Load/Strength analysis of wave energy components

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Abstract

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Components in wave power applications are subjected to large forces and irregular loading. Existing dimensioning practise are usually developed for other applications, with e.g. uniform running loading. In fatigue, large uncertainties are present requiring safety large factors to be used.

In this report, fatigue design principles are utilized to estimate service life and capacity of wave power components with the special circumstances they are subjected to. With the developed fatigue evaluation method, the uncertainty of the results is quantified with the Variation Mode and Effect Analysis (VMEA ). This allows proper safety factors to be established with regard to a required service life or strength. The factors that cause the most uncertainty can also be identified giving opportunity to reduce the uncertainty, which can lead to more efficient and optimized products.

Two cases are analysed for two wave power companies: Corpower and Ocean Harvesting Technologies. For the first company, a rope that transfers large forces from a buoy is analysed. For the second company, a roller chain or a rack and pinion that transmits power to a large weight in a gravity accumulator is analysed.

Key words: Wave power, reliability, fatigue, VMEA

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Preface

The project "From Buoy To Grid" is aimed at through applied research in Ocean Energy Centre (OEC) industry research programs in Marine Renewable Energy Technology (RMET) support the Swedish development company operating in Ocean Energy Centre as well as other companies in the offshore energy sector. Specific applied research that contributes to effective technological development of equipment for the production of marine energy is of great importance because the development in this field is rapid and global. The companies that succeed in developing cost effective and reliable solutions and concepts, and win market confidence will have a major impact on the global market for their products.

The goal of the project is to support Swedish companies active in ocean energy with knowledge. The target audience is primarily the Swedish development companies linked to the OEC but knowledge is generally available to all companies in the area. Principal and the beneficiary is Chalmers who along with SP and SSPA and the participating technology companies also contribute to the project. The project will consist of applied research and practical experiments and measurements. SP will be the project manager and participate in all subprojects. Chalmers will mainly participate in matters relating to electrical systems while the SSPA will focus its work on safety in the marine environment.
Summary

Components in wave power applications are subjected to large forces and irregular loading. Existing dimensioning practise are usually developed for other applications, with e.g. uniform running loading. In fatigue, large uncertainties are present requiring safety large factors to be used.

In this report, fatigue design principles are utilized to estimate service life and capacity of wave power components with the special circumstances they are subjected to. With the developed fatigue evaluation method, the uncertainty of the results is quantified with the Variation Mode and Effect Analysis (VMEA ). This allows proper safety factors to be established with regard to a required service life or strength. The factors that cause the most uncertainty can also be identified giving opportunity to reduce the uncertainty, which can lead to more efficient and optimized products.

Two cases are analysed for two wave power companies: Corpower and Ocean Harvesting Technologies. For the first company, a rope that transfers large forces from a buoy is analysed. For the second company, a roller chain or a rack and pinion that transmits power to a large weight in a gravity accumulator is analysed.
1 Introduction

In order to find a suitable procedure for solving future reliability assessments we will here evaluate the VMEA procedure for two company applications within the Buoy to grid group. The goal is to combine the system and mechanism knowledge at the companies with the competence of statistics and fatigue mechanism at SP. The two companies are Corpower and Ocean Harvesting Technologies (OHT).

OHT is developing a generic solution for wave energy conversion suitable to different buoy concepts for wave energy capture. The OHT Power take-off system uses a simple power balancing mechanism comprising a clever gearbox and a gravity accumulator to convert the highly intermittent and irregular mechanical power captured from the ocean waves into a smooth torque and angular velocity suitable for a conventional generator. The component connecting the gravity accumulator is subjected to fatigue and wear and the strength with respect to these failure mechanisms will be the primary problem to study in this report.

For Corpower, the mooring rope is subjected to large repeated tensile forces and tensile spectrum fatigue is the subject of the study.

The studies are based on available data from the companies and from literature. These data are quite limited and the reliability evaluation will therefore include a lot of uncertainties. However, using the VMEA approach all uncertainties can be taken into account regardless their magnitudes and sources and these first analyses can, firstly point out areas that demands more knowledge, and secondly assign proper safety factors for the present knowledge.

2 The VMEA approach to reliability

In (Thies, Smith och Johanning 2012) the following is stated

“The main source of failure rate uncertainty for Marine Energy Converters is the lack of knowledge with regard to inadequate or missing experimental and operational data.”

This makes design difficult and a reliability method must be able to take also “lack of knowledge” into account. The Variation Mode and Effect Analysis method includes tools for this. Namely, by simplifying the statistical modelling to second moment statistics it is possible to quantify and compare all identified sources of uncertainty, even if some of them must be based on solely engineering judgements.

2.1 The load/strength approach to reliability

For reliability regarding mechanical failure we use the concept of load/strength interaction. This means that we study separately a) the outer load acting on the structure to be designed and b) the strength, or resistance, of the structure. The aim is to design the structure to assure that the strength exceeds the load for future usage.

Engineering design for reliability must take into account a lot of uncertainties. They include

1. random variation and uncertainties in material, external loads and geometry,
2. possible model errors,
3. vaguely known sources of deviations from expected performance

In Figure 1, these uncertainties are illustrated schematically. The structure to be designed is drawn as an irregular picture, the model of the structure as a square and the resulting model errors as red deviations. The inputs to the structural performance are illustrated as
arrows, external loads from below, internal controlled features from the right and uncontrolled influences from above. For the defined load and strength variables the approximation symbols illustrate the randomness and uncertainty in these inputs and the barrier from above illustrates the absence of rational models for these influences.

Using nominal values on all inputs and the best physical models available, results in estimates of model strength and model load. All the present uncertainties need then to be considered in order to find how much the model strength need to exceed the model load, in the figure denoted as the safety factor $\gamma$.

![Diagram illustrating uncertainty in the design process](image)

**Figure 1 Illustration of the uncertainty influence in the design process.**

There are numerous methods to find a proper safety factor which can be categorized in two groups:

1. Combining safety factors on essential sources by calculating a worst case, based on worst cases for all essential inputs.
2. Assign statistical distributions to all essential sources, perform a probabilistic evaluation and use a pre-determined low probability of failure to find a proper safety factor.

In engineering practice, the most common methodologies belong to the first group. Advantages of these methods include a) the power of the lot of former experience collected in established safety factors for components and b) the simplicity of application. One apparent drawback is the tendency of over design by combining a large number of "worst cases", a combination that often has very low probability of occurrence. Other drawbacks are the lack of knowledge of the actual probability of failure and what measures that could be taken to reduce it.

The second group of methods have the principal advantage to monitor the whole process of uncertainty propagation through the models for load and strength, and result in quantified probabilities of failure. However, this ideal situation can only be achieved in case all the essential uncertainty causes can be assigned statistical distributions, which seldom is the case. In engineering practice this unfortunately often results in the
application of advanced statistical methods with inputs that contain considerable subjectivity.

As illustrated in Figure 1 there are many sources of uncertainty that apparently cannot objectively be assigned a statistical distribution. For some of the random variation sources, this can perhaps be achieved, but for possible model errors, uncertainties in observation representativity or the vaguely known sources statistical distributions are unknown, and the best that can be done is to assess rough estimates of their uncertainty.

2.2 The actual VMEA approach

The VMEA (Variation Mode and Effect Analysis, see (Chakhunashvili, Johansson och Bergman 2004) approach belongs to the second group. The problem of weak knowledge of input statistical distributions is considered by reducing the statistical complexity to second moment statistics. This means that the needed input information about the sources of uncertainty is reduced to a minimum, namely to a scalar measure of the standard deviation of each source.

For many engineering properties, such as fatigue life, standard deviation is a bad measure of variation since it is not constant for various lives. This is highly improved by working in logarithmic scale. For instance, fatigue life as a function of load level is usually modelled as a linear relationship in logarithmic scale, with experimental scatter with approximately constant variance for all interesting load levels. A lot of engineering models are linear in logarithmic scale and the log transformation should therefore be a rule rather than an exception.

An extra advantage of using logarithmic scale is that if we use the natural logarithm, the standard deviation can be interpreted as a coefficient of variation (the relative standard deviation) for the original variable, a standard deviation of $\ln\text{life}$ in fatigue of 0.12 is interpreted as a 12% coefficient of variation.

