulation is paused it can either be restarted again or stepped one time step at a time by pressing the *Step* button.

By checking the *Show Parameters* under *Options* in the main menu a window with all relevant parameters will be shown when the simulation is started, Fig. 4:13. The contents in this window can be printed on the active printer by clicking with the right mouse button in the window.

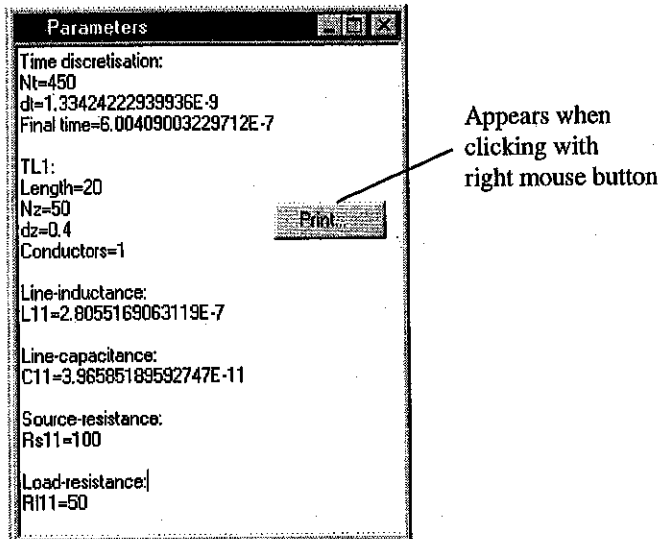


Fig. 4:13. The parameter window.

## 4.6 Presentation of computed results

Two options to visualise simulated results are available, visualisation in real time of voltages and currents along any of the conductors in a transmission line and visualisation of currents and voltages at terminations.

Before a simulation is started the responses to visualise should be defined. This is done through *Save End-Responses in File* and *Show Line-Responses on Screen*, both of which are located under *Options* in the main menu. When selecting any of these options a dialog box will appear, Fig. 4:14.

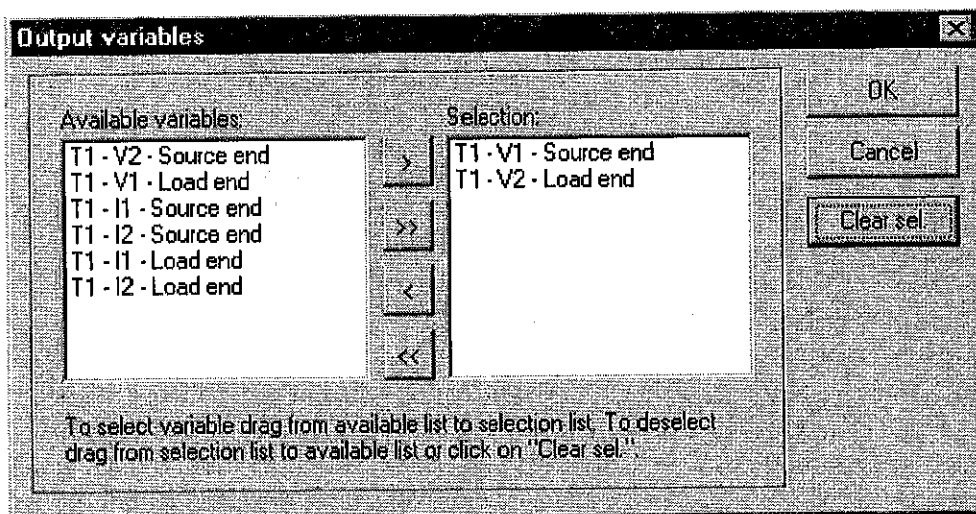


Fig. 4:14. Dialog box for defining end-responses to save in file. T1 refers to transmission line number 1 and, e.g., V2 refers to voltage on conductor 2.

Referring to Fig. 4:14, to select end-responses to save in file the wanted responses should be taken from the available variables list and placed in the selection list. This can be done by dragging variables, double clicking on them or by using the arrow buttons in-

between the lists. For the case of line responses to show on screen a similar dialog box will appear.

### 4.6.1 Voltages and currents as a function of position and time

During a simulation it is possible to visualise any of the currents and/or voltages on a conductor in a transmission line. In principle it is possible to show an unlimited number of line response windows on the screen, but of course the simulation will be slowed down due to the necessary updating of each window.

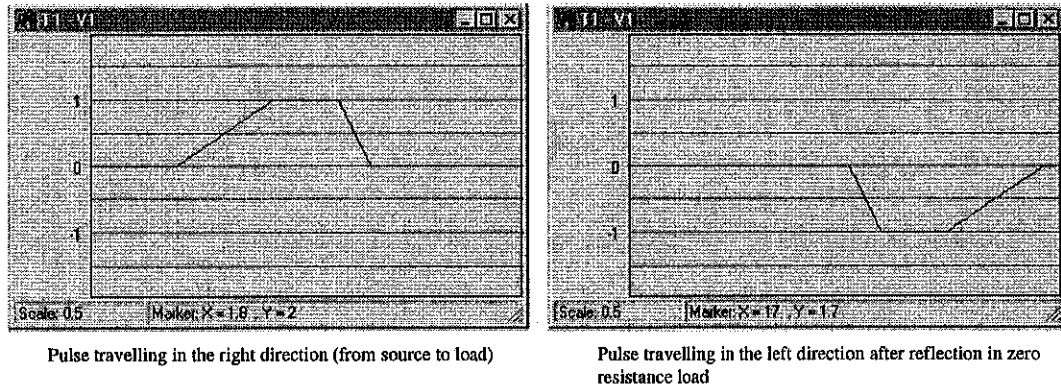


Fig. 4:15. Example of line response window when a trapezoidal voltage pulse is travelling back and forth on conductor 1 in transmission line 1.

When the cursor is moved over a line response window the corresponding position along the line and the amplitude of the presented response can be seen at the bottom of the window. When the line response window is active the amplitude scale factor can be increased and decreased with a factor of two by pressing Ctrl++ and Ctrl+-, respectively. By clicking the right mouse button in a line response window the scale factor can be set to any value and the grid lines can be turned on or off.

### 4.6.2 End-responses

Voltages and/or currents at the ends of a transmission line can be saved in files during a simulation for later visualisation and further processing. The format and names for these files are explained in appendix. End-response files can be showed in a window by using *Open End-Response File* under *Options* in the main menu, Fig. 4:16.

The plot in the end-response window can be re-scaled either by clicking one of the buttons *Set X-Scale* or *Set Y-Scale* or by holding down the left mouse button and dragging out a rectangle over the interesting area. When the cursor is moved over the plot window the coordinates can be seen at the bottom of the window. If the right mouse button is held down also the distance the cursor is moved can be seen (delta marker).

The responses can also be viewed in the frequency domain by clicking the *FFT* button. When doing so, a window showing some details about the FFT are shown, e.g. the achieved frequency resolution. The frequency response can also be normalised to a normalisation file, e.g. in order to get the frequency response due to a source with a constant amplitude for all frequencies. A normalisation file for different waveforms can be created through the *Generate Normalisation File* under *Utilities* in the main menu.

