

## Structural analysis of concrete members with shear failure

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### ABSTRACT

The paper aims to improve the knowledge of the shear response in reinforced concrete and the influence of aggregate interlock using non-linear finite element analysis. The influence of aggregate types on shear response of concrete was studied with finite element analysis of reinforced concrete beams failed in shear. The influence of aggregate interlock was investigated using different smeared crack approaches, i.e. multi directional, fixed and rotating crack models. The results from analysis are verified with the result from mechanical testing with respect to stiffness, capacity and crack pattern.

**Key words:** RC beams, shear failure, finite element analysis, crack model.

### 1. INTRODUCTION

Infrastructures represent a large capital in all countries. In order to establish a sustainable development in a society, it is of great importance that infrastructures generate a return and the investments result in safe structures with predictable response, and that the structures can be utilized during the entire lifetime. Non-linear finite element analysis (FEA) has proved to be capable of describing the behaviour of reinforced concrete in a comprehensive way, provided that appropriate constitutive models are adapted. However, the reliability of modelling methods used to simulate shear fracture in concrete structures are yet to be verified /Broo 2008/.

Smeared crack model was introduced for reinforced concrete more than four decades ago. Since then several improvements were made and the approach has gained high attention. One of these improvements was the introduction of shear retention, i.e. reduced shear stiffness due to cracking. The reduction in shear stiffness is introduced in the smeared crack model using a shear retention factor,  $\beta$ . The concept of a smeared crack model with a constitutive model based on total strain is that stresses are described as a function of strains. One commonly used total strain approach is the *Rotating crack model* in which the stress-strain relationships are evaluated in the principal directions of the strain vector ( $\beta = 1$ ). More appealing to the physical nature of cracking is the *fixed crack model* in which the stress-strain relationships are evaluated in a fixed coordinate system which is fixed upon cracking ( $0 < \beta < 1$ ). The concept of a smeared crack model with strain decomposition, *multi-directional fixed crack approach*, gives the possibility of modelling cracks that simultaneously occur ( $0 < \beta < 1$ ). In general, a smeared crack model is specified as a combination of tension cut-off, tension softening and shear retention.

In an ongoing project the shear response of concrete is studied based on an understanding of micro and meso scale material properties. In an earlier part of the project, tests were carried out on beams with two types of aggregate and two w/c ratios. Crack propagation was monitored during the loading by Digital Image Correlation (DIC) and Acoustic Emission (AE). One goal was to develop a methodology for detailed structural analysis of concrete, which consist of (a) material and structural testing, (b) DIC and AE, (c) microscopical analysis, and (d) numerical analysis. This paper focuses on the latter part and aims to improve the knowledge of the shear response in reinforced concrete members and the influence of aggregate interlock using FEA.

## 2. EXPERIMENTS

Here only a summary of shear tests on RC beams is given; more details concerning the tests can be found in /Flansbjerg *et al.* 2011/. The test series included four concrete recipes with a w/c of 0.38 or 0.9 and fine aggregate of natural or crushed rock material. The test set-up is illustrated in Figure 1. The summary of concrete mixture and results from material and structural testing for the beams is shown in Table 1. For each recipe four RC beams were manufactured and tested.

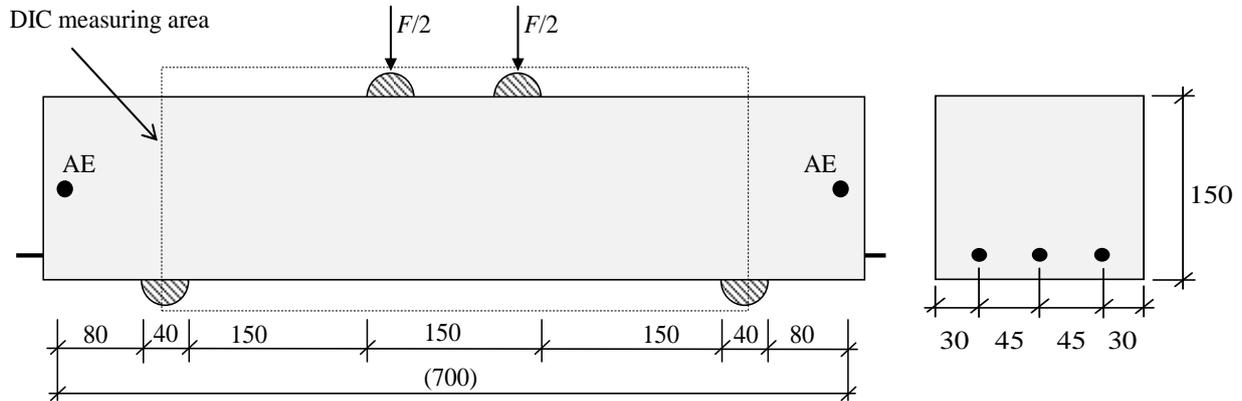


Figure 1 – The RC beams geometry and test set-up; dimension are in mm.