Using the logarithmic transformation in the load strength case, we aim to calculate the overall standard deviation of the difference between $\ln\text{strength}$ and $\ln\text{load}$. Once the standard deviations (and if relevant, correlation coefficients) for the uncertainty sources are available they can be combined to such an overall uncertainty by using the "Gauss approximation formula". In the simplest case, when no correlations need to be taken into account, this means that each standard deviation is squared and multiplied with the square of its sensitivity coefficient to form an uncertainty component. All these components are summarized to an overall variance. Taking the square root of this variance results in a final estimate of the uncertainty in the load/strength interaction.

In order to justify the use of standard deviations for the influential variables, they should be constant within the magnitudes of interest. In case the variation of a variable is proportional to its value, then a logarithmic transformation should be used for the variable. For instance, a geometrical tolerance that is symmetric around its nominal value, may be used as it is, since deviances from the nominal value can be assumed to be good described by the linear tolerance interval. A mechanical strength variable on the other hand is usually give with a tolerance interval by means of a percentage and it should be transformed to its logarithm in the VMEA evaluation.

The sensitivity coefficient is a number that relates the actual influential variable to the variable under investigation, the difference between $\ln\text{strength}$ and $\ln\text{load}$. In case the relationship has a mathematical expression, the sensitivity coefficient is the partial derivative with respect to the actual influential variable. In other cases the sensitivity coefficient is determined by numerical or experimental differentiation.
Reducing the relationship to a single number is an approximation that assumes a fairly linear relationship within the interval of interest, which often is fulfilled to enough precision in the logarithmic scale.

There is a powerful theorem in statistics, the central limit theorem, that says that if many random variables of the same order of magnitude are combined to a sum, then the sum tends to a normal distribution. This means, that if the pre-requisites of this theorem are fulfilled in our case, then the sum of all uncertainties would be normally distributed and our estimate of the overall standard deviation would totally determine the distribution around the nominal value. However, this is usually not the case, but the total uncertainty is often dominated by only one or a few sources. Still, it has been recognized, that the theorem result may be approximately valid in the central part of the distribution. This means, that it usually is not possible to predict the distribution far out in the tails, in our case for small probabilities of failure, but the result is indeed applicable down to, say, 5% probability of failure, corresponding to the mean value reduced by 1.64 standard deviations.

To summarize so far, the VMEA approach combines all sources of uncertainties to an overall uncertainty in the difference between logarithmic strength and load. This uncertainty makes it possible to find a safety factor that assign at most 5% failure rate.

The lack of knowledge about the true distribution of many uncertainty sources makes it necessary to complement the statistical analysis with extra safety considerations. Namely, in case the structure needs to be designed for less failures than 5%, the statistical tool gives no trustful answers. The VMEA approach need to be extended, which must be done based on other considerations than statistical. Therefore, we have introduced an extra safety factor that is chosen on engineering judgement, based on the magnitude of safety demands, economy, and other considerations, (Svensson och Johannesson 2013).

For the actual approach, reliability of sea based energy converters, there may be sufficient to design for the failure rate of 5% since risk for serious human injury should be negligible and the cost of maintenance may be less than the cost of a safer design.

2.3 Modelling fatigue for VMEA

As outlined above, the VMEA approach evaluates the uncertainty in the difference between strength an load. In order to apply this for mechanical fatigue we must first define these properties: strength and load. In engineering applications the fatigue strength is often defined as the load range or amplitude that correspond to the fatigue life of a certain number of cycles. We adopt this definition here, see details in (Svensson och Johannesson 2013). The mathematical formulae can be found in appendix A.

Fatigue strength: The stress range that corresponds to the fatigue life of two million cycles, App.A (3).

Comparing strength with load we need a corresponding definition of load. If the target life for the structure was two million cycles, then the fatigue load variable simply would be equal to the stress range expected to occur in service. But, if the target life is different, then the service stress range must be transformed to correspond to two million cycles to be comparable. Therefore we use the following definition:

Fatigue load: The structure stress range scaled by a factor equal to the “beta root” of the target life in cycles over two million, where beta is the fatigue exponent, App. A (4).

These definitions assumes a constant stress range over the whole life of the structure. In the sea based energy converter case we must also take the variability in stress into account. This is done by using the Palmgren-Miner rule for damage accumulation. Using
this rule it is possible to find a convenient representation of a time series of stresses. First the time series is counted with respect to its damaging cycles, using the Rain Flow Count procedure, and then a stress norm is defined corresponding to the constant amplitude stress range.

**Fatigue beta norm:** The beta root of the average of stress ranges raised to the power of beta, App. A (1)

Both load and strength by these definition will be affected by the fatigue exponent beta. Therefore, in the evaluation of the VMEA for variable amplitude fatigue, the uncertainty components of load and strength are calculated for a fixed beta and the uncertainty in beta is added as a separate uncertainty component.

### 3 The Corpower case

#### 3.1 Component under study

For the Corpower case the weak link to be studied is the rope that connects the buoy with the hydraulic energy converter. The rope is subjected to a tensile force that varies with wave height in combination with the controlled counter-force from the energy converter.

As a basis for the reliability study the company has provided force measurements on the rope for different controlled scaled wave scenarios. These measurements have been performed with buoy control close to the intended service usage for different significant wave heights. Using established scale factors, these measurements may be used to get an estimate of the service force time history.

Strength experiments have not been performed for this specific application, but estimates must be found from the open literature. This is done for two different types, a polyester rope and a steel wire.

#### 3.2 Reliability evaluation

In order to evaluate the reliability we use the load/strength concept outlined above, i.e. the quotient between strength and load should exceed unity, or after logarithmic transformation, the difference between the log-strength and the log-load should be positive.

First, the nominal external load and component strength will be estimated. The load will be estimated from the pool experiments, scaled to the real environment, and weighted by the expected frequency of sea states. The strength will be estimated from data found in literature.

##### 3.2.1 Assess uncertainties

The nominal values of load and strength will be uncertain to a large extent and we will assess these uncertainties by means of their standard deviations and further quantify their influence of the difference between log-strength and log-load.

Some of the uncertainties are regarded as scatter, meaning that the source is random, and therefore usually unavoidable. Other uncertainties are due to lack of knowledge and are thereby possible to reduce by further investigations.

#### 3.2.2 Design for twenty year survival

Using the concept of equivalent strength and load, outlined above, we will estimate the uncertainty of the predicted life for the chosen target of twenty year of usage. By
choosing the risk of failure to 5% we will then calculate the necessary rope or wire dimension for a reliable design.

3.3 Nominal load

Scaled tests have been performed in a pool with artificially generated waves. A buoy, controlled by an anchoring wire and equipped with a number of measurement gages was studied. Two series of measurements are available here,

1. a buoy is locked in different positions and subjected to different controlled wave heights in order to find the maximum force that is expected to occur (scale factor 60),
2. a buoy is controlled in a manner that is similar to the intended service situation, called latching (scale factor 30), for the Corpower equipment in order to find the force distribution for different wave heights.

![Example of force signal for three latch cycles.](image)

In order to find an estimate of the equivalent load that a rope in service is subjected to, we need to

1. estimate the equivalent load for each measured wave height,
2. use scale factors to find the corresponding service load and interpolate to get the equivalent load as a function of wave height,
3. use an estimated distribution of wave heights in service to get a life time equivalent load.

3.3.1 Pool measurements

A special equipment has been arranged in a pool in order to study the behaviour of the latch work cycles including a buoy and an artificial counterforce. Waves have been generated corresponding to different significant wave heights, the equipment has been “latched” in different ways, and data been collected. The data used here are according to Table 1, row numbers 15, 17, 19 and 21.
In order to find a reasonable load representation for our strength/load investigation we have used the measured load data. A short part of a typical force time signal is illustrated in Figure 2, covering approximately three latch cycles.

Commonly used fatigue theory for metals suggests a time independence for damage accumulation, i.e. only the sequence of minima and maxima is assumed to influence the damaging process. For the case when the component at stress is a textile fibre rope, it may be doubtful to use established fatigue theory. Perhaps the leading damage mechanism is rather due to wear and ageing. However, our nominal analysis will be based on classical fatigue methodology, since there are no apparent alternatives available and the doubt be a part of the uncertainty assessment.