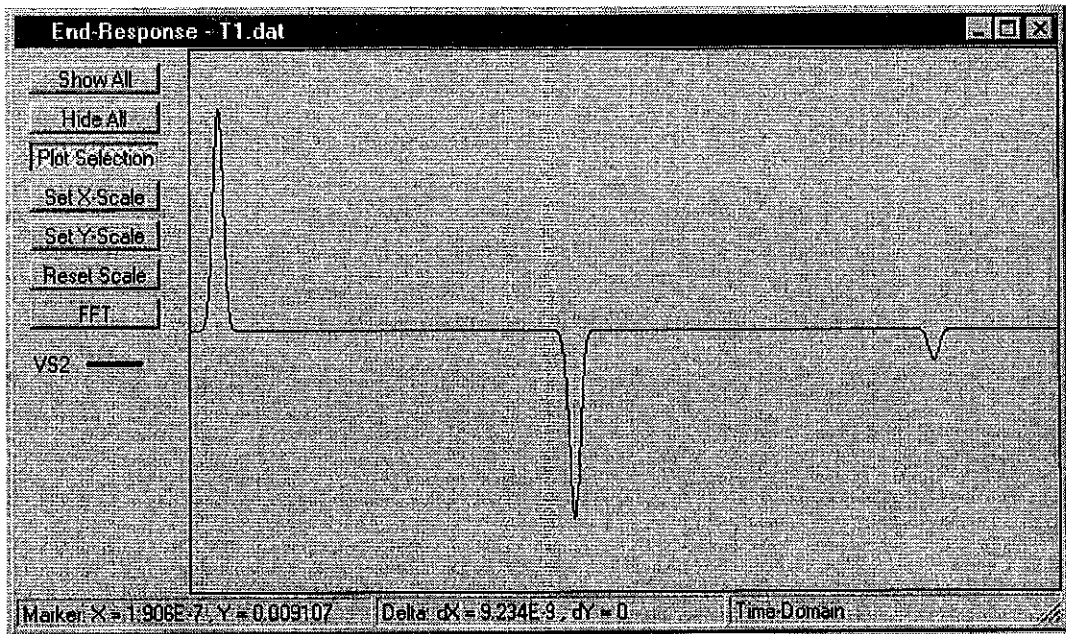


Fig. 4:16. Example of end-response window.

## 5 Line characteristics, L and C matrices

The characteristics for a transmission line is described for the program by inductance and capacitance matrices. These matrices contain information about the cross section of the transmission line, i.e. geometrical aspects as well as material information. The unit for these matrices are in per unit-length quantities, i.e. H/m and F/m, respectively.

### 5.1 Definition

The transmission line equations that the program is based upon assume the field distribution in the transmission line to be that of a TEM wave. This means that the per unit-length parameters, L and C, simply can be determined by solving the static Laplace equation for the cross section of the transmission line. Several numerical methods can be used for solving this equation for complicated cross sections. For some simple cases, see next chapter, even analytical solutions can be found. A program that is based upon a finite difference scheme that can be used when the cross section can be described by rectangles (i.e. rectangular conductors), such is the case on printed circuit boards for example, is the program LC-Calc [6].

For a homogeneous cross section the L and C matrices are related through:

$$\mathbf{L} = \mu\epsilon\mathbf{C}^{-1}, \text{ where } \mu \text{ and } \epsilon \text{ are material constants.}$$

This relation is often used when computing the per unit-length parameters in a way such as only one type of per unit-length parameter, L or C, has to be computed. Often it is easier to compute the capacitance matrix for a homogeneous cross section, e.g. free space, and determine the inductance matrix through the relation above. When this is done, dielectric material, if present, is inserted and the capacitance matrix is computed once again. This is the method LC-Calc use to compute the per unit-length parameters.

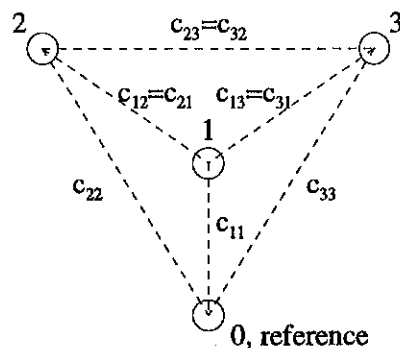


Fig. 5:1. Illustration of per unit-length capacitance for a transmission line with three conductors (reference not counted).

The capacitance matrix for the transmission line in Fig. 5:1 can be written as:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^3 c_{1k} & -c_{12} & -c_{13} \\ -c_{21} & \sum_{k=1}^3 c_{2k} & -c_{23} \\ -c_{31} & -c_{32} & \sum_{k=1}^3 c_{3k} \end{bmatrix}$$

In principle, the choice of reference conductor is arbitrary but often a particular choice will result in simpler calculations, e.g. a ground plane should be chosen as the reference conductor in order to simplify the calculations. It should also be noted that the per unit-length matrices will be different for different choices of reference conductor, although it is the same transmission line. It is therefore important to be aware of the choice of reference conductor.

## 5.2 Simple formulas

In this chapter formulas for the per unit-length parameters for a few simple geometries are given.

### 5.2.1 Two-wire line

Two parallel wires at a certain distance and not necessarily with the same radius.

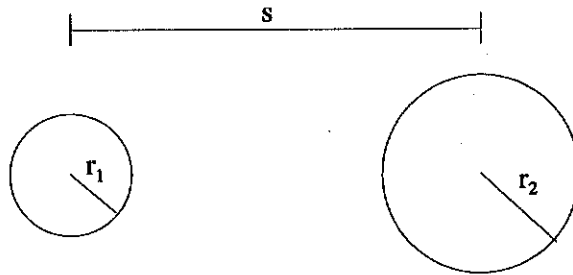


Fig. 5.2. Two-wire line.

$$L = \frac{\mu}{\pi} \cosh^{-1} \left( \frac{s^2 - r_1^2 - r_2^2}{2r_1 r_2} \right) \text{ H/m}, \quad C = \frac{\pi \epsilon}{\cosh^{-1} \left( \frac{s^2 - r_1^2 - r_2^2}{2r_1 r_2} \right)} \text{ F/m}$$

These are the exact relations for the per unit-length parameters. When the distance,  $s$ , between the wires are much larger than each of the wire radius the following simplified relations can be used:

$$L = \frac{\mu}{2\pi} \ln \left( \frac{s^2}{r_1 r_2} \right) \text{ H/m}, \quad C = \frac{2\pi \epsilon}{\ln \left( \frac{s^2}{r_1 r_2} \right)} \text{ F/m}$$

### 5.2.2 Coaxial cable

Two concentric conductors with radius  $r_1$  and  $r_2$ , respectively. The space in-between the two conductors is filled with a material with the dielectric constant  $\epsilon$ .

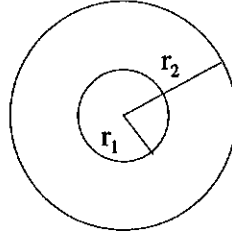


Fig. 5:3. Coaxial cable.

$$L = \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \text{ H / m}, \quad C = \frac{2\pi\epsilon}{\ln\left(\frac{r_2}{r_1}\right)} \text{ F / m}$$

### 5.2.3 Single wire over ground plane

A single circular wire over an infinitely large and perfectly conducting ground plane. By using image theory [7, Sec. 3-4] this configuration can be transformed into that of a two-wire line with conductors with the same radius.

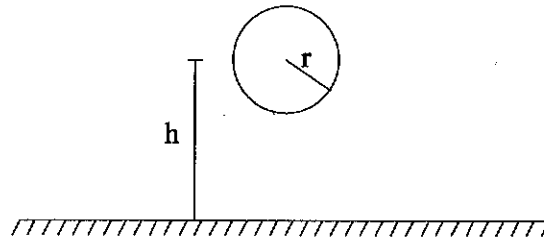


Fig. 5:4. Single wire over infinite, perfectly conducting ground plane.

$$L = \frac{\mu}{2\pi} \cosh^{-1}\left(\frac{h}{r}\right) \text{ H / m}, \quad C = \frac{2\pi\epsilon}{\cosh^{-1}\left(\frac{h}{r}\right)} \text{ F / m}$$

These are the exact relations for the per unit-length parameters. When the height,  $h$ , is much larger than the wire radius the following simplified relations can be used:

$$L = \frac{\mu}{2\pi} \ln\left(\frac{2h}{r}\right) \text{ H / m}, \quad C = \frac{2\pi\epsilon}{\ln\left(\frac{2h}{r}\right)} \text{ F / m}$$

### 5.2.4 $(n+1)$ wires

Simulating for instance a cable, although the conductors are placed in a homogenous medium and the distance between any two of the conductors much be large compared to the conductor radius.

