STATENS
PROVNINGSANSTALT
National Testing Institute

Fire Technology

Reprinted from FIRE SAFETY JOURNAL, 5 (1985) 281-285

Ulf Wickström

Temperature Analysis of Heavily-insulated Steel Structures Exposed to Fire

Technical Report SP-RAPP 1985:33 ISSN 0280-2503
Borås, Sweden 1985

Fire Safety Journal, 9 (1985) 281 - 285

Temperature Analysis of Heavily-insulated Steel Structures Exposed to Fire

U. WICKSTRÖM

Swedish National Testing Institute, Borås (Sweden)

(Received April 11, 1984; in final form November 26, 1984)

SUMMARY

An exact analytical one-dimensional solution of the temperature response of insulated steel structures is derived and a closed form solution is given for a structure exposed to the ISO 834 standard curve.

Alternative approximate solutions are also given where all the heat capacity is assumed lumped in the steel core; only one third of the insulation heat capacity is then considered. It is also shown that approximate solution schemes given elsewhere strongly underestimate the temperature rise in structures protected with heavy insulation systems. In addition the theoretical background of a proposed NORDTEST test method on how to obtain the thermal properties of an insulation system is outlined. Finally a numerical calculation scheme is recommended.

INTRODUCTION

Temperature in an insulated steel structure, as shown in Fig. 1, exposed to fire, may be estimated by a one-dimensional analysis if the exposure and the insulation are equal on all exposed surfaces and the corner effects are neglected. In addition, as the thermal diffusivity of steel is very high, the steel temperature may be assumed uniformly distributed which substantially simplifies the analysis. Figure 2 shows the simplified thermal model to be studied.

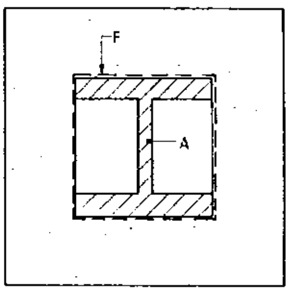

Fig. 1. Insulated steel structure with a steel core volume *A* and a surrounding area *F*.

If the insulation heat capacity is small and may be neglected, a simple well-known exponential expression of the steel temperature response may be derived. For the more general case approximatively similar solutions will be suggested where the insulation heat capacity is lumped and one third is added to the steel core heat capacity. Furthermore it is shown for the first time that the approximations may be improved by shifting the time scale by a value depending on the ratio between the heat capacities of the insulation and the steel core.

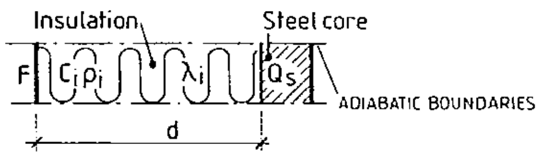

Fig. 2. One-dimensional heat conduction model.

Exact and approximative solutions will then be given of the thermal response of steel structures exposed to the standard fire according to ISO 834. The theoretical basis is also given for interpreting test results so that the thermal resistance of insulation materials and systems may be calculated and finally a numerical scheme is recommended to be used when the material properties vary with temperature.

TEMPERATURE RESPONSE OF STEPWISE CHANGE OF SURFACE TEMPERATURE

Consider the case shown in Fig. 2 and calculate the temperature θ as a function of

0379-7112/85/$3.30

time. (The solution of the problem may be found in several textbooks, e.g., [1].) Assume all material properties constant. A lumped heat capacity Q_s is assumed at $x = d$. The governing equation is

$$\lambda_i \frac{\partial \theta^2}{\partial x^2} - c_i \rho_i \frac{\partial \theta}{\partial t} = 0 \qquad (1)$$

where λ_i is the thermal conductivity, c_i specific thermal heat capacity, and ρ_i density, respectively, of the insulation material.

Assume the following boundary conditions.

$$\theta(x, 0) = \theta_0 \qquad \text{for } t = 0 \qquad (2)$$

$$\theta(0, t) = 0 \qquad \text{for } x = 0 \qquad (3)$$

and

$$F\lambda_i \frac{\partial \theta}{\partial x} + Q_s \frac{\partial \theta}{\partial t} = 0 \qquad \text{for } x = d \qquad (4)$$

where F is the section area.