### 3.3.2 Rain flow count

The first step in the load analysis is to extract damaging load cycles from the time signal. The best practise with respect to fatigue is to use the *Rain Flow Count* procedure, extracting hypothetical evolving hysteresis cycles. Figure 3 illustrates the result for *Rain Flow Count* of the time signals from around 100 latch cycles in the four different sea states.

![Figure 3 Results from the rain flow count: The number of cycles larger than specified force.](image)

The diagram is a cumulative count of cycle ranges and shows, for instance for latch 15, a distribution of about 85 large cycles around the range 80 N, but also a, probably not negligible, number of cycles with ranges below 40 N.
3.3.3 Possible mean value influence

The range representation, used here\(^1\), omits the mean values of the cycles, which may be important for the damaging process. In fatigue the mean value condition is often characterized by the R-ratio, defined as the ratio between the minimum stress and the maximum stress for a cycle. An overall R-ratio is here defined based on the minimum and maximum registered force for the whole measured time signal, which in our case it will reflect the position of the main latch cycle. For the four cases we found the following R-ratios:

<table>
<thead>
<tr>
<th>Latch 15</th>
<th>Latch 17</th>
<th>Latch 19</th>
<th>Latch 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>0.12</td>
<td>0.28</td>
<td>0.02</td>
</tr>
</tbody>
</table>

3.3.4 Transform to fatigue equivalent load

The waves and the buoy size in the pool experiments were chosen in scale 1:30 compared to service environment and the corresponding scaling must be done for the measured force.

Also, in order to compare load and strength for variable fatigue we must relate the measurements to the target life.

3.3.5 Scaling

The theory for scaling is summarized in Table 3 of Froude scale factors and for the present case this means that \(s\) in the table equals 30. The force then scales by the scale factor raised to three and the wave period to the square root of the scale factor.

Table 3 Scale factor transformation for different properties in wave analysis.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>wave height and length</td>
<td>(s)</td>
</tr>
<tr>
<td>wave period</td>
<td>(s^{0.5})</td>
</tr>
<tr>
<td>wave frequency</td>
<td>(s^{-0.5})</td>
</tr>
<tr>
<td>power density</td>
<td>(s^{2.5})</td>
</tr>
<tr>
<td>linear displacement</td>
<td>(s)</td>
</tr>
<tr>
<td>angular displacement</td>
<td>1</td>
</tr>
<tr>
<td>linear velocity</td>
<td>(s^{0.5})</td>
</tr>
<tr>
<td>angular velocity</td>
<td>(s^{-0.5})</td>
</tr>
<tr>
<td>linear acceleration</td>
<td>1</td>
</tr>
<tr>
<td>angular acceleration</td>
<td>(s^{-1})</td>
</tr>
<tr>
<td>mass</td>
<td>(s^{3})</td>
</tr>
<tr>
<td>force</td>
<td>(s^{3})</td>
</tr>
<tr>
<td>torque</td>
<td>(s^{4})</td>
</tr>
<tr>
<td>power</td>
<td>(s^{3.5})</td>
</tr>
<tr>
<td>linear stiffness</td>
<td>(s^{2})</td>
</tr>
<tr>
<td>angular stiffness</td>
<td>(s^{4})</td>
</tr>
<tr>
<td>linear damping</td>
<td>(s^{2.5})</td>
</tr>
<tr>
<td>angular damping</td>
<td>(s^{4.5})</td>
</tr>
</tbody>
</table>

\(^1\)The rain flow count procedure also gives the mean value for each counted cycle, but in order to use this information for damage calculation a reasonable theory for the mean value influence is needed, which is absent in our case.
3.3.6 Equivalent fatigue load

Based on the rain flow count of the four latch force signals we calculate the fatigue design properties according to formula (1) for the fatigue norm and formula (4) for the equivalent load. Apart from the force signal we used the following numbers:

- The fatigue exponent $\beta$ is (see the strength chapter below) 5.45 for the polyester rope and 4.8 for the steel wire.
- The target life is put to 20 years.
- The duration for each counted force signal is 2000 seconds which scaled with the square root of 30 corresponds to 30.4 hours.

The resulting fatigue design properties for the load are:

<table>
<thead>
<tr>
<th></th>
<th>Latch 15 (MN)</th>
<th>Latch 17 (MN)</th>
<th>Latch 19 (MN)</th>
<th>Latch 21 (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norm ($\beta=5.45$)</td>
<td>0.78</td>
<td>1.04</td>
<td>0.60</td>
<td>1.28</td>
</tr>
<tr>
<td>L_{eq} ($\beta=5.45$)</td>
<td>2.59</td>
<td>3.45</td>
<td>2.01</td>
<td>4.26</td>
</tr>
<tr>
<td>Norm ($\beta=4.8$)</td>
<td>0.68</td>
<td>0.91</td>
<td>0.53</td>
<td>1.12</td>
</tr>
<tr>
<td>L_{eq} ($\beta=4.8$)</td>
<td>2.66</td>
<td>3.55</td>
<td>2.07</td>
<td>4.38</td>
</tr>
</tbody>
</table>

3.3.6.1 Expand with expected wave distributions

The equivalent loads in the table above differ depending on the sea state, here represented by the two numbers “significant wave height” and “wave period”. In service, the energy converter will be subjected to a large spectrum of sea states and the long term equivalent load is the average effect on these states. In order to find this overall equivalent load we proceed as follows:

1. Estimate a model for equivalent load as a function of significant wave height and period.
2. Apply this model to each cell of a measured two dimensional distribution of sea states.
3. Pool the sea states to a proper weighted average by formula (5) in appendix A.

Using our four experimental results we estimated the parameters in a simple linear model: The logarithm of the equivalent load is modelled as a linear function of the logarithm of significant wave height and the logarithm of period, see the output from the statistical software R below.

```
Call: lm(formula = log(y) ~ log(T) + log(H))

Residuals:
      1      2      3      4
  0.06867 -0.11221 -0.05329  0.09683

Coefficients: Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.1672   1.5122  3.4358  0.158
log(T)       1.4777   0.5814 -2.5424  0.239
log(H)       1.5152   0.9995  1.5152  0.213

Residual standard error: 0.1718 on 1 degrees of freedom
Multiple R-squared: 0.9088, Adjusted R-squared: 0.7264
F-statistic: 4.961 on 2 and 1 DF, p-value: 0.302
```
In the model the equivalent load is denoted $y$, the significant wave height $H$ and the wave period $T$. The deviations between the model loads and the measured loads gives a rough estimate of the model error. The standard deviation for this error is 0.17, i.e. 17%.

The expected sea state distribution is based on measurements outside Portugal and is shown in Table 4 and the linear model was used to predict the equivalent load for each cell. The overall equivalent load was then estimated by weighting these values with their relative frequency according to the table. This is done with formula (5) in appendix A with the result:

