



# Progress report on low flow measurements based on optical characterization of gas density in vacuum



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## **Abstract**

### **Progress report on low flow measurements based on optical characterization of gas density in vacuum**

With the aim to achieve a new primary standard for calibration of ultra-low gas flows (leaks) the research in determination of gas density using optical refractometry has progressed, resulting in an evaluation of the stability of the prototype and a refined theoretical analysis. Furthermore, a renewed literature search has revealed more relevant work and further knowledge is gained from these papers. The assembled prototype was known to be of proportions far from optimum, however the assumption that it would be sufficient to reach applicable results was too optimistic. The conclusion is to continue with the efforts, however with a new and improved prototype and an adjusted choice of mirror parameters, and this report summarizes the work as of spring 2010.

Key words: Traceability, reference, leaks, Fabry-Perot, interferometry, refractometry,

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## **Förord / Preface**

Since 2007, a project has been running by the department of Measurement Technology, the vacuum laboratory in collaboration with the radiometry laboratory, to find an alternative method to create a reference leak. It is based on the theoretical analysis previously published in collaboration with Uppsala University. The aim is to enhance the accuracy and dynamic range of the calibration services, and to gain wider international recognition for advanced and innovative research in metrology.

During the years the results have been presented on one conference and in 2009 and beginning of 2010 a student was employed to make some optical measurements as his thesis work for Master och Science. This report is the concatenation of these presentations and report, in addition to a summary of the laboratory work and preparation.

## **Sammanfattning / Summary**

The setup of an experimental verification of a new technique to achieve a traceable reference for very low gas flows in vacuum is described. The main result of the reported work is the list of improvements for further work, nevertheless the background and preparatory work is described more detailed than elsewhere published.

The report is based on two unpublished extended papers from EVC-10, and one report for the degree of Master of Science.

## **1 Background**

### **1.1 Theoretical background**

The researchers in metrology on pressure and gas flow have identified the need for a calibration method of very small gas flows, usually denoted as reference leaks, which is traceable to the realization of appropriate SI-units. Therefore, work is performed on detecting gas density using optical refractometry, enabling traceability through optical frequency and eventually to the SI realization of 1s, as an alternative method compared to existing ones. The need to find alternative references for very low gas flows is also acknowledge in other national metrology institutes [1-10].

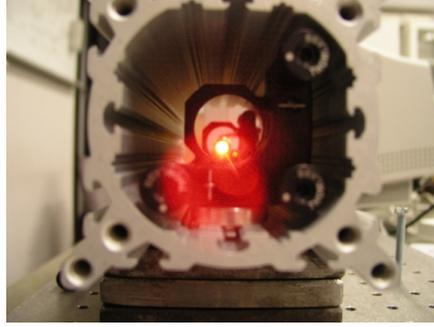
The theoretical analysis was made and published previously [11-13], resulting in an estimated accuracy sufficient for the requested performance. With the inclusion of parameters related to the laboratory equipment and the prototype, an overview was presented [appendix 1].

### **1.2 Prototype assembly**

The first prototype was assembled from standard, of the shelf, components. Through inserting the Fabry-Perot cavity within the loop of an erbium-doped fiber ring laser, the emitted wavelength would be tuned by the refractive index of the cavity, and the analysis would be limited to heterodyne mixing of the emitted wavelength with a stable reference laser beam. This reference would preferably be extracted from the wide spectrum of the output of an optical comb, which is locked to a synchronization signal from a high performance Cs-oscillator. Then the wavelength will be determined with traceable uncertainty from the realization of 1s.

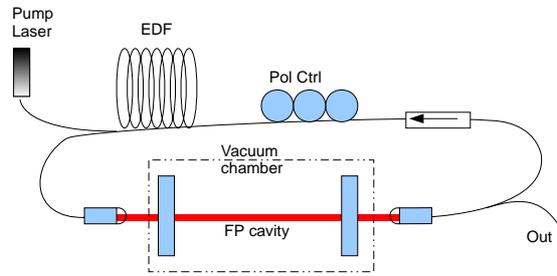
The only available high performance mirrors were the Newport Ultra Low-Loss Supermirrors™, with a specified reflectivity of  $R > 99,99\%$  and transmission  $T > 0,002\%$ . They were available at diameter of 25,4 mm (1") and as flat or concave with focal length  $f = 1\text{m}$ , where two of the latter was used. Since the cavity was much shorter than 1 m, a non-confocal FP interferometer was constructed. Furthermore, the mirrors were mounted in opto-mechanical stages for angular adjustment. Even though these devices could be purchased for use in vacuum, where the standard grease is replaced with vacuum grease and the aluminum is not anodized, it was assumed that even the vacuum grease might contaminate the vacuum system and the mirrors. As an alternative, standard opto-mechanical components was purchased, fully disassembled and cleaned before reassembly. This will decrease the life expectancy of the components, and a small chance of wear on the threads may emit metal particles. Nevertheless, an analysis of the performance with respect to de-gassing was made with satisfactory result [appendix 2].

