A generative mobility model

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1 Motivation

Many applications in the mobile radio network domain employ simulations to explore e.g. parameter configurations, robustness of protocols and buffer allocation algorithms. User mobility is (together with traffic and radio propagation models), one of the main components of such simulations, and has large impact on e.g. load distribution, cell handover frequency, signal fading and interference. In many simulations, detailed user mobility is as crucial as the physical infrastructure, where the exact position affect fading and reflections, but in others, e.g. load balancing, handover management and radio resource scheduling, coarser models are often sufficient. But even in these cases, properties of the trajectories of individual users will affect the results of the simulation, e.g. where distribution of positions, rest times, and displacement magnitudes and velocities, need to be considered.

2 Characterising human mobility

In a series highly cited of papers [1, 3, 2] Barabasi and his collaborators makes the following observations:

1. Displacement magnitudes and dwell times distributions are heavy-tailed.

2. The radius of gyration RoG (variance of position) of users are also heavy-tailed, and there is a strong correlation between the RoG of an individual user and the distribution of displacement magnitude for any given period

3. The number of places visited by an individual user tends to be Zipf-distributed

The first observation is in agreement with Levý walks, as observed by e.g. gracing animals and predators. The second two however are not.

3 Elements of a generative model

The centre of gyration (CoG) for any individual (e.g one carrying a mobile terminal) over a fixed period is the mean position over that period. At any
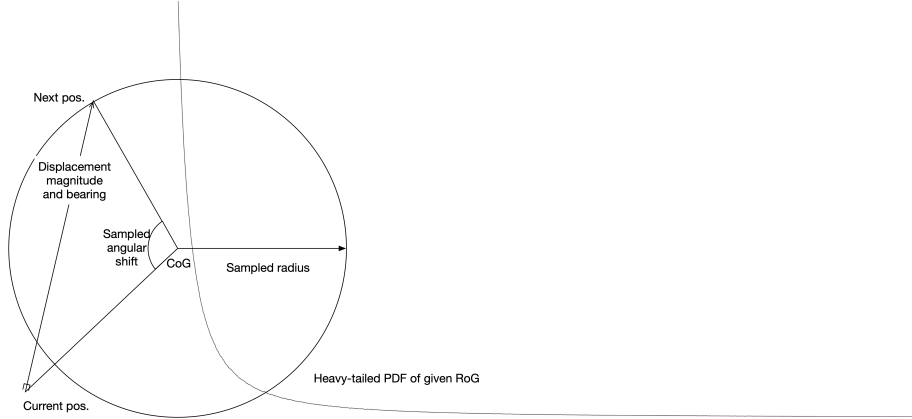
Figure 1: Displacement generation overview

given time within that period, the user is found at a given polar coordinate relative to its CoG. We can study the distribution of the polar radii magnitudes, and it turns out that for any (suitably large) duration, the radii distribution is also heavy-tailed, and strongly correlated with the RoG for that period. The the polar angles are limited to that of a circle, but the shifts in the polar angles between subsequent observations tend to have a very sharp decay, also reminiscent of heavy tailed distributions.

4 Displacement generation

Based on these ideas and observations, we can construct a mobility generator by first sampling CoC and CoG for each individual, according to some density distribution. Successive positions for the individual can then be obtained by sampling polar coordinates relative to the CoC. The polar radius magnitude is sampled from a function fitted to the RoG, while the *change* in polar angle is sampled from independent function fitted to some observed angular shift. This process is illustrated in Figure 1. The empirical distributions of observed radii and angular shifts are plotted in Figure 2

The temporal properties of a sequence of observations of a trajectory are not captured by these mechanism, nor is any dependence of the two polar variates. Instead, the *time* of the next observation, is obtained from the displacement magnitude and a range of reasonable speeds typical for a given means of transportation, but currently independently of any underlying infrastructure. This gives a random walk constrained by the CoG, which is non-dispersing without a fixed border, but individual displacements may be "unrealistic" since autocorrelation between displacement bearings is not taken into account.