**Overall equivalent load for fatigue exponent 5.45: 3.62 MN**

**Overall equivalent load for fatigue exponent 4.8: 3.50 MN**

<table>
<thead>
<tr>
<th>$H_{reg}$</th>
<th>$H_s$</th>
<th>$H_{reg}$</th>
<th>$H_s$</th>
<th>$H_{reg}$</th>
<th>$H_s$</th>
<th>$H_{reg}$</th>
<th>$H_s$</th>
<th>$H_{reg}$</th>
<th>$H_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.5</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>0.71</td>
<td>0.0357</td>
<td>0.033</td>
<td>0.0312</td>
<td>0.0317</td>
<td>0.174</td>
<td>0.594</td>
<td>0.329</td>
<td>0.377</td>
<td>0.107</td>
</tr>
<tr>
<td>1.06</td>
<td>1.0351</td>
<td>0.050</td>
<td>0.018</td>
<td>0.025</td>
<td>0.0114</td>
<td>0.024</td>
<td>0.009</td>
<td>0.019</td>
<td>0.018</td>
</tr>
<tr>
<td>1.14</td>
<td>2.0347</td>
<td>0.067</td>
<td>0.024</td>
<td>0.033</td>
<td>0.142</td>
<td>0.403</td>
<td>0.251</td>
<td>0.275</td>
<td>0.210</td>
</tr>
<tr>
<td>1.77</td>
<td>2.0595</td>
<td>0.085</td>
<td>0.029</td>
<td>0.042</td>
<td>0.82</td>
<td>0.293</td>
<td>0.235</td>
<td>0.103</td>
<td>0.064</td>
</tr>
<tr>
<td>2.12</td>
<td>2.0707</td>
<td>0.100</td>
<td>0.035</td>
<td>0.050</td>
<td>0.52</td>
<td>0.105</td>
<td>0.079</td>
<td>0.098</td>
<td>0.010</td>
</tr>
<tr>
<td>2.47</td>
<td>3.0895</td>
<td>0.117</td>
<td>0.041</td>
<td>0.058</td>
<td>0.47</td>
<td>0.140</td>
<td>0.081</td>
<td>0.075</td>
<td>0.042</td>
</tr>
<tr>
<td>2.83</td>
<td>4.0792</td>
<td>0.133</td>
<td>0.047</td>
<td>0.067</td>
<td>0.10</td>
<td>0.505</td>
<td>0.070</td>
<td>0.029</td>
<td>0.006</td>
</tr>
<tr>
<td>3.18</td>
<td>5.0086</td>
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<td>0.053</td>
<td>0.075</td>
<td>0.10</td>
<td>0.527</td>
<td>0.070</td>
<td>0.029</td>
<td>0.006</td>
</tr>
<tr>
<td>3.54</td>
<td>5.3178</td>
<td>0.167</td>
<td>0.059</td>
<td>0.083</td>
<td>2.16</td>
<td>0.249</td>
<td>0.042</td>
<td>0.019</td>
<td>0.016</td>
</tr>
<tr>
<td>3.89</td>
<td>5.6364</td>
<td>0.183</td>
<td>0.065</td>
<td>0.092</td>
<td>3.16</td>
<td>0.198</td>
<td>0.019</td>
<td>0.016</td>
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<tr>
<td>4.24</td>
<td>6.1414</td>
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<td>0.071</td>
<td>0.100</td>
<td>1.01</td>
<td>0.219</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>4.68</td>
<td>7.0153</td>
<td>0.217</td>
<td>0.077</td>
<td>0.108</td>
<td>6.01</td>
<td>0.413</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>5.05</td>
<td>7.4649</td>
<td>0.230</td>
<td>0.082</td>
<td>0.117</td>
<td>3.2</td>
<td>0.232</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
</tr>
</tbody>
</table>

**Table 4 The used distribution of sea states.**

3.4 Nominal strength

We have no specific test results for the fatigue strength of the components under study, but we must find a first estimate from literature data.

3.4.1 Polyester rope

In (Flory och Banfield 2010) we find some strength assessments for polyester ropes. Experiments show that there is a significant mean load dependence with shorter lives for low R-ratios. This is because the movement between the individual fibres increases at low R-ratios causing more wear. Complete fatigue life curves for different R-ratios are not available. We will here use the Wöhler curve for a mean load equal to 40% of the braking load as a first approximation. From the diagram in the paper we estimate the slope to 5.45 and we can then can extract the equivalent load (according to formula (3) in appendix A): 45% of the ultimate tensile strength. The coefficient of variation in life is estimated as 60%, based on 23 observations.

**The nominal fatigue strength for polyester rope is 45% of UTS**

For a polyester rope with the ultimate tensile strength equal to 8.1 MN the nominal fatigue strength would be 3.65 MN and just exceeds the nominal equivalent load found above.
3.4.2 Steel wire

For steel wires at sea there are large experiences from the off shore and ship applications. From the DNV (Det Norske Veritas) (Det Norske Veritas Offshore Standard 2010) recommendations we find the following excerpt:

**F 200 Fatigue properties**

201 The following equation can be used for the component capacity against tension fatigue:

\[ n_c(s) = a_D s^{-m} \]

This equation can be linearised by taking logarithms to give:

\[ \log(n_c(s)) = \log(a_D) - m \cdot \log(s) \]

\[ n_c(s) = \text{the number of stress ranges (number of cycles)} \]
\[ s = \text{the stress range (double amplitude) in MPa} \]
\[ a_D = \text{the intercept parameter of the S-N curve} \]
\[ m = \text{the slope of the S-N curve} \]

The parameters \( a_D \) and \( m \) are given in Table F1 and the S-N curves are shown in Fig.6.

<table>
<thead>
<tr>
<th>Table F1 S-N Fatigue Curve Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-N curve</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>Stud chain</td>
</tr>
<tr>
<td>Studless chain (open link)</td>
</tr>
<tr>
<td>Stranded rope</td>
</tr>
<tr>
<td>Spiral rope</td>
</tr>
</tbody>
</table>

For spiral rope we then obtain the equivalent strength (using formula (3) in appendix A) 189 Mpa and the slope 4.8.

As far as we can understand, the DNV standard curve represents a “design load”, which is constructed two standard deviations below the median curve. Since we want to find the nominal strength for our reliability evaluation, we need to transform the curve back to the median. However, the standard deviation is not given. To solve this problem, we found a fatigue test result for spiral strands in the open literature, showing the standard deviation 0.54 for the natural logarithm of life. This standard deviation for log life transforms to log stress by division with the fatigue exponent 4.8 giving the standard deviation 0.11 for the log strength. We transform the given DNV design equivalent strength to the nominal equivalent strength by increasing it with two standard deviations:

**The nominal equivalent strength for the steel spiral rope is 237 MPa**

Since we want to compare to our equivalent load as force in MN, the equivalent strength above must be multiplied by the sectional area. For a rope with 138 mm diameter, for instance, the equivalent strength by means of force is 3.54 MN and would exceed the nominal equivalent load found above.
3.5 Uncertainties

For a safe design it is not sufficient to use the nominal values found above, but scatter and uncertainties must be taken into account to find proper safety factors. Using the VMEA approach we here make assessments of the sources of uncertainty that are judged to be dominating. Each source is given a coefficient of variation, either as a measure of scatter or uncertainty.

The influence of the source on the actual property of interest, the difference between log strength and log load, is estimated by means of a sensitivity coefficient. However, when possible, the coefficient of variation is judged directly by means of the property of interest, making the sensitivity coefficient equal to unity.

When uncertainty components are based on judgements, they are assumed to be judgements of a “worst case” deviance. In order to transform such a measure to a statistical standard deviation we assign a uniform distribution to the judgement and use the “worst case” half range divided by the square root or three to get a standard deviation.

When uncertainty components are based on data, then the statistical t-distribution is used to take also the standard deviation uncertainty into account. This is done by multiplying the standard deviation with correction factor based on the number of samples behind the estimate.

3.5.1 Load

Our nominal equivalent load is based on scaled experiments and extrapolated to real environments by scaling, modelling and summation over sea states. We can identify the following sources of uncertainty.

Scaling and experimental equivalence. The scale factors used are judged to be sufficiently accurate, but the artificially generated waves differ of course from waves in service. This uncertainty in load due to this influence is judged to be max 2%.

Friction. The force measurements in the experiments are not taken immediately next to the buoy, but a wheel influenced by friction disturb the measures. The influence of this is not available at this stage. Therefore we must add an uncertainty for the frictional influence, which is roughly judged to be max 5%.

Extrapolation model. In order to calculate the equivalent load for other sea states than the ones measured, we used a simple linear model with an calculated model error of 17%. However, since our overall equivalent load is based on a large range of sea states, this model is assumed to be averaged out to a large extent. However, it is difficult to find a good measure of this. An indication is found by calculating the error if only the observed cells are used. We then find a deviance between model overall load and actual overall load of 4%, which we use directly as an maximal uncertainty component.

Relevance of rain flow count. The force time signals have been analysed in accordance with established fatigue theory and reduced to a simple count of force ranges. For the steel wire case, this theory is judged to be relevant, and the model error be negligible. For the polyester rope, the “fatigue” mechanisms are probably dominated by wear and ageing and the established fatigue theory is doubtful. The uncertainty is therefore judged to be 20% for the polyester rope case.

Sampling error in experiments. The sampling error in the pool measurements may be estimated by comparing the calculated equivalent load for two halves of each force signal. By calculating the standard deviation of the results for each pair of results we obtain the following coefficients of variation.
<table>
<thead>
<tr>
<th>Latch 15</th>
<th>Latch 17</th>
<th>Latch 19</th>
<th>Latch 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.052</td>
<td>0.004</td>
<td>0.045</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Taking the pooled standard deviation (the square root of the mean of squares) as a “sampling error” estimate gives 0.04. This estimate is based on four degrees of freedom (8-4), resulting in a t-correction factor of 1.3.