Finally, a vacuum-chamber was used where the mirror holders could be securely attached at the inside. The chamber is shown in throughput in figure 1, where a red laser is used for illustration and the mirrors and windows are not installed.



**Figure 1:** Photo of tube for vacuum chamber, without mirrors. Red laser is used for illustration.

The planned setup is shown schematically in figure 2. The beam travels through transparent windows made of standard glass for vacuum viewports, which was verified from datasheets to have low absorption at 1550 nm. Within the FP-cavity, the interference wavelength depends on the gas density and the temperature, which requires the latter to be stable. The polarization controller is set to compensate for any changes within the fiber ring, to achieve zero net polarization change after one round-trip.



**Figure 2:** Ring laser with FP-cavity in vacuum chamber.

The 20 m erbium doped fiber (EDF) is pumped by a 160 mW laser at 980 nm (pump laser) which should induce a gain of 15 – 20 dB. The polarization controller (Pol Ctrl) is adjusted for zero net polarization change, and the isolator ( $\leftarrow$ ) defines the direction of the lasing. A 90/10 power splitter leaves 90% of the light within the ring and 10% is emitted at the output (Out). The light is confined within the fiber, except when coupled into the vacuum chamber which is achieved through a pair of collimators.

Preliminary results indicated that it was possible to lock the ring laser to an interference wavelength of the interferometer, but since the gain spectrum of the erbium-doped fiber is wide the laser emission would jump between different peaks. A wider filter would therefore be necessary, however when this was inserted the intracavity losses became too high and the unit ceased lasing. Furthermore, the stability was additionally decreased from the length of the ring-laser, and the influence of vibrations within the fan-cooled bench-top EDF Amplifier (EDFA) was presumed to be limiting the performance.

The gain medium was thus changed to a customized EDFA consisting of 20 m of erbium-doped fiber, wound on a copper spool for temperature stability, and pumped by a 980 nm laser. Unfortunately this design did not deliver sufficient gain, and even though the mechanical stability was better it was not possible to achieve lasing.

The third setup was based on wavelength locking an external laser to the FP-cavity. In this setup a bench-top external cavity laser was seeding the cavity, while scanning the output frequency. When the interferometer is operable, the output will emit light at constructive interference while staying dark at all other wavelengths. The setup of this measurement is described more extensively [appendix 3].

## 2 Prototype characterization

Except for the preliminary evaluations made during alignment, the rigorous characterization was made within the scope resulting in the attached report [Appendix 3]. This characterization was based on the third setup, and substantial efforts were made to construct the locking electronics for the laser. Furthermore, the main difficulty proved to be the fringes from the FP-cavity, which were dominated by interference from other surfaces, and not dependent on the refractive index in the chamber.

## 3 Conclusion

The experiments have given valuable experience for further development, in addition to the proposition of a new metrology utilization of an optical comb. The project will proceed with the following major changes:

- 1 a customized FP-etalon with much smaller dimensions and fixed proper alignment must be ordered
- 2 The operating wavelength will be changed to shorter wavelength. The choice of wavelength was based on the availability of fiber based gain media, and the dimensions of single mode fiber core. For an increased sensitivity, a free-space solution with visible light should be considered.

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## 5 Appendices

### **Appendix 1: Optical method for density determination in vacuum.**

Viktoria Jonasson, Fredrik Arrhén, Måns Ackerholm, Per Olof Hedekvist, Leslie Pendrill

Presentation at 10th European Vacuum Conference (EVC-10), Lake Balaton, HU Sept. 22-26, 2008. Paper not published previously.