Gyration radius distribution, ~127k UEs, Fr-Mn, ~13112k obs.

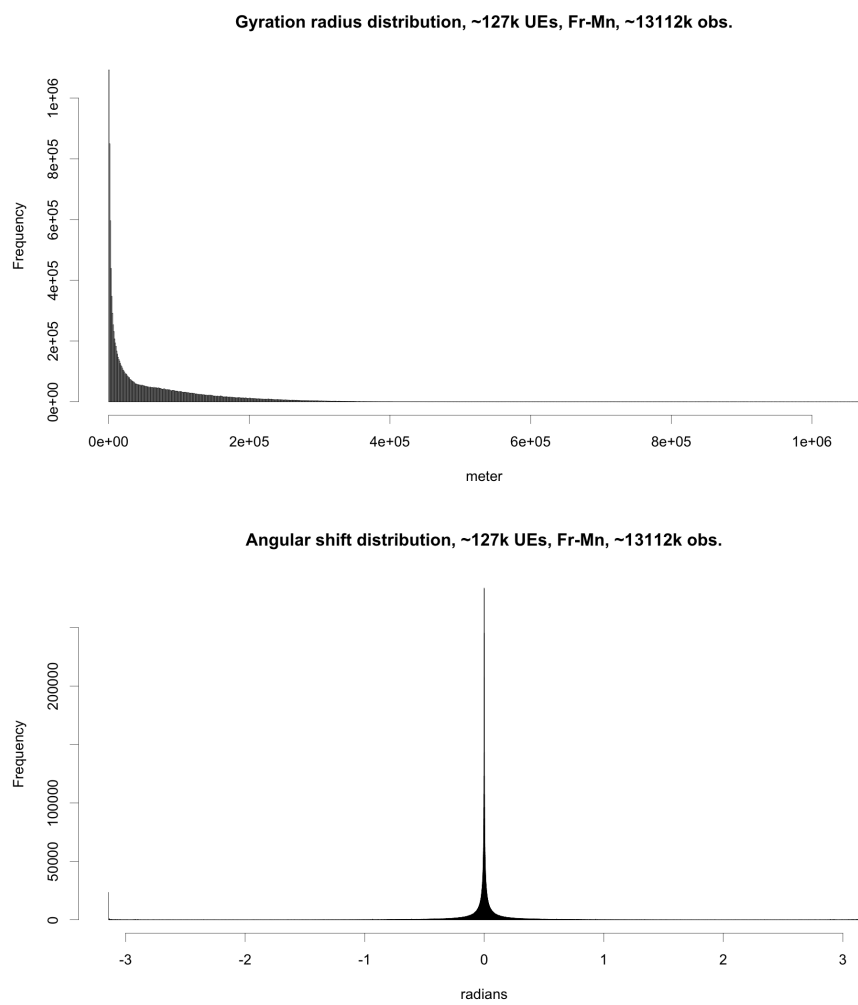**Angular shift distribution, ~127k UEs, Fr-Mn, ~13112k obs.**

Figure 2: Radii magnitude and angular shift distributions

5 Fits to recorded data

Radii observed in a data set of telecom users are well modelled by common long-tailed distribution, if we separate groups of individual within ranges of observed RoG, but not as well for the entire population. Figure 3 show a log-normal fit applied to the whole population, and for the group of individuals within the 6:th percentile of the observed RoGs. Similar fit was obtained for the other percentiles, and using Pareto distributions, and not shown.

Angular shift is obtained from a slightly truncated log-normal distribution fit to absolute values of observed shifts as shown in Figure 4. When used to *generate* angular shift, a random *sign* is also applied to the angular shift.

The choice of a log-normal fit here is perhaps a bit arbitrary, but it does give a very strong preference for small angular changes. Note however that for large radii, even small angular changes can represent large linear displacements.

6 Results

By studying statistics of recorded trace, we have been able to fit radius distributions and angular shifts to heavy-tailed distributions in a way that lends itself to generative mobility modelling. Log-normal distributions appear to be sufficiently "heavy-tailed" to obtain good fits for this type of data, and are much easier to calculate (and often more accurate) than Pareto, truncated Pareto or truncated power-law distributions.