**Sampling of service environment.** The equivalent load is based on measurements of wave characteristics outside Portugal, which is judged to be of sufficient accuracy.

### 3.5.2 Strength

**Scatter.** The scatter in fatigue strength is always large and in our case it can was found from our literature sources.

For the polyester rope the estimated scatter coefficient of variation in life is 60%. The estimate is based on 23 observations giving 21 degrees of freedom and a t-correction factor of 1.06.

From the model for fatigue life, formula (2) in appendix A, it is seen that the sensitivity coefficient for log load is $1/\delta L$ since the formula is linear in log scale.

For the steel wire the coefficient of variation in fatigue life was estimated to 54%, based on 24 observations (Alani and Raoof (1997). The t-correction factor is then 1.06 and the sensitivity as for the polyester rope, $1/\delta L$.

**Parameter uncertainty.** The uncertainty in the estimated fatigue strength from test results is connected to the scatter standard deviation. According to statistical theory it is the scatter standard deviation divided by the number of observations, which can be applied here for the polyester case. An additional uncertainty is the relevance of the literature data, which is judged to be max 20% in life.

For the steel wire case, we found the estimated fatigue strength from the DNV standard. We judge the uncertainty in this value to be max 20% in life.

**Model error, linearity.** The fatigue strength is here modelled by the Basquin equation, formula (2) in appendix A, which is linear in log scale. This is a simplification but the resulting model error will to a certain extent be reflected in the estimated scatter around the fitted curve. We here assume that additional errors are negligible.

**Model error, Palmgren-Miner.** The used theory for damage accumulation is well known to give considerable errors for fatigue in steel.

For steel it is a common practice to assign an uncertainty of at least 50% in life for the damage accumulation model error. However, the loads caused by the latching procedure can be regarded as dominated by the largest cycles, see Figure 3, and may be regarded as a “almost” constant amplitude load, in particular since the fatigue exponent is large, namely about five. For steel we therefore judge the model error to be max 10% in life.

For polyester no information about the cumulative damage mechanisms is available at this stage, but with the same argument as above, we almost have a constant amplitude load. Still, the lack of knowledge force us to estimate the error to max 50% in life.

**Model error, mean value influences.** As mentioned above the mean value influence for polyester rope is considerable and the force measurements show R-ratios down to almost zero. This suggest a large uncertainty for this type of model error, here judged to max 200% in life.
For the steel wire, we do not expect the mean value influence to be large, but judge that the uncertainty of max 10% in life is reasonable.

**Laboratory uncertainty.** The components under study will be used in salt water and parts of the rope will be used in the vicinity of the sea surface, which could be expected to be the most aggressive environment. The strength curves that we have used are established in laboratory for sea conditions and should to a certain extent reflect the influence of aggressive environment. Still there is an uncertainty for the equivalence between laboratory conditions and service which we judge to be 5% in load.

### 3.5.3 Reliability evaluation

Using the nominal values and the uncertainty components we can now evaluate the reliability by the VMEA approach, see the spread sheet for the polyester rope below:

#### Load-Strength Evaluation for a wave power component

Polyester rope connection between buoy and energy converter  
Target life: 20 years

**Evaluation of Uncertainties**

<table>
<thead>
<tr>
<th>Uncertainty components</th>
<th>Sensitivity coefficient</th>
<th>Uncertainty</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Palmgren-Miner error</td>
<td>x</td>
<td>0.183</td>
<td>0.115</td>
</tr>
<tr>
<td>Mean value influence</td>
<td>x</td>
<td>0.183</td>
<td>0.115</td>
</tr>
<tr>
<td>Laboratory uncertainty</td>
<td>x</td>
<td>1.000</td>
<td>0.029</td>
</tr>
<tr>
<td><strong>Total Strength uncertainty</strong></td>
<td></td>
<td>0.117</td>
<td>0.221</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pool measurements, scatter</td>
<td>x</td>
<td>1.000</td>
<td>0.040</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>x</td>
<td>1.000</td>
<td>0.023</td>
</tr>
<tr>
<td><strong>Total Load uncertainty</strong></td>
<td></td>
<td>0.052</td>
<td>0.204</td>
</tr>
<tr>
<td>Wöhler Exponent</td>
<td>x</td>
<td>0.200</td>
<td>0.500</td>
</tr>
<tr>
<td><strong>Total uncertainty</strong></td>
<td></td>
<td>0.128</td>
<td>0.317</td>
</tr>
</tbody>
</table>

**Reliability Evaluation**

<table>
<thead>
<tr>
<th>Input</th>
<th>Result</th>
<th>Result (log-scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median strength [MN]</td>
<td>6.35</td>
<td>Safety factor 1.75</td>
</tr>
<tr>
<td>Median Load [MN]</td>
<td>3.62</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Evaluation - Extra safety factor</th>
<th>Variation safety factor 1.75</th>
<th>Extra dist. 0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required extra safety factor</td>
<td>Extra safety factor 1.00</td>
<td>Variation dist. 0.56</td>
</tr>
</tbody>
</table>
The uncertainty components are here listed for strength, load and the Wöhler exponent and added together quadratically to give the overall uncertainty which is seen in the row for Total uncertainty and equals 0.342. This number represents the uncertainty in the difference between the logarithms of nominal strength and load and can be interpreted as 34% uncertainty in the quotient between strength and load.

Assuming a normal distribution for the difference the lower 5% quantile in the distribution is 1.64 times this uncertainty, giving 0.56, denoted as “Variation distance” and this is the distance we need for 95% probability of survival between log strength and log load. Taking the anti-log gives the corresponding “Variation safety factor” that equals 1.75.

We choose not to add any extra safety and find that the demanded strength is 6.35 MN. Since the nominal fatigue strength was found to equal 45% of the UTS strength, this means that we demand the UTS to be 14.1 MN. This is a very high strength that probably is unrealistic to achieve. Designing for 20 years may be unrealistic.

### 3.5.4 Design for one year

The approach for fatigue load/strength interaction used here can easily be adjusted to another target life. As seen from formula (4) in appendix A the equivalent load is proportional to the target life raised to the inverse of the fatigue exponent. This means that if we want to adjust our reliability calculation to one year we just need to divide the 20 year equivalent load with 20 raised to one over 5.45 which gives a factor 1.72. The median equivalent load decreases from 3.62 MN to 2.09 MN and with the safety factor 1.75 we then demand the UTS: 8.1 MN.

**Demanded ultimate tensile strength for a polyester rope for 95% survival for one year: 8.1 MN**

This is still a high number and we next make the corresponding calculation for a steel wire:
Load-Strength Evaluation for a wave power component

Steel wire connection between buoy and energy converter

Target life: One year.

Evaluation of Uncertainties

<table>
<thead>
<tr>
<th>Uncertainty components</th>
<th>Sensitivity coefficient</th>
<th>Correction factor</th>
<th>Standard deviation</th>
<th>Scatter</th>
<th>Uncertainty</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Strength scatter</td>
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<td>1,060</td>
<td>0,540</td>
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<td></td>
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<tr>
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<td>1,000</td>
<td>0,200</td>
<td>0,042</td>
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<tr>
<td>Adjustment uncertainty</td>
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<td>1,000</td>
<td>0,100</td>
<td>0,021</td>
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<td>Reference data relevance</td>
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<td>0,100</td>
<td>0,100</td>
<td></td>
</tr>
<tr>
<td>Mean value influence</td>
<td>x</td>
<td>1,000</td>
<td>1,000</td>
<td>0,050</td>
<td>0,050</td>
<td></td>
</tr>
<tr>
<td>Laboratory uncertainty</td>
<td>x</td>
<td>1,000</td>
<td>1,000</td>
<td>0,029</td>
<td>0,029</td>
<td></td>
</tr>
<tr>
<td>Total Strength uncertainty</td>
<td></td>
<td>0,119</td>
<td></td>
<td></td>
<td></td>
<td>0,125</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0,172</td>
</tr>
<tr>
<td>Pool measurements, scatter</td>
<td>x</td>
<td>1,000</td>
<td>1,300</td>
<td>0,040</td>
<td>0,052</td>
<td></td>
</tr>
<tr>
<td>Scaling</td>
<td>x</td>
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<td>1,000</td>
<td>0,012</td>
<td>0,012</td>
<td></td>
</tr>
<tr>
<td>Distribution of Hf</td>
<td>x</td>
<td>1,000</td>
<td>1,000</td>
<td>0,014</td>
<td>0,014</td>
<td></td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>x</td>
<td>1,000</td>
<td>1,000</td>
<td>0,023</td>
<td>0,023</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td>x</td>
<td>1,000</td>
<td>1,000</td>
<td>0,029</td>
<td>0,029</td>
<td></td>
</tr>
<tr>
<td>Total Load uncertainty</td>
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<td></td>
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<td>0,066</td>
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<tr>
<td>Wöhler Exponent</td>
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<td>1,000</td>
<td>0,500</td>
<td>0,100</td>
<td></td>
</tr>
<tr>
<td>Total Exponent uncertainty</td>
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<td></td>
<td></td>
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<td>0,100</td>
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<tr>
<td>Total uncertainty</td>
<td></td>
<td>0,130</td>
<td></td>
<td></td>
<td></td>
<td>0,210</td>
</tr>
</tbody>
</table>