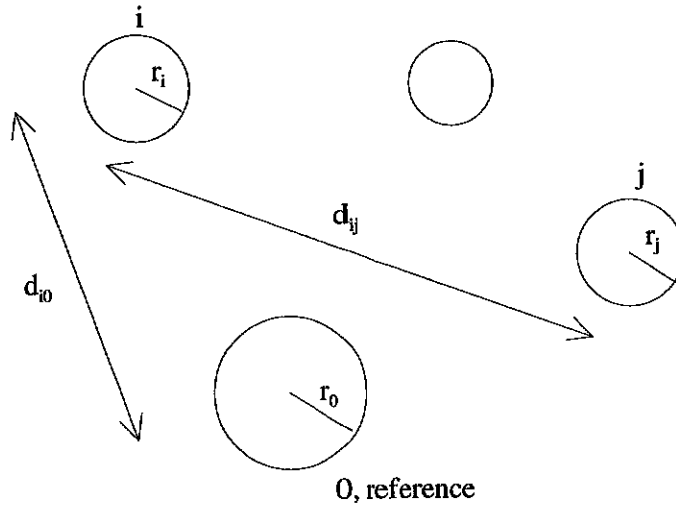


Fig. 5:5.  $(n+1)$  wires.

For the configuration in Fig. 5:5 the entries in the inductance matrix are given by:

$$L_{ii} = \frac{\mu}{2\pi} \ln \left( \frac{d_{i0}^2}{r_0 r_i} \right) \text{ H / m}, \quad L_{ij} = \frac{\mu}{2\pi} \ln \left( \frac{d_{i0} d_{j0}}{d_{ij} r_0} \right) \text{ H / m}$$

These relations assume the distances between wires to be much larger than each wire radius. Capacitance matrix can be obtained by using:  $\mathbf{C} = \mu\epsilon\mathbf{L}^{-1}$ .

### 5.2.5 $n$ wires over ground plane

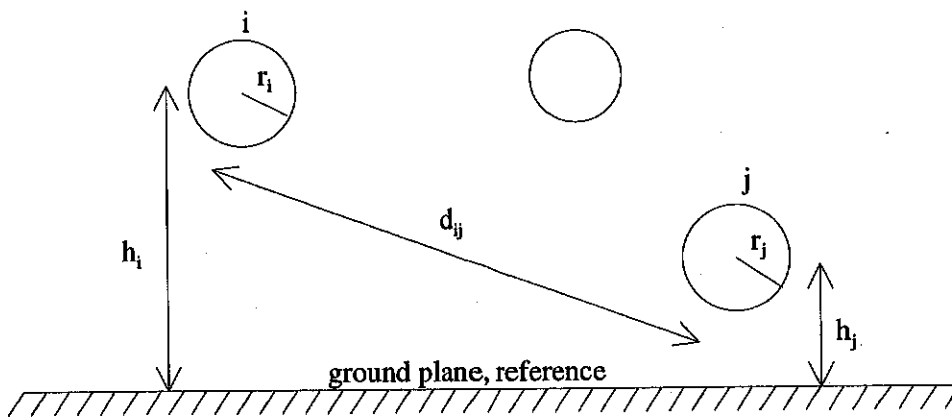


Fig. 5:6.  $n$  wires over infinite, perfectly conducting ground plane.

For the configuration in Fig. 5:6 the entries in the inductance matrix are given by:

$$L_{ii} = \frac{\mu}{2\pi} \ln\left(\frac{2h_i}{r_i}\right) \text{ H / m}, \quad L_{ij} = \frac{\mu}{4\pi} \ln\left(1 + \frac{4h_i h_j}{d_{ij}^2}\right) \text{ H / m}$$

These relations assume the heights over the ground plane to be much larger than each wire radius. Capacitance matrix can be obtained by using:  $\mathbf{C} = \mu\epsilon\mathbf{L}^{-1}$ .

## 6 Sample simulations, validation

In this chapter a few examples are given in order to show how the code can be used. The examples can be found in the folder "Examples" that will be created when the program is installed. Filenames are "Ex" followed by digits corresponding to the number of the chapter. File extension is "MTL".

### 6.1 Pulse propagation on a simple transmission line

In this example we consider a pulse propagating on a simple coaxial line. To follow the example using the program open the corresponding file in the folder "Examples".

#### 6.1.1 Line characteristics

For the simulations the following parameters were used:

- Length = 100 m
- Number of conductors = 1
- Number of elements = 200
- $L = 0.5 \mu\text{H/m}$
- $C = 22.2 \text{ pF/m}$
- Time step =  $1.6658 \cdot 10^{-9}$  s (magic time step)
- Number of time steps = 1000
- Excitation in left end, trapezoidal pulse  $t_r = t_f = 20 \text{ ns}$  ,  $t_w = 50 \text{ ns}$

#### 6.1.2 Propagation on matched line

When the line is matched it should be terminated by resistances, in both ends, with a value that is equal to the characteristic impedance of the line (for lines with several conductors this will be a matrix). The characteristic impedance is given by

$Z_C = \sqrt{\frac{L}{C}} = 150 \Omega$ . Thus, for this example we use resistances in both ends with a value of 150 Ohm.

By showing the line voltage on the screen we can see the pulse propagating along the line and observe that it disappears when it reaches the termination at the right end, as it should, since the line is matched. We can also observe that the amplitude of the propagating voltage pulse is 0.5 V although the exciting voltage source has an amplitude of 1 V. This is because the voltage feed into the line is divided through the line impedance and the impedance of the source. For this example the two impedances are equal and equal to 150 Ohm.

By also saving the voltage at the right end in an end-response file we can after the simulation look at the end-response. We can then observe that the pulse has reached the termination after approx. 333 ns which corresponds to the length of the line and the propagation speed which is that of speed of light in free space.

### 6.1.3 Propagation on shorted line

For this example we use the same line as above but now we terminate with a short circuit in both ends (0 Ohm).

Looking at the line voltage in a line response window we can observe that the amplitude of the propagating pulse now is equal to 1 V since it is directly connected to the conductor. We can also see that the pulse is reflected in both ends and that the pulse changes polarity at the reflection. This is a consequence of the load conditions which requires the total voltage to be equal to zero. By also opening a line response window for the current on the line we can see that the current pulse doesn't change polarity at the reflections.

### 6.1.4 Propagation on open line

The same line as in 6.1.3 but now we keep the short circuit in the generator end ( $z=0$ ) and changes the termination in the load end to open. Since we cannot define an infinite resistance we make it large, e.g. 1 MOhm.

Once again observing the line voltage in a line response window we can observe that the pulse changes polarity in the generator end but not in the load end. The current behaves in the opposite way, i.e. changes polarity at the open end but not at the shorted end.

## 6.2 Crosstalk between parallel conductors

In this example the crosstalk between two parallel conductors that are terminated to a common ground plane is considered, Fig. 6:1. The computed response is also compared with a full-wave formulation implemented in a method of moments program called PCB-MoM [1].

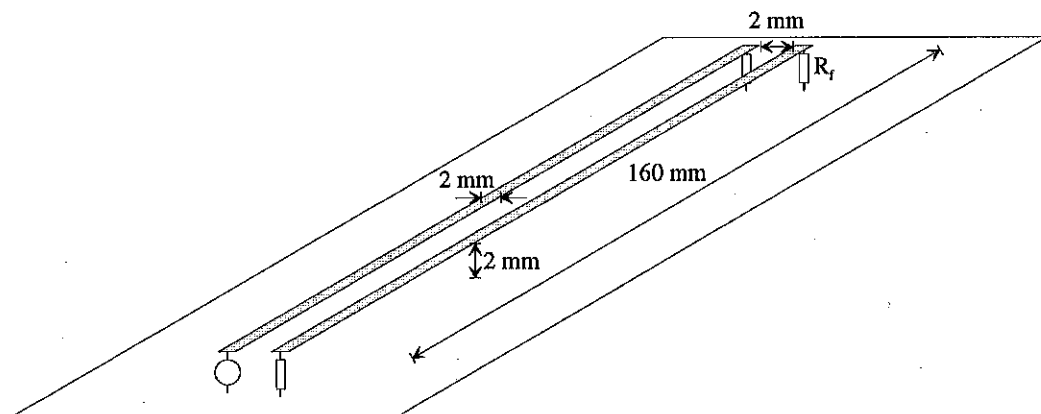


Fig. 6:1. Geometry for computation of crosstalk between two parallel conductors that are terminated to a common ground plane. All resistors are 1 kOhm.