Now assume the solution

$$\theta(x, t)/\theta_0 = \Sigma \mathrm{K}_n \exp(-\beta_n t) \sin(\alpha_n x) \qquad (5)$$

Equations (1) and (3 - 5) give

$$\theta(x, t)/\theta_0 = \Sigma \mathrm{K}_n \exp(-a\alpha_n^2 t) \sin(\alpha_n x) \qquad (6)$$

where $a = \lambda_i / c_i \rho_i$. The eigenvalues α_n are obtained from the following equation which is derived from eqn. (4).

$$(\alpha_n d)/\cot(\alpha_n d) = \mu \qquad (7)$$

where $\mu = Fdc_i\rho_i/Q_s = Q_i/Q_s$. The coefficients K_n are obtained from the initial condition $\theta_0 = \theta$ for $t = 0$. Thus the temperature $\theta_s(t)$ at $x = d$ is

$$\theta_s(t)/\theta_0 = \sum_{n=1}^{\infty} \mathrm{K}_n \exp\left[-\frac{(\alpha_n d)^2}{RQ_i/F} t\right] \sin(\alpha_n d) \qquad (8a)$$

where [1]

$$\mathrm{K}_n = \frac{2[(\alpha_n d)^2 + \mu^2]}{(\alpha_n d)[(\alpha_n d)^2 + \mu^2 + \mu]} \qquad (8b)$$

and the thermal resistance $R = d/\lambda_i$. (Note that when $\mu \to 0$, the first eigenvalue $\alpha_1 d \to \sqrt{\mu}$ and $\alpha_n d \to n\pi$ for $n \geqslant 2$. Thus $\theta_s(t)/\theta_0 \to \exp(-tF/RQ_s)$ which may be derived directly if Q_i is neglected in the analysis.)

Equation (8) may be approximated as

$$\theta_s(t)/\theta_0 = 0 \qquad \text{for } t \leqslant \bar{t} \qquad (9a)$$

$$\theta_s(t)/\theta_0 = \exp\left[-\frac{t - \bar{t}}{\tau}\right] \qquad \text{for } t > \bar{t} \qquad (9b)$$

where

$$\tau = R(Q_s + Q_i/3)/F \qquad (9c)$$

By comparison with the exact solution the time shift $\bar{t}$ is estimated as

$$\bar{t} = \mu\tau/8 \qquad (9d)$$

In Fig. 3 the value of θ_s/θ_0 has been plotted as a function of the non-dimensional group t/τ for a few values of μ. As an example it is shown that the approximate solution, according to eqns. (9a) and (9b), and the exact solution nearly coincide for $\mu = 0.41$.

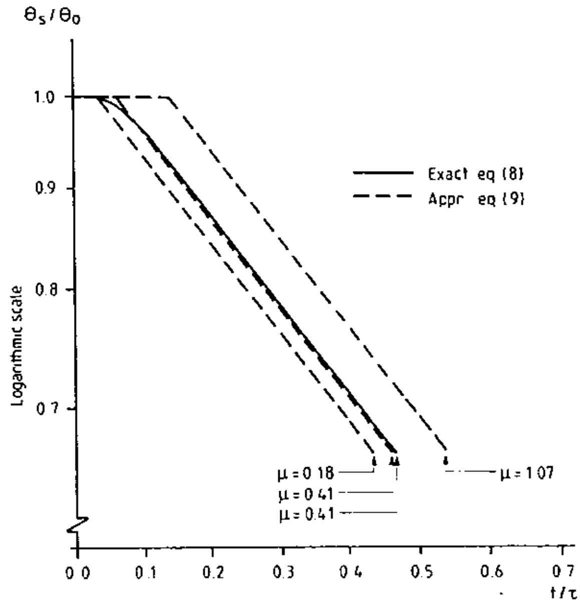

Fig. 3. A comparison of the exact and approximate solutions of the stepfunction θ_s/θ_0 versus dimensionless time t/τ for various μ, where t is time, $\tau = (Q_s + Q_i/3)d/(\lambda_i F)$ and $\mu = Q_i/Q_s$, respectively.

According to eqn. (9c) one third of the insulation heat capacity Q_i is added to the steel core. In an approximation formula, not including the time shift $\bar{t}$, Lie [2] suggests that a slightly different portion, 0.3 Q_i, should be added to the heat capacity of the core.

ANALYTICAL SOLUTION OF FIRE EXPOSED INSULATED STEEL STRUCTURE

If one-dimensional conditions are assumed and the thermal properties are constant, the temperature rise of a fire exposed insulated steel structure may be obtained by superposition (Duhamel's theorem [1]). Thus the steel temperature is obtained as

$$T_s(t) = \int_0^t T(t - \xi)\Phi'(\xi)d\xi \qquad (10)$$

where T is the imposed fire curve and the response function $\Phi = (1 - \theta_s/\theta_0)$; θ_s/θ_0 is given by eqn. (8) or (9).