Reliability Evaluation

Median strength [MN] 2,25
Median Load [MN] 1,60

Evaluation - Extra safety factor

Here the total uncertainty is lower due to better knowledge about the failure mechanisms, and the demanded safety factor is 1.41. Recalculation of the equivalent load to one year as above and using no extra safety results in a demand of a 110 mm diameter steel wire.

3.6 Extreme loads

The fatigue investigation above studies the endurance of the component in question. It is also important to study the reliability with respect to immediate failures caused by extreme wave conditions.

In order to do this, experiments have been performed in scale 1:60 under different conditions. One example of sampled force signals has been scaled with the factor 60 raised to three and are shown below.

The buoy has been forced to different positions, for the first part in time it is damped, similar to the latching procedure, for the second part it is fixed in a high position, almost
above the surface most of the time, in the third part it is fixed half way under the surface and in the last part it is fixed mainly below the surface.

![Figure 4](image)

**Figure 4** The colours represent different wave conditions. The green represent a *design wave*, the red a *storm wave* and the black an *extreme wave*.

It is interesting to see that the maximum forces are similar for storm and extreme waves except for the position above surface, and they do not exceed the design wave too much. A first indication of this picture is that extreme wave conditions are not critical for the design, the fatigue criteria outlined above should be safe also for immediate failure under extreme conditions.

This problem is not the focus for the actual study, but need more thorough analysis for drawing correct conclusions.

### 3.7 Discussion

The required safety factors depends on the combination of a lot of uncertainties in the calculations. In the two pie charts below the distribution of uncertainty components are illustrated for the two cases, polyester rope and steel wire.

For the polyester case the uncertainty is dominated by the factors “mean value influence” and “Relevance of rain flow counting”, but also “strength scatter” and “fatigue exponent” have considerable influence. The two largest components and the fatigue exponent uncertainty are results of the lack of detailed knowledge about the fatigue mechanisms in a polyester rope. This could possibly be reduced by further literature studies, or more complete experimental investigations.

The fatigue strength scatter can be judged to be quite normal and may be seen as a benchmark for the goal in reducing model uncertainties.
For the steel wire case the uncertainty is dominated by three sources, “Strength scatter”, “Wöhler exponent” and “reference data relevance”. The two latter can, just as in the polyester rope case, certainly be reduced by more investigations, while the strength scatter component probably cannot be reduced. However, regarding this as a benchmark shows that the steel wire case is under better control since the scatter source is the largest.
4 The Ocean Harvesting Technologies case

The Ocean Harvesting Technologies (OHT) concept is a gear box together with a gravity energy accumulator that temporally stores energy to produce an even power output to the electric power grid. The input power comes from a buoy or similar device that takes energy of incoming sea waves. As the waves are highly irregular, power input is also uneven.

In principle, the OHT gearbox transmits power to the gravity accumulator when the incoming power from the waves is large and otherwise when the incoming wave power is small the gravity accumulator instead transmits power to the gearbox. The effect is that the power output to generator is smoothed.

The weight in the accumulator compensates for the variable speed input to the gearbox and provides the generator with a close to constant torque and speed. The speed of the generator is slowly tuned to match the generated power output to the average captured power in the current sea state. The OHT concept is further described in (Sara Sahlin 2013). The weight is put in a housing sealed from the surrounding sea, c.f. Figure 7.

Figure 7.a OHT gearbox and gravity accumulator located in a central hub together with the electrical generator. The hub serves a number of wave buoys on this layout.
Figure 7.b Weather protection removed to give an idea of how the gearbox and the accumulator looks like. The weight is moving up and down in the gravity accumulator by means of a rack and pinion connected to the gear-box.

This study is made for a 100 kW rated unit with a Capacity Factor of 50% meaning that the average produced power is 50 kW during a year. OHT is planning for further scaling up of the device to 400 kW and 1000 kW at a later stage. Such a device is shown in Figure 7 above.
In Figure 8 is the incoming and output power, calculated with a simulation tool, given where the smoothing effect is evident. From the simulations, the history of the position of the gravity accumulator can also be determined. An example is given in Figure 9.

**Figure 8** Incoming power from waves (upper figure) and output power to the generator (lower figure).

**Figure 9** Incident wave position (upper figure) and position of gravity accumulator position (lower figure).

Two different components are considering for the OHT transmission that elevates and lowers the gravity accumulator. These feature a roller chain or a rack which are depicted
in Figure 10. These components, rack and roller chain are identified as weak points for fatigue failure and are analysed in the following.

Figure 10 Two different designs for the transmission to gravity accumulator, featuring either a roller chain or a rack.

4.1 Fatigue analysis
Analysing the mechanism with respect to fatigue is here the problem of estimating the life of the components. The load that affects the transmission moving the gravity accumulator (GA) is the force acting on the GA. It is realized that this force is the static weight of the GA when neglecting inertia effects caused by the acceleration that is assumed to be small. The component (rack or roller chain) is affected by fatigue damage on the part (in this case rack tooth or chain link) that is involved in the transfer of force to GA transmission. That implicates that fatigue damage is localized only on part of the length along the component.

For roller chains and gears the damage mode is also what can be categorized as wear which will, in the method described here, be included in the fatigue evaluation. The constant loading (GA weight) has the effect that fatigue damage is constant per cycle. Cycles are in the case of rack or roller chain, teeth or chain links transferring force via the sprocket/pinion. With the constant force assumption the number of these cycles is proportional to the transmitted work per length (rack or roller chain).

Dimensioning of components involved in power transmissions is often performed in terms of transmitted power and a service life. This can be reformulated as fatigue life in transmitted energy per component length. Fatigue life of the component is achieved by dividing the fatigue life in energy per length with the transmitted power per length. As transmitted power can vary along the length of the component, fatigue life will also vary along the component.

4.2 Fatigue life of OHT transmission to the GA
Examples of the fatigue life for both rack and roller chain used as a transmission component in the OHT concept are performed here. By analysing the movement of the GA and using standard dimensioning tools for rack and roller chain, the service life will be estimated.

OHT has a simulation tool for their gearbox together with the GA. The simulation outputs the position of the GA. The position of the GA is $x_k$ where index $k$ represents the points in time for which results are stored. The work transmitted at the time points is calculated as

$$W_k = F|x_{k+1} - x_k|$$

where $F$ is the weight from the GA. To have the work distributed along the length span $L$
that the GA moves along, the length is divided into $N$ sections with equal length $L/N$. The energies $E_n$ are total work transmitted when the GA position is in the span $(n - 1)L/N$ to $nL/N$ for $n = 1 \ldots N$. All $W_k$ is then added to the $E_n$ that corresponds to the GA position $x_k$. With the total time $T$ of the simulation, the average power transmitted per length is then

$$p_n = \frac{E_n}{(L/N)T}$$

for the position $y_n$ in the middle of the sections, i.e

$$y_n = (n - 1/2)(L/N)$$

Now the expected life for each section is $E_k/C$ where $C$ is the component life in energy per length. Determining $C$ is performed with standard dimensioning methods for the used component. The dimensioning is performed by setting parameters involved in the dimensioning to values that best corresponds to the OHT case. Most important in fatigue is the loading magnitude, in this case this is the weight $F$. Accordingly, dimensioning is performed for the load $F$ and operational and environment variables identified to correspond to this case.