### 6.2.1 Line characteristics

Since the parallel conductors have a rectangular cross-section the program LC-Calc [6] is well suited for computing the line characteristics. Using LC-Calc the following inductance and capacitance matrices were obtained:

$$\mathbf{L} = \begin{bmatrix} 0.383 & 0.067 \\ 0.067 & 0.383 \end{bmatrix} [\mu\text{H}/\text{m}], \quad \mathbf{C} = \begin{bmatrix} 30.0 & -5.25 \\ -5.25 & 30.0 \end{bmatrix} [\text{pF}/\text{m}]$$

For the computation the transmission line was divided into 50 elements and 8192 time steps were used. As excitation a Gaussian pulse was used. The interesting quantity was the current through the  $R_f$  resistor in conductor 2 in Fig. 6:1.

## 6.2.2 End-response in frequency domain

Since we would like to compare the computed current with the current obtained by the frequency domain program PCB-MoM we must transform the computed current in the time domain to the frequency domain. Since the amplitude of the source used in PCB-MoM has an amplitude that is constant for all frequencies we must also normalise the transformed data obtained by the BMTL program. Thus, before we perform the FFT of the computed current we must generate a normalisation file that can be used when the FFT is done.

When all of the above mentioned steps were done the results shown in Fig. 6:2 were obtained. As can be seen the agreement between the two methods is good. For the computation done with PCB-MoM the conductors were divided into 20 current elements along the length and 4 in the transverse direction.

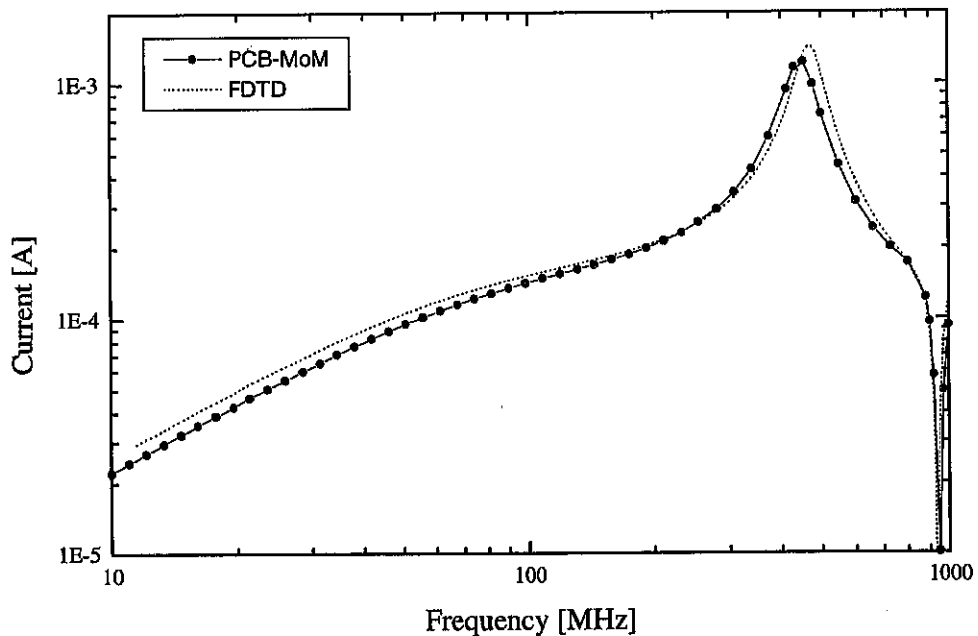


Fig. 6:2. Computed current through the  $R_f$  resistor in Fig. 6:1. Computations done with BMTL, dotted line, and PCB-MoM, solid line with circles.

## 6.3 Propagation on a transmission line network

The transmission line network considered in this example is shown in Fig. 6:3. The network consists of three one-conductor transmission lines (e.g. coaxial cables) with the same characteristic impedance. Two of the lines are 10 m long and one is 30 m. One of the shorter lines is terminated with a short circuit while the other two are terminated with matched loads.

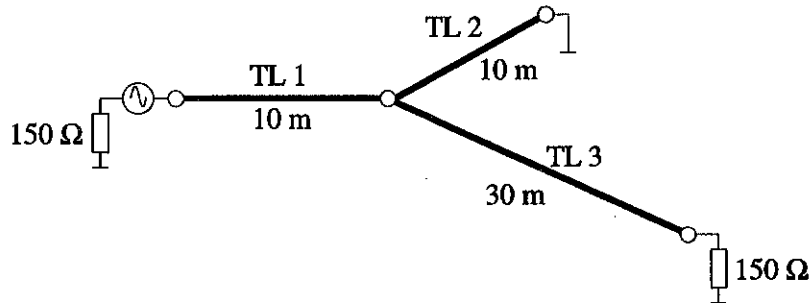


Fig. 6:3. Simple transmission line network considered in this example.

### 6.3.1 Line characteristics

All three transmission lines in Fig. 6:3 have the same characteristic impedance, 150 Ohm ( $L = 0.5 \mu\text{H/m}$ ,  $C = 22.2 \text{ pF/m}$ ). All lines are divided into elements with the length 0.2 m and for the computation 1000 time steps were used with the time step given by the optimum value (magic time step). As excitation a Gaussian pulse with the amplitude 1 V was used.

### 6.3.2 Computed response

The computed quantity was the voltage at the end of transmission line TL3, i.e. over the terminating 150 Ohm resistor. The computed result is shown in Fig. 6:4.

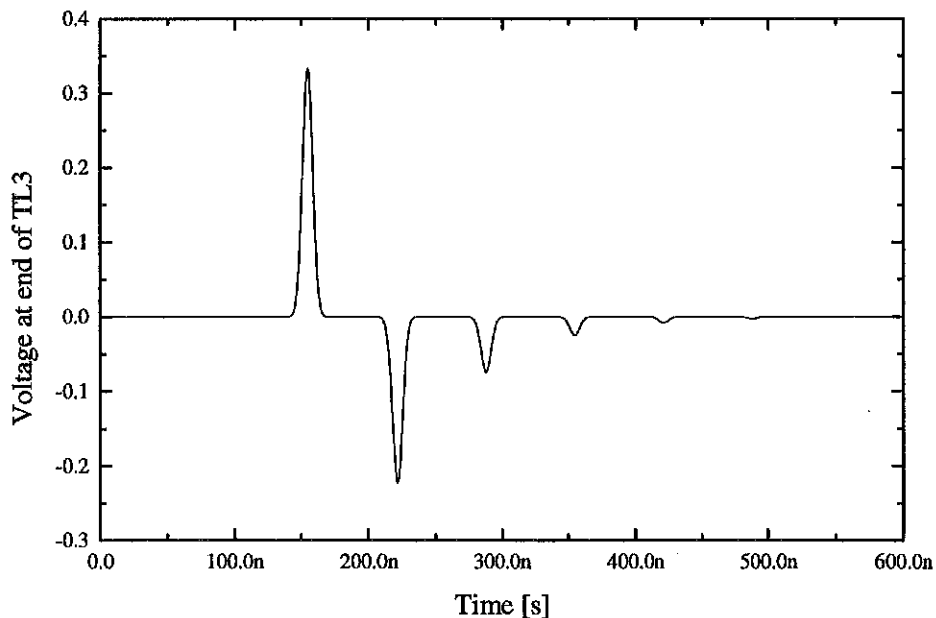


Fig. 6:4. Computed voltage at the end of transmission line TL3.