Applying distribution parameters to outlined model, we can generate displacements that are non-dispersive (in contrast to Levý walks) and conforms to the observed radii magnitudes. Generated displacement magnitudes tend to be larger than recorded ones, probably because the angular shift model is too simple. Moreover, the model fails to reproduce trajectories with sufficiently consistent (positively auto-correlated) displacement bearings. Figure 5 shows examples of recorded trajectories where successive displacement display consistent bearings over large parts of their trajectories, while the generated ones lacks this property.

References

[1] Marta C. González, Hidalgo César A., and Albert-László Barabasi. Understanding individual human mobility patterns. *Nature*, 453(7196):779–782, June 1988.

[2] Chaoming Song, Tal Koren, Pu Wang, and Albert-Laszlo Barabasi. Modelling the scaling properties of human mobility. *Nature Physics*, 6:818–823, October 2010.

[3] Chaoming Song, Zehui Qu, Nicholas Blumm, and Albert-Laszlo Barabasi. Limits of predictability in human mobility. *Science*, 327:1018–21, February 2010.
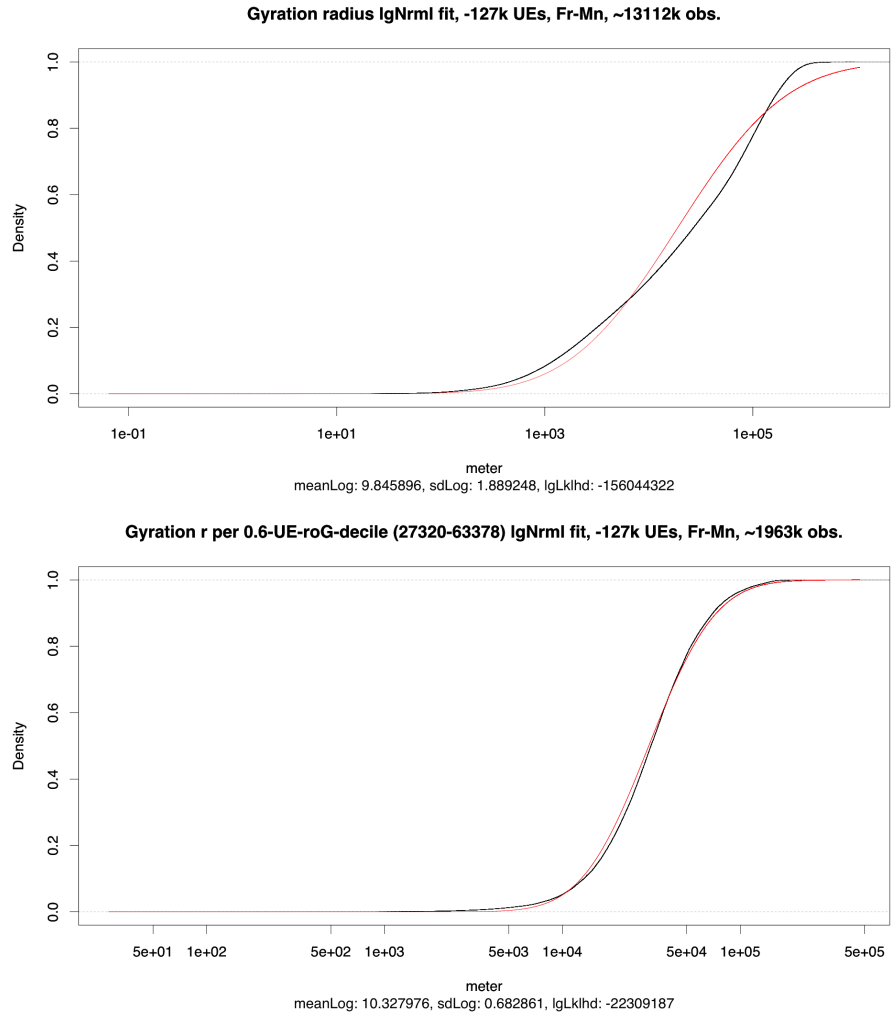
Gyration radius lgNrml fit, -127k UEs, Fr-Mn, ~13112k obs.