### **Appendix 2: The function in vacuum of optical standard components**

Viktoria Jonasson, Fredrik Arrhén, Måns Ackerholm, Per Olof Hedekvist, Leslie Pendrill

Poster presentation at 10th European Vacuum Conference (EVC-10), Lake Balaton, HU Sept. 22-26, 2008. Paper not published previously.

### **Appendix 3: Gas density determination with heterodyne refractometry for low flow measurements**

Nikolaj Henriksson

Master's Thesis in Engineering Physics, Mars 15, 2010  
Umeå universitet

## Appendix 1: OPTICAL METHOD FOR DENSITY DETERMINATION IN VACUUM

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### 1 Abstract

At present there are no NMIs or accredited laboratories calibrating reference leaks in the Nordic countries. The methods used by other laboratories today have relatively large uncertainties and there is a need to improve this.

A feasibility study of a new method for density determination of gases in vacuum has been made at SP Technical Research Institute of Sweden. It is an optical method using laser refractometry and this will now be experimentally verified.

This study for determining gas density promises small uncertainties and with a known density of the gas it may be possible to use the method to determine the mass flow of gases. The method is based on the proportionality between density and the refractivity of a gas.

A first prototype using a high-finesse optical resonator for vacuum use has been built and is at present under evaluation. A principal challenge is the thermal robustness of the laser refractometer.

Hopefully a new method to determine the gas density will lead to better uncertainties than today's methods.

### Keywords

Mass-flow, density, laser refractometry, calibration, reference leaks, gas flow, metrology

### 2 Introduction

For a lot of industrial and research applications leaks are a big problem. These leaks have to be measured and quantified. For this to be done reference leaks with established values are used. These reference leaks have to have their actual leak rate determined by calibration against some kind of reference.

Neither accredited laboratories nor the national laboratories calibrate reference leaks in the Nordic countries today. The aim of this project is to enable the possibility to calibrate reference leaks and to develop a method with small measurement uncertainties. The purpose of this report is to analyze different methods of calibration of reference leaks as a basis for a future purchase of equipment.

We have chosen a dynamic comparative primary method, with a constant pumping speed and a well-known conductance through the gas-flow meter. An advantage with this method is the possibility to use the same system as a primary method for calibration of vacuum gauges.

The gas flow in such a system is one of the primary factors to be determined and this gives a large contribution to measurement uncertainty. Contactless measurement methods of gas flow measurement can promise smaller measurement uncertainties. Therefore a new optical method using refractometry to measure the density of the gas in the flow meter was chosen.

### 3 Reference leaks

A reference leak is based on delivering gas at a known rate. It is also called calibrated leak, standard leak, transfer leak, leak artifact or test leak. It is usually used as a transfer standard to measure the sensitivity of a tracer gas leak detector, but it can be used to calibrate vacuum gauges or measure the pumping speed of vacuum pumps. They are also used in calibration of mass spectrometers and leak detectors. The main tracer gas used in reference leaks is Helium even if other gases may be used.

There are different kinds of reference leaks based on different techniques, such as permeation leaks, short orifice leaks, capillary or membrane leaks. Most reference leaks have a fixed value, but also adjustable reference leaks exist. Some of the disadvantages of the adjustable leaks are large uncertainties and poor repeatability.

Nowadays it is possible to manufacture reference leaks by permeation through polymers. These polymers work with different gases, such as argon, sulphur hexafluoride and refrigerants which enhance the needs of calibration resources for gases other than helium.

### 4 Leak calibration

Calibrations of reference leaks can be made in several ways. A combination of several methods has to be used to cover a large range which can cause problems.

Calibration with primary methods is arduous, so calibrations of reference leaks are often done by comparison with another calibrated leak. The measurement of leak rate is mainly done in vacuum or atmospheric pressure.

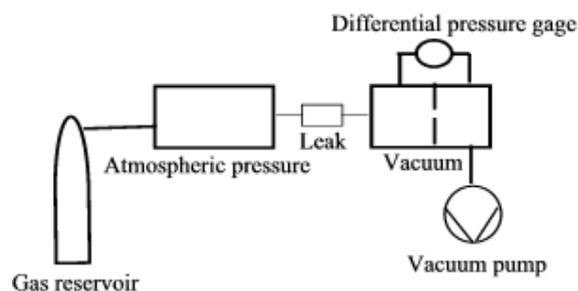


Figure 1: Traditional flow calibration

The well-known method to measure the flow of a test leak by a pressure drop across a well-known conductance is recalled [7, 8].