The result in Fig. 6:4 can be understood by the following reasoning:

- The voltage pulse starting at the source and travelling along TL1 will have an initial amplitude of 0.5 V since the generator voltage is divided by the source and line impedances.
- When the pulse is arriving at the junction of the three transmission lines it will be partly reflected and partly transmitted into the two other lines. The reflection coefficient seen by this pulse will be  $\Gamma = \frac{75-150}{75+150} \approx -0.33$  and therefore the amplitude of the reflected pulse travelling back towards the source will be approx. 0.17 V. Since the source is matched to the line the pulse will disappear when it reaches the source. The amplitudes of the pulses transmitted into the other two lines will be the same on both lines. The amplitudes will be  $0.5(1+\Gamma) \approx 0.33$  V. Thus, the first pulse arriving at the end of TL3 will have an amplitude of approx. 0.33 V. The pulse will arrive at a time given by the total length, which is 10+30 m, and the speed of propagation on the lines. Since the speed is that of light in free space the time will be approx. 133 ns (note that this is the time to the first part of the pulse).
- The pulse propagating along TL2 will be totally reflected at the short circuit at the end of the line. Thus, the amplitude of the pulse that is now travelling towards the junction again will be approx. -0.33 V.
- When this pulse is reaching the junction it will be partly reflected back again and partly transmitted into TL1 and TL3. As before, the amplitudes of the transmitted pulses will be equal and are given by  $-0.33(1+\Gamma) \approx -0.22$  V. Thus, the second pulse arriving at the end of TL3 will have an amplitude of approx. -0.22 V. This pulse will arrive at a time given by the length travelled, 10+10+10+30 m, and the speed of light. The time will be 200 ns.
- The other pulses at the end of TL3 can be obtained by continuing the above argumentation.

## 6.4 End-responses due to an incident plane wave

As another comparison with a full-wave formulation the induced current in a conductor over a ground plane due to an incident plane wave was considered, Fig. 6:5.

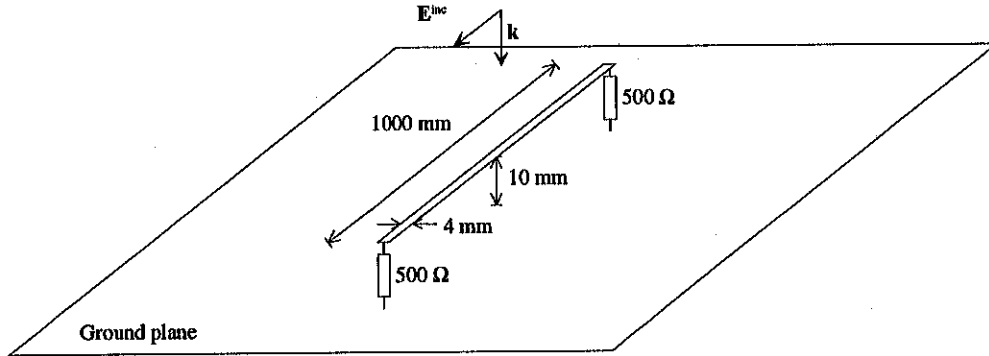


Fig. 6:5. Terminated conductor over a ground plane. Excitation with an incident plane wave from above with E-field polarised along the line.

### 6.4.1 Line characteristics

The program LC-Calc [6] was used for computing the per unit-length parameters. The capacitance was found to be 21.0 pF/m and the inductance 0.53  $\mu$ H/m. For the computation the conductor was divided into 50 elements and 4096 time steps were used.

### 6.4.2 Exciting plane wave

The excitation of the conductor was an incident plane wave generated internally in the BMTL program. The following parameters were used for generating the field:

$$\left\{ \begin{array}{l} \theta = 0 \\ \phi = 90 \\ \text{Pol.} = 90 \\ \text{Ground plane} = \text{Checked} \\ \text{x-coord.} = 10 \text{ mm} \\ \text{y-coord.} = 0 \text{ mm} \\ \text{delay} = 0 \end{array} \right.$$

### 6.4.3 Computed response

The computed quantity was the induced current through one of the terminating resistors. The computed current in the time domain was transformed to the frequency domain by using the internal FFT routine and a normalisation file generated by the program.

The results from the computations done by the programs BMTL and PCB-MoM [1] are shown in Fig. 6:6 where we can observe a good agreement between the two approaches. For the computation done with PCB-MoM the conductor was divided into 28 current elements along the length and 4 in the transverse direction.

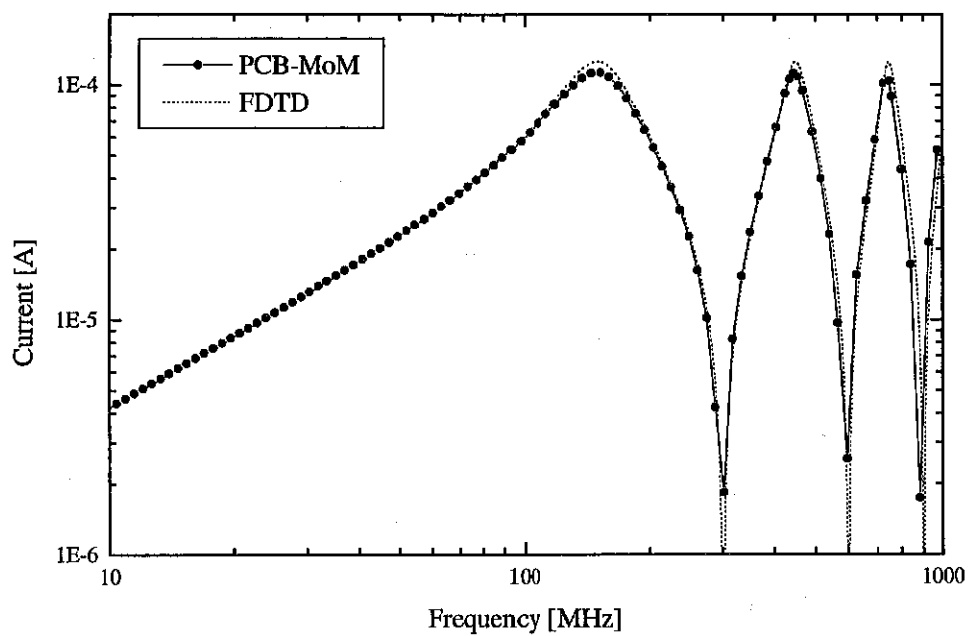


Fig. 6.6. Induced current through one of the terminating resistors in Fig. 6:5. Computations done with BMTL, dotted line, and PCB-MoM, solid line with circles.

## 7 Theory

In this chapter the theoretical basis for the BMTL program is explained. We start by formulating the equations for a multi-conductor transmission line expressed in the time domain. After that the equations is solved by using an iterative finite difference method as described in [2]-[4]. Finally, we generalise the method in order to also be able to treat connections of transmission lines (junctions).

### 7.1 Multi-conductor transmission line equations

The responses of the transmission lines are computed by solving the ordinary multi-conductor transmission line equations in the time domain. Thus, the solution is restricted to lines that fulfils the requirements for these equations to be valid. In particular this means that the cross section of the transmission line should be uniform along the length of the line (can be overcome by cascading several lines with different cross sections) and the extent of the cross section (i.e. distance between the conductors) must be small compared to the wavelength.

The transmission line equations for a lossless multi-conductor transmission line expressed in the time domain are (line assumed to be in z-direction), see e.g. [3]:

$$\begin{aligned} \frac{\partial}{\partial z} \mathbf{V}(z,t) + \mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z,t) &= -\frac{\partial}{\partial z} \mathbf{E}_T(z,t) + \mathbf{E}_L(z,t) \\ \frac{\partial}{\partial z} \mathbf{I}(z,t) + \mathbf{C} \frac{\partial}{\partial t} \mathbf{V}(z,t) &= -\mathbf{C} \frac{\partial}{\partial t} \mathbf{E}_T(z,t) \end{aligned} \quad (7:1)$$

where, if we assume the line to have  $(n+1)$  conductors (i.e. including the reference conductor):

$\mathbf{V}$  and  $\mathbf{I}$  are  $n \times 1$  vectors of the line voltages (with respect to the reference conductor) and currents, respectively.

$\mathbf{L}$  and  $\mathbf{C}$  are the inductance and capacitance matrices of dimension  $n \times n$  describing the line characteristics.

$\mathbf{E}_T$  and  $\mathbf{E}_L$  are  $n \times 1$  vectors describing the incident field excitation.