If the ISO 834 standard fire curve is approximated by a sum of exponential terms [3] as

$$T = \sum_{j=0}^{3} B_j \exp(-\beta_j t) \qquad (11)$$

where B_j and β_j are as given in Table 1, then the steel temperature is obtained from eqn. (10) as

$$T_s = \sum_{n=1}^{\infty} \sum_{j=0}^{3} \frac{B_j K_n \sin(\alpha_n d)}{\left[1 - \dfrac{\beta_j Q_i R/F}{(\alpha_n d)^2}\right]} \times \left\{\exp(-\beta_j t) - \exp\left[\frac{(\alpha_n d)^2}{R Q_i/F} t\right]\right\} \qquad (12)$$

where α_n and K_n are given by eqns. (7) and (8b), respectively. A similar expression of the exact solution of the steel temperature has been derived by Lie [2], who used a different approximation formula of the ISO standard fire curve.

If instead the approximate solution according to eqns. (9a) and (9b) is substituted, the following expression is obtained for $t > \bar{t}$.

$$T_s = \sum_{i=0}^{3} \frac{B_i}{1 - \beta_i \tau} \{\exp[-\beta_i(t - \bar{t})] - \exp[-(t - \bar{t})/\tau]\} \qquad (13)$$

TABLE 1

Constants in the exponential expression of the ISO 834 standard curve

j	0	1	2	3
B (°C)	1325	−430	−270	−625
β (h^{-1})	0	0.2	1.7	19

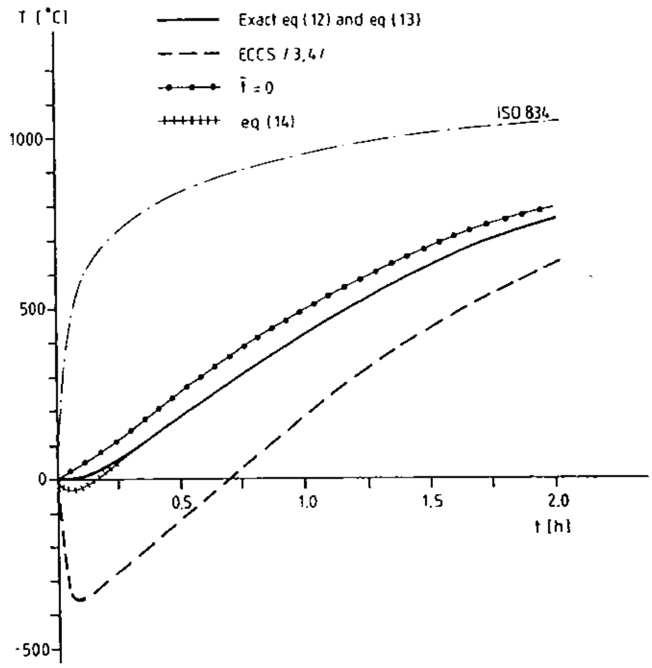

Fig. 4. Example of calculated temperature versus time of an insulated steel structure exposed to fire according to ISO 834. The exact solution is plotted for comparison with various approximative solutions. Input values representing a length of one meter of the structure: Q_s = 14 000 Ws K^{-1}. Q_i = 15 000 Ws K^{-1} and $\lambda_i F/d$ = 5 W K^{-1}.

where τ and $\bar{t}$ are given by eqns. (9c) and (9d), respectively.

An example of calculated temperature of an insulated steel structure exposed to fire according to ISO 834 is shown in Fig. 4. The exact solution, eqn. (12), as well as the approximate solution according to eqn. (13) are plotted in full line. The differences between them are not noticeable in the chosen scale. However, when employing the approximative numerical calculation procedure as recommended in refs. 4 and 5, a much lower temperature is predicted as indicated by the dotted line. A warning must therefore be expressed for using that calculation formula for design purposes. A third graph is also plotted in Fig. 4 showing the steel temperature obtained if the influence of the time shift $\bar{t}$ is neglected.

In most cases the change of material properties at elevated temperature must be considered. Then a numerical scheme based on the difference form of eqn. (13) may be employed. However, as that expression contains the time shift $\bar{t}$, an alternative formula is suggested. By assuming the response function according to eqn. (9b) valid for the *entire* interval $t \geqslant 0$ and setting $\bar{t} = \mu\tau/10$, an alternative slightly less accurate closed form solution for constant material properties may be derived as:

$$T_s = \sum_{i=0}^{3} B_i \left\{ \frac{\exp(-\mu/10)}{1-\beta_i\tau} [\exp(-\beta_i t) - \exp(-t/\tau] - \exp(-\beta_i t) \times [\exp(-\mu/10) - 1] \right\} \quad (14)$$

As seen in Fig. 4 this equation gives quite accurate temperatures although negative values are obtained at short exposure times.