meanLog: 9.845896, sdLog: 1.889248, lgLklhd: -156044322

Gyration r per 0.6-UE-roG-decile (27320-63378) lgNrml fit, -127k UEs, Fr-Mn, ~1963k obs.

meanLog: 10.327976, sdLog: 0.682861, lgLklhd: -22309187

Figure 3: Log-normal fit to radii global distribution not perfect, but restricted to deciles of user RoG it improves

5

Angular shift lgNrml fit, -127k UEs, Fr-Mn, ~13112k obs.

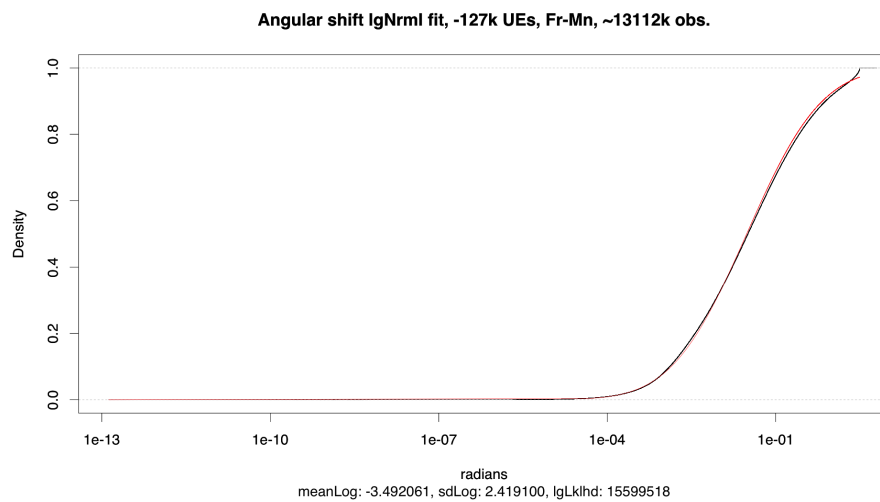meanLog: -3.492061, sdLog: 2.419100, lgLklhd: 15599518

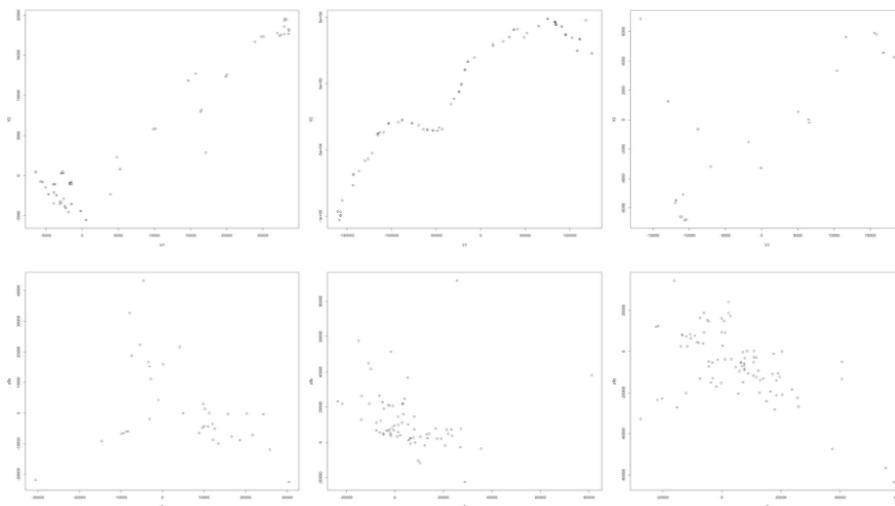Figure 4: Truncated log-normal fit to absolute values of observed angular displacement

Figure 5: Lack of "directionality" in generated trajectories