## 7.2 Solution by using FDTD

The solution to the equations (7:1) is found by using an iterative finite difference procedure [2]-[4]. In using this procedure the transmission line is divided into small elements along the length of the line and similarly the time is divided into small steps, Fig. 7:1.

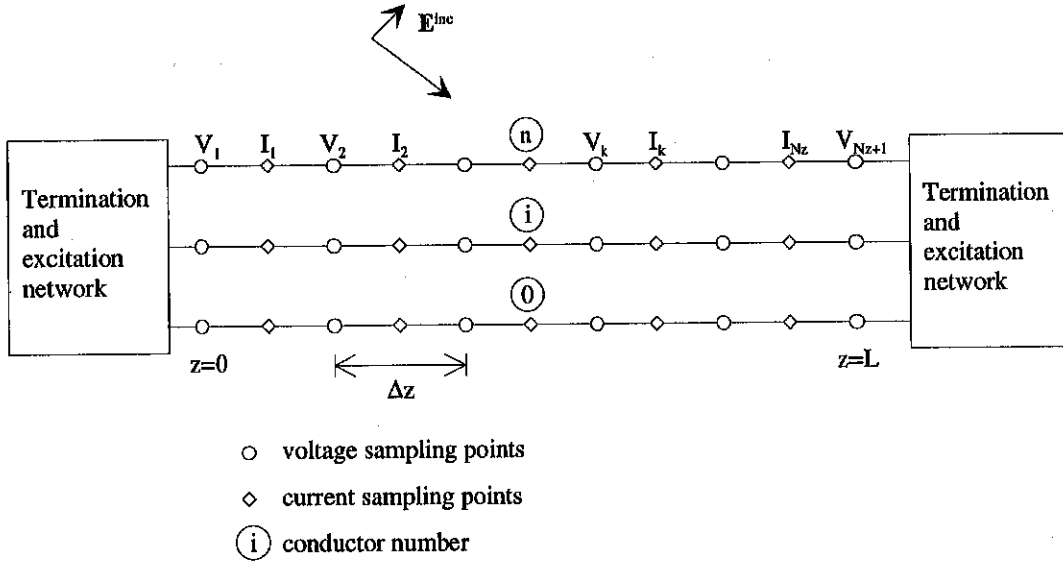


Fig. 7:1. Division of a transmission line with  $(n+1)$  conductors into  $Nz$  elements, each with a length of  $\Delta z$ .

The termination and excitation networks at the ends of the transmission line in Fig. 7:1 are described by the following equations (generalised Thévenin equivalents):

$$\begin{aligned}
 z = 0: \quad \mathbf{V}(0, t) &= \mathbf{V}_S(t) - \mathbf{R}_S \mathbf{I}(0, t) \\
 z = L: \quad \mathbf{V}(L, t) &= \mathbf{V}_L(t) + \mathbf{R}_L \mathbf{I}(L, t)
 \end{aligned} \tag{7:2}$$

where:

$\mathbf{V}_S$  and  $\mathbf{V}_L$  are  $n \times 1$  vectors of open circuit voltages (with respect to the reference conductor) of the excitation sources.

$\mathbf{R}_S$  and  $\mathbf{R}_L$  are the resistance matrices of dimension  $n \times n$  describing the terminating resistances.

Approximating the derivatives in equation (7:1) by finite differences and using the discretisation in Fig. 1 taking the termination and excitation networks into account gives the following equations for the voltages and currents along the line:

$$\begin{aligned}
\mathbf{V}_1^{n+1} &= \left( \frac{\Delta z}{\Delta t} \mathbf{R}_S \mathbf{C} + \mathbf{1} \right)^{-1} \left[ \left( \frac{\Delta z}{\Delta t} \mathbf{R}_S \mathbf{C} - \mathbf{1} \right) \mathbf{V}_1^n - 2\mathbf{R}_S \mathbf{I}_1^{n+1/2} + \mathbf{V}_S^n + \mathbf{V}_S^{n+1} + \right. \\
&\quad \left. + \frac{\Delta z}{\Delta t} \mathbf{R}_S \mathbf{C} (\mathbf{E}_{T,1}^n - \mathbf{E}_{T,1}^{n+1}) \right] \\
\mathbf{V}_k^{n+1} &= \mathbf{V}_k^n - \frac{\Delta t}{\Delta z} \mathbf{C}^{-1} (\mathbf{I}_k^{n+1/2} - \mathbf{I}_{k-1}^{n+1/2}) + \mathbf{E}_{T,k}^n - \mathbf{E}_{T,k}^{n+1} ; k = 2 \dots N_z \\
\mathbf{V}_{N_z+1}^{n+1} &= \left( \frac{\Delta z}{\Delta t} \mathbf{R}_L \mathbf{C} + \mathbf{1} \right)^{-1} \left[ \left( \frac{\Delta z}{\Delta t} \mathbf{R}_L \mathbf{C} - \mathbf{1} \right) \mathbf{V}_{N_z+1}^n + 2\mathbf{R}_L \mathbf{I}_{N_z}^{n+1/2} + \mathbf{V}_L^n + \mathbf{V}_L^{n+1} + \right. \\
&\quad \left. + \frac{\Delta z}{\Delta t} \mathbf{R}_L \mathbf{C} (\mathbf{E}_{T,N_z+1}^n - \mathbf{E}_{T,N_z+1}^{n+1}) \right] \\
\mathbf{I}_k^{n+3/2} &= \mathbf{I}_k^{n+1/2} - \frac{\Delta t}{\Delta z} \mathbf{L}^{-1} (\mathbf{V}_{k+1}^{n+1} - \mathbf{V}_k^{n+1}) - \\
&\quad - \mathbf{L}^{-1} \left[ \frac{\Delta t}{\Delta z} (\mathbf{E}_{T,k+1}^{n+1} - \mathbf{E}_{T,k}^{n+1}) - \frac{\Delta t}{2} (\mathbf{E}_{L,k}^{n+3/2} + \mathbf{E}_{L,k}^{n+1/2}) \right] ; k = 1 \dots N_z
\end{aligned} \tag{7.3}$$

where:

$t = n\Delta t$  and the index  $n$  should be increased from zero in step of one until the desired solution time is reached.

$z$ -coordinates for the voltage sampling points are given by  $z_k = (k-1)\Delta z$

$z$ -coordinates for the current sampling points are given by  $z_k = \left(k - \frac{1}{2}\right)\Delta z$

$$[\mathbf{E}_{T,k}^n]_i = x_i E_x^{inc}(x_i, y_i, (k-1)\Delta z, n\Delta t) + y_i E_y^{inc}(x_i, y_i, (k-1)\Delta z, n\Delta t)$$

$$[\mathbf{E}_{L,k}^n]_i = E_z^{inc}\left(x_i, y_i, \left(k - \frac{1}{2}\right)\Delta z, n\Delta t\right) - E_z^{inc}\left(0, 0, \left(k - \frac{1}{2}\right)\Delta z, n\Delta t\right)$$

coordinates  $x_i$  and  $y_i$  describe the locations of the individual conductors as defined in Fig. 7.2.

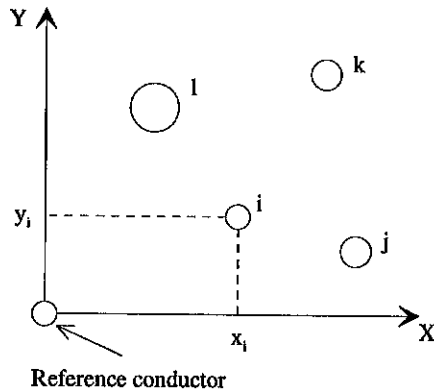


Fig. 7.2. Cross section for a multi-conductor transmission line, definition of coordinates for the conductors.

It should be noted that the excitation due to fields should be computed as the incident field, thus all conductors should be removed when computing the  $\mathbf{E}_{T,k}$  and  $\mathbf{E}_{L,k}$  vectors.