The crack propagation was registered at one side of the specimens during the testing by the use of the optical full-field deformation measurement system based on Digital Image Correlation (DIC). The macroscopic analysis was performed on lapped concrete slabs of the approximate size  $450 \times 150 \text{ mm}^2$ . The slabs were impregnated with epoxy glue containing fluorescence dye.

## 3. FE ANALYSIS AND RESULTS

The shear tests on RC beams were analysed in detail using non-linear three-dimensional FE models. Due to symmetry, the FE model included half of a beam length and half of a beam cross section. Isoparametric continuum elements were used for the concrete and longitudinal reinforcements. Different constitutive models based on non-linear fracture mechanics using a smeared crack approach were used for concrete: (a) rotating crack model based on total strain denoted TS-rotating, (b) fixed crack model based on total strain denoted TS-fixed, and (c) multi-directional fixed crack approach denoted Multi-dir /Diana 2009/. The shear retention factor was varied ( $\beta = 0.1, 0.4$  and  $0.8$ ) in the analyses with TS-fixed and Multi-dir constitutive models. The crack band width was assumed to be equal to the element size, 5 mm.

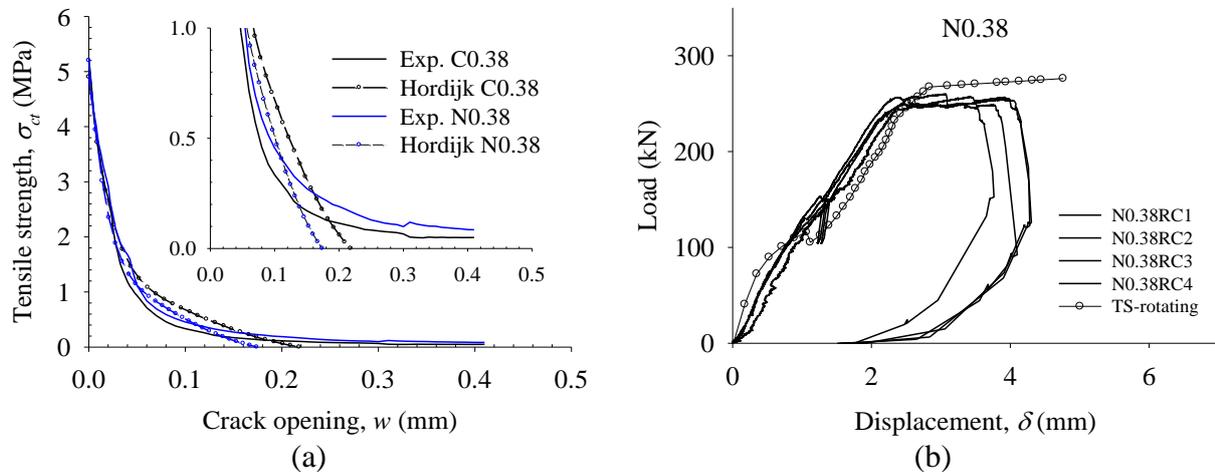
For the concrete in compression, the model by /Thorenfeldt *et al.* 1991/ was adopted. The tension softening curves calculated from direct tensile tests, Figure 2(a), were not significantly influenced by the aggregate type. The softening behaviour given by /Hordijk 1987/ was a good representation of the experimental softening curves and were used in the analysis. The reinforcing steel was modelled according to an isotropic plastic model with the Von Mises yield criterion. The interaction between concrete and rebar was described base on a frictional model describing the relations between stresses and deformations; the model was earlier developed in /Lundgren 2005/. The material properties given in Table 1 were used in the analysis. The yield and ultimate stresses of steel bars were  $f_{sy} = 500 \text{ MPa}$  and  $f_{su} = 600 \text{ MPa}$  and the elastic modulus was  $E_{s0} = 222 \text{ GPa}$ . An incremental static analysis was made using an explicitly specified load step size and a Newton-Raphson iterative scheme to solve the non-linear equilibrium equations. In a phased analysis, the self-weight load was first applied. Thereafter, the beam was subjected to the external load as a prescribed displacement at the loading point.

*Table 1 – Summary of concrete mixture and results from material and structural testing.*

Mixture/properties	Unit	Source	C0.38	N0.38
w/c	NA	NA	0.38	0.38
Fine aggregate 0-8 mm	[kg/m <sup>3</sup> ]	NA	1064	900
Coarse aggregate 8-16 mm	[kg/m <sup>3</sup> ]	NA	708	900
Water	[kg/m <sup>3</sup> ]	NA	164	162
Cement	[kg/m <sup>3</sup> ]	NA	429	426
Water reducer	[kg/m <sup>3</sup> ]	NA	1.1	0.7
$f_{c,cyl}$ (calculated as $f_{c,cube} \cdot 0.85$ )	[MPa]	Compression test	71.83	82.37
$f_t$	[MPa]	Direct tensile test	4.9	5.2
$E$	[GPa]	Eurocode 2	39.49	41.33
$G_F$	[N/m]	Direct tensile test	208.7	175.5
Shear strength	[MPa]	Direct shear tests	7.04	8.20
Load capacity	[kN]	RC beam tests	256.4	260.2
Load capacity	[kN]	Eurocode 2	257.3	269.4

The results from numerical analyses and experiments in terms of load-displacement response are presented for N0.38 and C0.38 in Figures 2(b) and 3(a), respectively. The FE analyses based on TS-rotating constitutive model for both RC beams (N0.38 and C0.38) showed a shear compression failure; this agrees well with the failure mode observed in the tests. Web-shear cracks initiated across the neutral axis at a load level of about 120-170 kN. With increased load, the reinforcement yielded at approximately 250kN and the final failure was triggered with crushing of concrete adjacent to the loads.