### 4.2.1 Load estimates

With the OHT simulation tool and the wave spectrums (Josefsson 2011), the position of the GA is computed during 3 hours. Three different wave spectrums are used: small, medium and large waves. In Figure 11 are the average powers transmitted along the position of GA. In addition to the three wave spectrums, a combined average power transmitted is given where the powers of the small, medium and large wave spectrums are added with weight factors 0.3, 0.4 and 0.3 respectively. These weight factors are interpreted as the fraction of total time that each wave spectrum is assumed to occur. The waves can be categorized as:

- Small wave  
  H1: $H_s = 1.25$ m, $T_z = 5.0$ s
- Medium wave  
  H2: $H_s = 2.00$ m, $T_z = 6.3$ s
- Large wave  
  H3: $H_s = 4.75$ m, $T_z = 8.3$ s

With $H_s$ the height and $T_z$ the period of the Pierson-Moskowitz spectrum of the waves. Time series of the power have provided by OHT, being obtained from their simulation models for a scaled prototype. The average captured energy is $P_{H1} = 36$ kW, $P_{H2} = 52$ kW and $P_{H3} = 102$ kW. Losses are not included in the model, which means that the average generated electrical power is assumed to be the same as the captured power for the different sea states.
Figure 11 Power per length transmitted along the GA transmission for different wave spectrums.
4.2.2 Rack
The construction based on the GA connected to a rack that is elevated by a rotating pinion, is analysed. In this case one rack connected is to one pinion. To show how standard dimensioning can be utilized dimensioning guidelines from the supplier Kedjetechnik is utilized (Kedjetechnik AB 2013). Data for the setup is given in Table 5.

<table>
<thead>
<tr>
<th>Module</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth face width</td>
<td>80 mm</td>
</tr>
<tr>
<td>Pinion number of teeth</td>
<td>40</td>
</tr>
<tr>
<td>Rack material hardness</td>
<td>300 H$_B$</td>
</tr>
</tbody>
</table>

Table 5 Data for transmission setup featuring rack and pinion.

The pinion is the component that is the weakest link and is likely fail first since its teeth suffer from many more cycles than the rack teeth. The pinion makes several revolutions when traversing the rack, causing uniform damage on its teeth. The method of using the transferring power along component length is therefore unnecessary, making the pinion subject to standard dimensioning. The pinion is there for excluded from this case study.

Here the dimension is performed with respect to surface fatigue of the flanks. The dimensioning guidelines here involve a determination of a required flank strength factor $K_{1000}$ that is read of a chart for particular material hardness values and speed of the gear. The subscript 1000 refers to the life 1000 hours that the strength factor corresponds to. No dimensioning is given for racks explicitly but it is realized that from Hertzian contact theory the contact pressure is equal on the rack teeth and the pinion teeth. Maximum power for the pinion is thus calculated and reformulated in fatigue life in energy per component (rack) length. The gear pitch circle diameter is given by the module times the number of gear teeth i.e. 10·40= 400 mm. For the force $F = 137340$ kN the flank strength factor is determined to $K_{1000} = 0.76$. For the material hardness 300 H$_B$ this value gives a maximum allowed rotational velocity of 200 rpm, giving maximum allowed power 575.3 kW. With the life 1000 h and pinion circumference 1256.6 mm, the energy per rack length life is 457 800 kWh/m.

The resulting life (minimum value along the GA position) is 126 715 hours. The life along the whole rack is given in Figure 12.
Figure 12 Service life along the GA position for different wave spectrums.

The evaluation of uncertainty and safety factors is performed with the VMEA method similarly to the Corpower case. In Table 6 is the VMEA analysis presented. The total uncertainty in logarithmic life is 0.809 (81%). The variations and sensitivities of the sources of uncertainty are mostly estimated. For the effect of dynamic loading, a service factor $k_y$ is used. In a smooth drive $k_y = 1.0$ while with the driving machine moderately non-uniformly driven $k_y = 1.25$. The life is with $k_y = 1.25$ reduced from the previously determined 126 715 h to 63 358 h. The sensitivity coefficient is then

$$\frac{\ln(126 715) - \ln(63 358)}{1.25 - 1.0} = 2.8$$

The standard deviation for $k_y$ is set to 0.25.
Table 6 VMEA table of uncertainties in the service life estimation of the rack transmission.

<table>
<thead>
<tr>
<th>Uncertainty components</th>
<th>Uncertainty components</th>
<th>Scatter</th>
<th>Uncertainty components</th>
<th>Scatter</th>
<th>Uncertainty components</th>
<th>Scatter</th>
<th>Uncertainty components</th>
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<th>Uncertainty components</th>
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<th>Uncertainty components</th>
<th>Scatter</th>
<th>Uncertainty components</th>
<th>Scatter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td><strong>Result</strong></td>
<td></td>
<td><strong>Input</strong></td>
<td><strong>Result</strong></td>
<td></td>
<td><strong>Input</strong></td>
<td><strong>Result</strong></td>
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<td><strong>Input</strong></td>
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<td><strong>Input</strong></td>
<td><strong>Result</strong></td>
<td></td>
<td><strong>Input</strong></td>
<td><strong>Result</strong></td>
</tr>
<tr>
<td>Strength</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>x</td>
<td>1.000</td>
<td>1.000</td>
<td>0.250</td>
<td>0.250</td>
<td>Supplier gear design guide</td>
<td>x</td>
<td>1.000</td>
<td>1.000</td>
<td>0.100</td>
<td>0.100</td>
<td>Alternative usage</td>
<td>x</td>
<td>1.000</td>
<td>1.000</td>
<td>0.100</td>
</tr>
<tr>
<td>Total Strength</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>uncertainty</td>
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<tr>
<td>Load</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Wave load spectrum</td>
<td>x</td>
<td>1.000</td>
<td>1.000</td>
<td>0.200</td>
<td>0.200</td>
<td>Force transfer to buoy</td>
<td>x</td>
<td>1.000</td>
<td>1.000</td>
<td>0.200</td>
<td>0.200</td>
<td>Fraction different wave loads</td>
<td>x</td>
<td>0.100</td>
<td>1.000</td>
<td>0.500</td>
</tr>
<tr>
<td>Dynamic loading</td>
<td>x</td>
<td>2.800</td>
<td>1.000</td>
<td>0.250</td>
<td>0.700</td>
<td>Total Load uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total Load uncertainty</td>
<td></td>
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<td></td>
<td>Total uncertainty</td>
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<td></td>
<td></td>
<td></td>
<td>Total uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 7 is a reliability evaluation performed where a required life is set to 30 000 h. The regular safety factor is

\[
\frac{\text{"Median life"}}{\text{"Required life"}} = 4.27
\]

The required safety factor when requiring 95 % of the expected life not to be too low is the Variation safety factor given by

\[\exp(\text{"Total uncertainty"} \cdot 1.64) = 3.77\]

The quotient

\[\frac{\text{"Median life"}}{\text{"Required life"} - \text{"Variation safety factor"}} = 1.13\]

gives the extra safety factor to the 95% safety.

Table 7 Reliability evaluation of the rack transmission.

<table>
<thead>
<tr>
<th>Input</th>
<th>Result</th>
<th>Result (log-scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median life (1000h)</td>
<td>128</td>
<td>Safety factor</td>
</tr>
<tr>
<td>Required life (1000h)</td>
<td>30</td>
<td>Life</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Required life</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance</td>
</tr>
</tbody>
</table>

Required safety factors

<table>
<thead>
<tr>
<th>Evaluation - Extra safety factor</th>
<th>Variation safety factor</th>
<th>3.77</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required extra safety factor</td>
<td>Extra safety factor</td>
<td>1.13</td>
</tr>
<tr>
<td>1.1</td>
<td>Extra dist.</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The contributions to the uncertainties are given in Figure 13. With this aid, the largest sources of uncertainties can be identified and it is clear that the possibility of dynamic loading is the largest contributor to the uncertainty.
4.2.3 Roller Chain

An alternative construction for GA transmission is based on roller chains. The setup conceived includes two roller chains connected to the GA. Detailed data is given in Table 8.

Table 8 Data for transmission setup featuring roller chains.

<table>
<thead>
<tr>
<th>Chain</th>
<th>40B-3 (63.5 mm pitch, triplex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprocket number of teeth</td>
<td>56</td>
</tr>
</tbody>
</table>

The analysis and dimensioning of the roller chain is performed with a chain design tool provided by the chain supplier Renold (Renold 2013). With this tool, the chains are set to work with a load of 68670 N (half of the weight of the GA).