Equations (7:3) should be solved in the given order and the index  $k$  should be increased before the index  $n$ , thus:

loop over index  $n$

    Compute  $\mathbf{V}_1^{n+1}$

    Compute  $\mathbf{V}_k^{n+1}$  for  $k = 2 \dots N_z$

    Compute  $\mathbf{V}_{N_z+1}^{n+1}$

    Compute  $\mathbf{I}_k^{n+3/2}$  for  $k = 1 \dots N_z$

end of loop over  $n$

In order to obtain accurate results when using equations (7:3) the length of the elements must not be too long in terms of the wavelength at the highest frequency of interest. As a rule of thumb the length should not be longer than one tenth of the wavelength (where the wavelength is related to the lowest mode velocity on the line,  $\Delta z \leq \frac{v_{\min}}{10f_{\max}}$  ). The

time step used for the iterative solution procedure must also be chosen carefully, otherwise the solution might be unstable. The stability criterion that should be used is that the time step should be less or equal to the element length divided by the highest mode velocity on the line, thus  $\Delta t \leq \frac{\Delta z}{v_{\max}}$  . This is the Courant stability criterion.

### 7.3 Extension to include junctions

Often we have not only one transmission line but several that are connected to each other in some way. In order to treat this situation as well, we start by considering a simple junction consisting of three connected transmission lines.

The expressions for the voltages and currents along a transmission line given in equations (7:3) can be obtained by modelling the line as a number of short sections consisting of lumped circuit elements. We can for instance use so called  $\pi$ -sections. By using such a model for the lines the equivalent circuit for the connection of three one-conductor transmission lines shown in Fig. 7:3 is obtained.

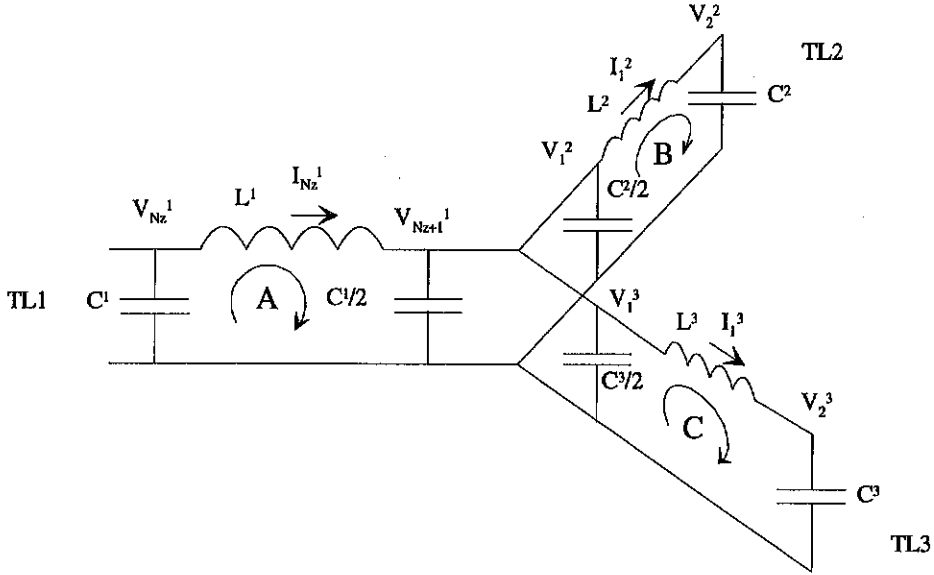


Fig. 7:3. Equivalent circuit for the connection of three one-conductor transmission lines.

In the figure superscripts denote the number of the transmission line and subscripts the element number. For the sake of plainness, the element length,  $\Delta z$ , is omitted in the figure.

Applying Kirchoff's voltage law to the loops A,B and C in Fig. 7:3 the following equations for the currents at the time  $t = (n + 3/2)\Delta t$  are obtained:

$$\begin{aligned} (I_1^{n+3/2})^i &= (I_1^{n+1/2})^i - \frac{\Delta t}{\Delta z L^i} \left( (V_2^{n+1})^i - (V_1^{n+1})^i \right); \quad i = 2,3 \\ (I_{N_z}^{n+3/2})^1 &= (I_{N_z}^{n+1/2})^1 - \frac{\Delta t}{\Delta z L^1} \left( (V_{N_z+1}^{n+1})^1 - (V_{N_z}^{n+1})^1 \right) \end{aligned} \quad (7:4)$$

Comparing equations (7:4) with the expression for the currents in equations (7:3), omitting the terms representing the field excitation, it is observed that they are equal. Thus, the current expressions for the transmission lines remain the same when we have a junction.

Now, applying Kirchoff's current law to the connection point in Fig. 7:3 the following expression for the voltage at the time  $t = (n + 1)\Delta t$  is obtained:

$$(V_1^{n+1})^{2,3} = (V_{Nz+1}^{n+1})^1 = (V_{Nz+1}^n)^1 - \frac{2\Delta t}{\Delta z(C^1 + C^2 + C^3)} \left( (I_1^{n+1/2})^2 + (I_1^{n+1/2})^3 - (I_{Nz}^{n+1/2})^1 \right) \quad (7:5)$$

Once again comparing with equations (7:3), now with the expression for the internal voltages (i.e.  $V_k^{n+1}$ ;  $k = 2 \dots Nz$ ) we observe that two things have changed. The first is that the expression for the capacitance has changed and the second is that the involved currents have changed.

Putting together the observations above we come to the following conclusions in order to treat connections of transmission lines:

- The expression for the currents for the lines remain the same
- The expression for the voltage at the connection has to be changed to that of (7:5)

Although the above has been derived for a simple junction consisting of only three one-conductor transmission lines it can be extended to the connection of several multi-conductor transmission lines. In determining the voltage at the junction we can simply use the expression for  $V_k^{n+1}$  in equation (7:3) if we modify the capacitance matrix and the current vectors.

## 8 References

- [1] J. Carlsson, "A method of moments program for radiated emission and susceptibility analysis of printed circuit boards", SP report 1998:03.
- [2] C. R. Paul, "Incorporation of terminal constraints in the FDTD analysis of transmission lines", IEEE Trans. Electromag. Compat., vol. 36, no. 2, pp. 85-91, May 1994.
- [3] C. R. Paul, *Analysis of multiconductor transmission lines*. New York: Wiley Interscience, 1994.
- [4] A. Orlandi and C. R. Paul, "FDTD analysis of lossy, multiconductor transmission lines terminated in arbitrary loads", IEEE Trans. Electromag. Compat., vol. 38, no. 3, pp. 388-399, Aug 1996.
- [5] R. Sherman et. al, "EMP engineering and design principles", Bell Telephone Laboratories, Technical publication department, Whippany, New Jersey, 1975.
- [6] J. Carlsson, "Crosstalk on printed circuit boards", SP report 1994:14, 2nd ed.
- [7] R. F. Harrington, *Time-harmonic electromagnetic fields*. New York: McGraw-Hill, 1961.

## Appendix File formats

All data files used or created by the program are in ASCII-format.

### A1 Project file

By using *Save* or *Save As...* under *File* in the main menu all parameters for the present transmission line network can be saved in a file that later can be opened and used again. When a project is saved all output files, field excitation files and normalisation files will be saved with names that are derived from the project name. The extension for project files is *MTL*. When the program is started the project name is by default *New*. This means that if the project is not saved under a new name (or not saved at all) output files will get names derived from the name *New*.

The contents in the project file is:

*Number of transmission lines*

*Data for all transmission lines starting with number one, i.e. :*

*Number of conductors*

*Number of elements*

*Length*

*Element length*

*Excitations, left end*

*Excitations, right end*

*Field excitation (0 or 1, i.e. false or true)*

*Source resistances*

*Load resistances*

*Line capacitances*

*Line inductances*

*Multiplication factors for excitations in left end*

*Multiplication factors for excitations in right end*

*Connected to left (0 or 1, i.e. false or true)*

*Connected to right (0 or 1, i.e. false or true)*

*Connection matrix, left end*

*Connection matrix, right end*

*Time discretisation parameters*

*Time step*

*Number of time steps*

*Final solution time*

### A2 L, C and R files

Line characteristics, inductance and capacitance, as well as termination resistances can be input either manually or imported from files. The required file format for these files is as follows.