The test leak is connected to the system and a steady flow is established. The partial pressure of the test gas in the upper chamber of the vacuum system is measured by a mass spectrometer. Then the test leak is replaced by the reference flow meter and when steady conditions are established the mass spectrometer is read again. The difference in partial pressure is a measure of the difference in generated flow from the two sources as long as the difference is quite small. Since the calibration is a quite time-consuming task

temperature stability is a major issue. As the reference flow meter might be a calibrated secondary standard or a primary flow meter.

Primary flow meters exist in several different techniques of which two will be described shortly here. They are based on gas flow from either a constant volume or a constant pressure gas reservoir. In the case of constant volume a small volume is filled with the test gas to certain pressure. During the calibration, the pressure drop in this reservoir is measured and this drop together with the total volume of the reservoir gives the amount of gas molecules flowing from the reservoir.

In the case of constant pressure, the reservoir is a variable volume and the pressure is to be kept constant. As gas flows from the reservoir, its volume is decreased to keep the pressure at desired level and the flow can be calculated from the change in volume.

Both methods have drawbacks of course. They both require constant temperature. For the constant volume method one of the major drawbacks is that the leak flow is decreasing with dropping pressure, The constant pressure method on the other hand have problems with determining the volume change and in some designs also undesirable leakage.

As an example the constant pressure flow meter will be described more in detail. Such a flow meter contains a variable volume, a reference volume, control system with pressure gauges and a mechanism to vary the volume. It varies the volume in the measure range, in order to keep the pressure in the variable volume constant. To enhance the sensibility the pressure difference between the reference volume and the variable volume is used in as feedback in order to adjust the variable volume. This differential pressure is measured with a CDG and from the volume change and the actual pressure used the flow can be calculated.

## 5 Gas flow and gas density

The basis of the proposed method of determining gas flow more accurately with the continuous expansion method is to replace measurement of the pressure in the flowmeter, as in case of traditional throughput systems with a measurement of the gas density,  $\rho$  using laser refractometry [11,12].

## 6 Refractometry and gas density

Laser refractometry makes it possible to measure the density of gases. Helium is one of the gases where the density is very well-known [10]. This offers a good opportunity to improve the uncertainty of the flow in a system with known conductance.

Gas pressure (and flow) is dependent on density as:

$$p = \rho RT ;$$

where  $\rho$  = density; R = Rydberg constant; T = temperature and density depends in the substance amount as:

$$\rho = \frac{N}{V}$$

$\rho$  = density

N = substance amount

V = volume

The substance amount can be measured with laser refractometry [9, 10] using the Lorenz-Lorentz formula as:

$$\rho = \frac{\sum_i x_i M_i n^2 - 1}{\sum_i x_i A_i n^2 + 2} \quad (\text{F. 6})$$

$\rho$  = density

$x_i$  = concentration of gas component i

$M_i$  = molecular mass

$A_i$  = molecular refractivity

$n$  = refractive index

## 7 Gas flow and laser refractometry

An expression for the gas flow in terms of change in density and refractive index can be got by differentiating the Lorenz-Lorentz formula (F.6), yielding:

(assuming  $V$  is constant and that  $n \approx 1$ ):

$$q = \frac{\Delta N}{\Delta t} = \frac{V}{\sum_i x_i A_i} \cdot \frac{2}{3} \frac{\Delta n}{\Delta t}$$

The basic equation for deducing a gas flow rate on the basis of laser heterodyne refractometry

$$q = \frac{\Delta N}{\Delta t} = \frac{V}{\sum_i x_i A_i} \cdot \frac{2n}{3\nu} \cdot \frac{\Delta \nu}{\Delta t} \quad (11)$$

is then derived in terms of the change in resonance frequency,  $\nu$ , of the optical resonator resulting from a change in the refractive index due to a change in gas density [11,12].

In order to obtain sufficient resolution, the optical resonator has to be built with supermirrors (reflectivity  $R = 99.999\%$ ). Typical projected relative resolution in gas density at  $10^3$  Pa are:

$$\frac{\Delta \rho_{\text{finesse}}}{\rho} = 8.3 \cdot 10^{-5} \text{ for nitrogen and } 71.7 \cdot 10^{-5} \text{ for helium.}$$

The high resolution of the new method promises reduced uncertainties in gas-flow measurements, compared with current state-of-the-art levels of about 0.1%. Also since the density itself is independent of temperature, the effects of temperature changes during the measurements will decrease.