The influence of aggregate interlock was investigated by varying the shear retention factor in the analyses based on TS-fixed and Multi-dir constitutive models. The results from analysis with  $\beta = 0.1$  and 0.4 are presented Figures 3(a). The stiffness started to differ for different  $\beta$  values right after the shear crack formed; however, the stiffness before and around the maximum load was not estimated correctly in these analyses (TS-fixed and Multi-dir). Different degrees of aggregate interlock also significantly influenced the capacity. The capacity was well estimated in all analyses except when high shear retention factor,  $\beta = 0.4$ , was adopted in the analysis with Multi-dir constitutive model. The failure mode was only correctly predicted in analysis with TS-rotating constitutive model. The failure mode in the analysis with TS-fixed constitutive model was triggered by crushing of concrete; no yielding of reinforcing steel took place. While, yielding of reinforcement was the only failure mode observed in the analysis with Multi-dir constitutive model.



*Figure 2 – (a) Tension softening and (b) load-displacement from experiment and analysis.*

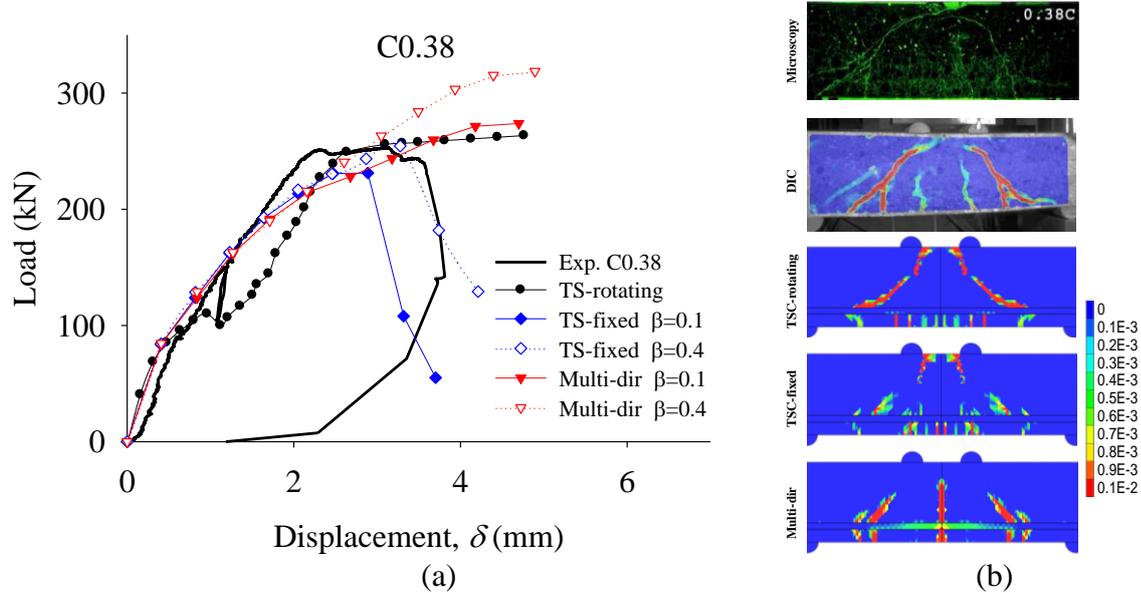


Figure 3 – (a) Load-displacement relations and (b) crack pattern from experiment and analysis.

The crack patterns shown in Figure 3(b) reveal more about each constitutive model and the extent to which they were capable of reproducing the experimental observations. Although, the best agreement with experiments, in terms of stiffness, capacity and failure mode, was observed in the analysis with TS-rotating and Multi-dir with  $\beta=1$ , the crack pattern and consequently the shear fracture process were not correctly captured in either of these analyses. This may resemble that the FE analysis at the structural level needs to include micro and meso scale material properties of aggregate, cement paste and their interaction in a more comprehensive way.

#### 4. CONCLUSIONS

The paper investigated the shear response in reinforced concrete and the influence of aggregate interlock using non-linear finite element analysis. The analysis showed that different degrees of aggregate interlock may highly influence the shear capacity. Out of the three constitutive models based on smeared crack approach used in this study, i.e. rotating, fixed and multi-directional crack models, the best predictions in terms of stiffness, capacity and failure mode were made by rotating crack model. However, none of the constitutive models were capable of well predicting the shear fracture process and crack pattern. Further implementation of micro and meso level material properties into analysis at structural level however remains for the future work.

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