*Row number Column number Value*

...

...

where row number and column number determine the place in the corresponding matrix, value is the actual number to place in that position.

The number of rows in the files should be as many as there are conductors (the reference conductor not counted) in the particular transmission line.

### A3 End-response files

If selected, the program will save end-responses in files, one for each selected transmission line involved in the simulation. The names of these files will be *Project Name* plus *Tx.DAT*, where *x* is the number for the transmission line.

The first two lines in the end-response files are for comments only. The first line contains a description of the contents in the file and the second line is blank. The information in the first line is used by the plotting routine which is called by choosing *Open End-Response File* under *Options* in the main menu. The file contains as many columns as the number of selected end-responses plus one, the first column contains the time.

Example of contents in an end-response file:

Time, VS1, IL2

```
0.000000000000000E+0000  0.000000000000000E+0000  0.000000000000000E+0000
1.33424222939936E-0009  4.06997478544093E-0001  0.000000000000000E+0000
2.66848445879872E-0009  7.43526828867703E-0001  0.000000000000000E+0000
4.00272668819808E-0009  9.51320875721022E-0001  0.000000000000000E+0000
```

...  
...

When a FFT is performed on one of the end-responses the computed frequency response is saved in a file with a name given by *Project Name* plus *RCxTy.FRQ*, where *RC* is a response code given below, *x* is the conductor number and *y* is the transmission line number.

Response codes (RC):

- VS voltage at source end (at  $z=0$ )
- VL voltage at load end (at  $z=L$ )
- IS current at source end (at  $z=0$ )
- IL current at load end (at  $z=L$ )

The format for frequency response files is similar to the format for end-response files. The first two lines contain comments and subsequent lines contain frequency and amplitude in two columns.

Example of contents in a frequency-response file:

Freq, VS1

```
0.000000000000000E+0000  2.66534814266252E-0010
9.15527801514015E+0005  6.39982638223727E-0004
1.83105560302803E+0006  1.27945695880490E-0003
2.74658340454205E+0006  1.91791767007468E-0003
```

...  
...

## A4 Normalisation file

When an FFT is performed the frequency response can be normalised to a file, the normalisation file. This file is a time domain file which is saved with the name *Project Name* but with the extension *NOR*. The format for the normalisation file is the same as for end-response files which means that it can be plotted by the internal plotting routine.

## A5 Field excitation files

Three different files are required in order to compute the response of a transmission line due to an incident electromagnetic field. The names of these files are:

- *Project Name* plus *ET1.x*
- *Project Name* plus *ET2.x*
- *Project Name* plus *EL.x*

where  $x$  is the identification number for the particular transmission line ( $x = 1..99$ ), *ET* stands for transverse and *EL* for longitudinal field components.

Using the definitions below and remembering that the transmission line is in the  $z$ -direction (i.e. the cross section is in the  $xy$ -plane) and the reference conductor is placed at  $(x, y) = (0, 0)$ :

$$\begin{aligned} \left[ \mathbf{E}_{T,k}^n \right]_i &= x_i E_x^{inc}(x_i, y_i, (k-1)\Delta z, n\Delta t) + y_i E_y^{inc}(x_i, y_i, (k-1)\Delta z, n\Delta t) \\ \left[ \mathbf{E}_{L,k}^n \right]_i &= E_z^{inc}\left(x_i, y_i, \left(k - \frac{1}{2}\right)\Delta z, n\Delta t\right) - E_z^{inc}\left(0, 0, \left(k - \frac{1}{2}\right)\Delta z, n\Delta t\right) \end{aligned}$$

$N_c$  is the number of conductors (reference not counted),  $N_z$  is the number of elements and  $N_t$  the total number of time steps minus one (i.e. final solution time is  $N_t\Delta t$ )

the formats for the different files can be described as follows.

### ET1.x

$$\begin{array}{ccc} \left( \left[ \mathbf{E}_{T,1}^0 \right]_1 - \left[ \mathbf{E}_{T,1}^1 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,1}^0 \right]_{N_c} - \left[ \mathbf{E}_{T,1}^1 \right]_{N_c} \right) \\ \cdots & \cdots & \cdots \\ \left( \left[ \mathbf{E}_{T,N_z+1}^0 \right]_1 - \left[ \mathbf{E}_{T,N_z+1}^1 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,N_z+1}^0 \right]_{N_c} - \left[ \mathbf{E}_{T,N_z+1}^1 \right]_{N_c} \right) \\ \left( \left[ \mathbf{E}_{T,1}^1 \right]_1 - \left[ \mathbf{E}_{T,1}^2 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,1}^1 \right]_{N_c} - \left[ \mathbf{E}_{T,1}^2 \right]_{N_c} \right) \\ \cdots & \cdots & \cdots \\ \left( \left[ \mathbf{E}_{T,N_z+1}^{N_t-1} \right]_1 - \left[ \mathbf{E}_{T,N_z+1}^{N_t} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,N_z+1}^{N_t-1} \right]_{N_c} - \left[ \mathbf{E}_{T,N_z+1}^{N_t} \right]_{N_c} \right) \end{array}$$

The file will have  $N$  columns and  $N_t(N_z + 1)$  rows.

ET2.x

$$\begin{array}{ccc}
\left( \left[ \mathbf{E}_{T,2}^1 \right]_1 - \left[ \mathbf{E}_{T,1}^1 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,2}^1 \right]_{N_c} - \left[ \mathbf{E}_{T,1}^1 \right]_{N_c} \right) \\
\cdots & \cdots & \cdots \\
\left( \left[ \mathbf{E}_{T,N_z+1}^1 \right]_1 - \left[ \mathbf{E}_{T,N_z}^1 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,N_z+1}^1 \right]_{N_c} - \left[ \mathbf{E}_{T,N_z}^1 \right]_{N_c} \right) \\
\left( \left[ \mathbf{E}_{T,2}^2 \right]_1 - \left[ \mathbf{E}_{T,1}^2 \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,2}^2 \right]_{N_c} - \left[ \mathbf{E}_{T,1}^2 \right]_{N_c} \right) \\
\cdots & \cdots & \cdots \\
\left( \left[ \mathbf{E}_{T,N_z+1}^{N_t} \right]_1 - \left[ \mathbf{E}_{T,N_z}^{N_t} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{T,N_z+1}^{N_t} \right]_{N_c} - \left[ \mathbf{E}_{T,N_z}^{N_t} \right]_{N_c} \right)
\end{array}$$

The file will have  $N$  columns and  $N_t N_z$  rows.

EL.x

$$\begin{array}{ccc}
\left( \left[ \mathbf{E}_{L,1}^{3/2} \right]_1 + \left[ \mathbf{E}_{L,1}^{1/2} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{L,1}^{3/2} \right]_{N_c} + \left[ \mathbf{E}_{L,1}^{1/2} \right]_{N_c} \right) \\
\cdots & \cdots & \cdots \\
\left( \left[ \mathbf{E}_{L,N_z}^{3/2} \right]_1 + \left[ \mathbf{E}_{L,N_z}^{1/2} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{L,N_z}^{3/2} \right]_{N_c} + \left[ \mathbf{E}_{L,N_z}^{1/2} \right]_{N_c} \right) \\
\left( \left[ \mathbf{E}_{L,1}^{1+3/2} \right]_1 + \left[ \mathbf{E}_{L,1}^{1+1/2} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{L,1}^{1+3/2} \right]_{N_c} + \left[ \mathbf{E}_{L,1}^{1+1/2} \right]_{N_c} \right) \\
\cdots & \cdots & \cdots \\
\left( \left[ \mathbf{E}_{L,N_z}^{N_t-1+3/2} \right]_1 + \left[ \mathbf{E}_{L,N_z}^{N_t-1+1/2} \right]_1 \right) & \cdots & \left( \left[ \mathbf{E}_{L,N_z}^{N_t-1+3/2} \right]_{N_c} + \left[ \mathbf{E}_{L,N_z}^{N_t-1+1/2} \right]_{N_c} \right)
\end{array}$$

The file will have  $N$  columns and  $N_t N_z$  rows.