The resulting life is 289 469 hours. As with the rack case, dynamic loading has a large impact on the service life. If similarly to the rack case, the driving machine is changed from smooth running to slight shocks, the service life is lowered to only 8 740 hours.

When using VMEA, the change in dynamic loading from smooth running to slight shock is quantified to a change of 1.0. This gives the sensitivity coefficient

\[
\ln(289 469) - \ln(8 740) \quad 1.0 = 3.5
\]

The standard deviation is set to 0.50.

The resulting VMEA table is given in Table 9. It is clear that the variation due to dynamic loading is extreme, also visualized Figure 14. When evaluating the safety factors the media life is evaluated as

\[
\exp\left(\ln(289 469) + \ln(8 740)\right) = 110
\]

The extremely large variation has the effect that using the regular safety factor of 3.67, 95 % safety is not achieved when taking uncertainties into account. This has the result that the extra safety factor is much smaller than 1.0 (equivalent to that the extra distance in log scale is negative). The uncertainty in the dynamic loading must accordingly be handled, in order to give an acceptable variation of service life.
Table 9 VMEA table of uncertainties in the service life estimation of the roller chain transmission.

<table>
<thead>
<tr>
<th>Input</th>
<th>Uncertainty components</th>
<th>Sensitivity scatter</th>
<th>t-correction factor</th>
<th>Standard deviation</th>
<th>Scatter</th>
<th>Uncertainty</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.250</td>
<td>0.250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Renold Roller chain tool</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.100</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative usage</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.100</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OHT Gearbox model</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.050</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Strength uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.250</td>
<td>0.150</td>
<td>0.292</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wave load spectrum</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.200</td>
<td>0.200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force transfer to buoy</td>
<td>x</td>
<td>1.00</td>
<td>1.00</td>
<td>0.200</td>
<td>0.200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction different wave loads</td>
<td>x</td>
<td>0.100</td>
<td>1.000</td>
<td>0.500</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic loading</td>
<td>x</td>
<td>3.500</td>
<td>1.000</td>
<td>0.500</td>
<td>1.750</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Load uncertainty</td>
<td></td>
<td></td>
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<td>0.050</td>
<td>1.761</td>
<td>1.762</td>
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<td></td>
<td>0.255</td>
<td>1.768</td>
<td>1.786</td>
</tr>
</tbody>
</table>

Table 10 Reliability evaluation of the roller chain transmission.

<table>
<thead>
<tr>
<th>Input</th>
<th>Result</th>
<th>Result (log-scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median life (1000h)</td>
<td>11</td>
<td>3.67</td>
</tr>
<tr>
<td>Required life (1000h)</td>
<td>0</td>
<td>Life 4.70</td>
</tr>
<tr>
<td>Required extra safety factor</td>
<td>Variation safety factor</td>
<td>18.7</td>
</tr>
<tr>
<td>Evaluation - Extra safely factor</td>
<td>Variation dist.</td>
<td>2.93</td>
</tr>
<tr>
<td>Required extra safety factor</td>
<td>Extra safety factor</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Extra dist.</td>
<td>1.63</td>
</tr>
</tbody>
</table>
4.3 Discussion

For the OHT application, a method for estimating the service life of the transmission to the gravity accumulator is proposed. The method is based on that the load in the component transferring power to the GA is constant, which allows for straightforward service-life estimation based on the total energy transferred. It also exemplified how standard dimensioning procedures can be utilized. The benefits are thus, that loading can be expressed in energy and power which in this application is natural engineering quantities and that regular service life analysis from component suppliers can be used. This should aid and simplify the evaluation of the technical and economic performance of the design.

In the examples evaluated for the OHT GA transmission it is noticed that the dynamical loading factor is the largest contributor to the uncertainty in service life. It is therefore recommended that when proceeding with the analysis, dynamic loading is studied in more detail. This would give a reduction in the resulting uncertainty, possibly making the technical life longer or transmission components smaller and cheaper.
5 Summary

Wave energy converters must be designed to withstand large forces in a tough environment. The present financial conditions put large demands on keeping costs to a minimum. It is therefore important to control both the expected fatigue performance, and to quantify uncertainties for relevant safety factors in design.

In this pilot study it has been demonstrated how the VMEA tool can be used for designing safety factors for specified fatigue life of marine energy converters. Two different applications have been studied with respect to components that are expected to be critical for fatigue.

In the Ocean Harvesting case a counter weight chain (or a rack) is subjected to fatigue and wear and the strength with respect to these failure mechanisms was analysed. The external loads used are not measured, but taken from the Ocean Harvesting simulation model. The strength investigation is to a large extent based on standard dimensioning tools for rack and roller chain, available from component manufacturers. These tools are often presented as “black boxes” and the extent of hidden safety factors is not always known for the user.

For both alternative design concepts (rack or roller chain), it is found that the effect of dynamic force is the dominating source of uncertainty.

For the Corpower case, the load analysis is based on extensive load experiments performed by Corpower, but the strength analysis lacks detailed information and relies on literature data.

The mooring rope is subjected to large repeated tensile forces and tensile spectrum fatigue was the subject of the study. The analyses show very large uncertainties resulting in large safety factors. The dominating sources of uncertainty are related to the weak knowledge of the component strength, in particular for the polyester rope case.
6 Future work

This pilot study points out the directions for further investigations in order to obtain designs that are more close to optimum with regard to the trade-off between mechanical safety and reasonable cost. The analyses that are the basis for the required safety factors show a large potential to reduce these factors by more detailed investigations.

Following parts can be addressed in a future project:

- For the Ocean Harvesting case, a study of what is behind the component manufacturers design rules could be a basis for controlling the “dynamic load” effect, that is the dominating uncertainty source.

- For the Corpower case, more specific data for material strength could reduce uncertainty and allow smaller safety factors.

- Further analyses will also be necessary for other weak components in different wave energy applications.

- Investigate cooperation within Europe with other actors in the field of wave and tidal power plants.
7 References


Appendix

A Mathematical formulae

A time series of load $S$ is represented by its $m$ counted ranges $\Delta S_i$, and for a certain fatigue exponent $\beta$ the load beta norm is defined:

$$\|S\|_{\beta} = \left( \sum_{i=1}^{m} \Delta S_i^\beta \right)^{1/\beta}.$$  \hfill (1)

The equivalent strength is defined as the load level that corresponds to the life of two million cycles. We start with the Basquin equation for high cycle fatigue,

$$N = \alpha \cdot \Delta S^{-\beta},$$  \hfill (2)

where the number of cycles to failure is modelled as a function of the load range with two material dependent parameters, the constant $\alpha$ and the fatigue exponent $\beta$. After a material test the parameters are estimated for the best fit giving:

$$N = \hat{\alpha} \cdot \Delta S^{-\hat{\beta}},$$

The equivalent strength is the number that fulfils

$$2 \cdot 10^6 = \alpha \cdot S_{eq}^{-\beta}.$$

This can be estimated from a specific test as

$$S_{eq} = \left( \frac{\hat{\alpha}}{2 \cdot 10^6} \right)^{1/\beta}.$$  \hfill (3)

The equivalent load is defined based on measurements in the field: If we measured a time series of load during time $t$ resulting in $m$ counted cycles and the target life is time $T$, then the equivalent load is estimated as:

$$L_{eq} = \left[ \frac{T}{t} \cdot \frac{m}{2 \cdot 10^6} \|S\|_{\beta}^{1/\beta} \right]^\beta,$$  \hfill (4)

If each sea state has the equivalent load $L_{eq,j}$ and is expected to appear at the frequency $c_j$, then the overall equivalent load is,

$$L_{eq, tot} = \left[ \sum_{j=1}^{m} c_j \cdot L_{eq,j}^\beta \right]^{1/\beta},$$  \hfill (5)

where

$$\sum_{j=1}^{m} c_j = 1.$$
B Figures

Figure 15 Force signal for part of the latch 15 experiment.

Figure 16 Force signal for part of the latch 17 experiment.

Figure 17 Force signal for part of the latch 19 experiment.
Figure 18 Force signal for part of the latch 21 experiment.
SP Technical Research Institute of Sweden

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