TIME DERIVATIVE OF THE STEEL TEMPERATURE

The time derivative of the steel temperature is obtained from eqn. (10).

If the approximate response function according to eqn. (9) is used, the following very simple expression is obtained.

$$\left.\frac{dT_s}{dt}\right|_{t=t} = \frac{1}{\tau}[T(t-\bar{t}) - T_s(t-\bar{t})] \quad (15)$$

CALCULATION OF THERMAL RESISTANCE R

The thermal resistance R may be obtained from fire tests where furnace and steel temperature are recorded. Equation (1) may then be employed but as the time shift $\bar{t}$ is dependent on R, the time derivative corresponding to eqn. (14) is recommended for this purpose:

$$1/R = \left\{\frac{dT_s}{dt} + [\exp(\mu/10) - 1]\frac{dT}{dt}\right\}(A/F)c_s\rho_s (1 + \mu/3)/(T - T_s) \quad (16)$$

Here A is the steel core volume. When appropriate, the thermal conductivity of the insulation material may be obtained as $\lambda_i = d/R$. The thermal resistance R or alternatively the thermal conductivity λ_i may then be calculated explicitly and given as a function of the average insulation temperature $T_m = (T + T_s)/2$.

A comprehensive study of various insulation systems has been reported by Andersen and Wickström [6] where the thermal properties were estimated on basis of eqn. (16). A formalized testing method is underway from NORDTEST*.

NUMERICAL CALCULATIONS

For non-linear cases when the material properties vary with temperature numerical calculations are required. Based on the same approximation as eqn. (14) the following approximative temperature time derivative is derived.

$$\Delta T_s/\Delta t = (T - T_s)/\tau - [\exp(\mu/10) - 1]\frac{dT}{dt} \quad (17)$$

To ascertain numerical stability of this forward difference or explicit time integration scheme the time increment must be chosen so that $\Delta t \leqslant \tau = (A/F)c_s\rho_s(1 + \mu/3)/\lambda_i$. However, to obtain reasonable accuracy Δt should not in addition exceed 60 s.

ACKNOWLEDGEMENTS

Stig Andersson was most helpful with the calculation work of this paper. His contribution to the design and plotting of the diagrams is also greatly appreciated.

LIST OF SYMBOLS

- A steel core volume (m^3)
- a thermal diffusivity ($m^2\,s^{-1}$)
- c heat capacity (Ws $kg^{-1}\,K^{-1}$)
- d length (m)
- F section area (m^2)
- Q heat capacity (Ws K^{-1})
- R thermal resistance ($m^2\,K\,W^{-1}$)
- t time (s, h)
- x length coordinate (m)

*NORDTEST is a joint Nordic body with the function of promoting development in the field of technical testing.

Greek letters

α_n constant (m^{-1})
K_n constant (—)
λ heat conductivity ($W\ m^{-1}\ K^{-1}$)
θ temperature (K)
ρ density ($kg\ m^{-3}$)
τ see equation (9c) (s)
μ Q_i/Q_s (—)

Subscripts

i insulation
s steel

REFERENCES

1 H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd edn., Oxford University Press, 1959.

2 T. T. Lie, Temperature of protected steel in fire, *Paper 8 of Symposium No 2. Behaviour of Structural Steel in Fire*, HMSO, London, 1968.

3 *SBN — Swedish Building Code 1975*, National Board of Physical Planning and Building, 1975.

4 *European Recommendations for the Fire Safety of Steel Structures — Calculation of the Fire Resistance of Load Bearing Elements and Structural Assemblies Exposed to the Standard Fire*, European Convention for Constructional Steelwork ECCS, Technical Committee 3, Fire Safety of Steel Structures, Elsevier Scientific Publishing Company, 1983.

5 S.-E. Magnusson, O. Pettersson and J. Thor, *Fire Engineering Design of Steel Structures*, Stålbyggnadsinstitutet, Stockholm, 1976.

6 N. Andersen and U. Wickström, Brandbeskyttende Isolation af Stålkonstruktioner, *NORDTEST projekt 275-81*, DANTEST, Copenhagen, 1984 (in Danish).