The disadvantages are that the method requires high-finesse optical resonators adapted to vacuum use; temperature variations can cause large measurements errors and the method has not got international acknowledgement yet.

## 8 The project

Reference leaks usually works in the range  $4.4 \cdot 10^{-14} - 4.4 \cdot 10^{-7}$  mol/s ( $10^{-10} - 10^{-3}$  Pa·m<sup>3</sup>/s at room temperature). SP is aiming to cover the range  $4.4 \cdot 10^{-12} - 4.4 \cdot 10^{-8}$  mol/s ( $10^{-8} - 10^{-4}$  Pa·m<sup>3</sup>/s at room temperature) which is assumed to be enough for most applications.

As medium for leak calibration, Helium is chosen. The choice is done taken into consideration that helium is the most common tracer gas used in leak detection. Helium is inert, non-toxic and non-flammable. Its content in atmosphere is very low (5 ppm) which gives a very low background noise when used for leak detection. The density for helium is very well-known as well as it's optical properties.

The possibility to calibrate leaks against a larger selection of gases in the future exists. One of the long-term aims of the project is to calibrate gas flow in a large assortment of gases.

A demand for low measurement uncertainty points to a method with stable results, concerning both repetition and reproduction. SP is examining calibration methods for helium molar flows under  $4.4 \cdot 10^{-8}$  mol/s ( $10^{-4}$  Pa·m<sup>3</sup>/s) in room temperature with low measurement uncertainty.

Methods used today for calibrating reference leaks are relatively uncertain. In the measurement range  $4.4 \cdot 10^{-15}$  to  $10^{-7}$  mol/s ( $10^{-11}$  to  $10^{-3}$  Pa·m<sup>3</sup>/s) the normal uncertainties are 0.1 – 10 %. The most important contribution to the uncertainty is the determination of changes of gas density in the gas flow gauge.

The gas density measurements will be used either in a constant pressure system to determine the change in density and thus be fed back to the change in volume or if the achieved sensitivity is good enough in a constant volume system. This is to be decided in a later stage of the project.

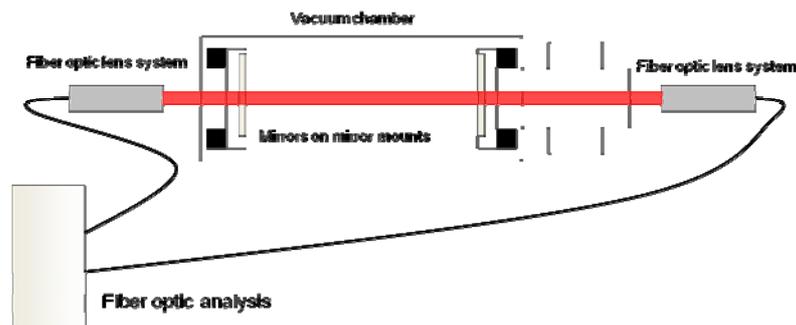


Figure 2: Schematic sketch of the optical connection into the vacuum chamber

## 9 Conclusion

A method for calibrating of leaks, which can give high quality and traceability to the calibrations has been sought. We chose a dynamic comparative method which also will give us the possibility to calibrate vacuum gauges in the same system. These methods are primary methods (both for reference leaks and vacuum gauges).

Constant pumping speed, a well-know orifice, pressure and temperature measurements are important in this method, but a large contribution to the measurement uncertainty comes from the measurement of the gas flow.

A new method for gas flow measurements is under development. Laser refractometry is used in the new gas flow measurement method. By a combination of the laser refractometry method with a traditional dynamic comparative method of leak calibration we hope to get a significant improvement in measurement uncertainty for leak calibrations.

The first prototype for the refractometry method is built and we have set great hope on the combined method.

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## Appendix 2: THE FUNCTION IN VACUUM OF OPTICAL STANDARD COMPONENTS

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### 1 Abstract

The selection of opto-mechanical components developed to suit vacuum systems is limited, expensive and usually with a long delivery time. It would therefore be very beneficial for everyone working with vacuum systems if it was possible to use standard, off-the-shelf components also in vacuum. To verify this issue, the possibility and feasibility to use standard detachable kinematic mirror mounts by a major international manufacturer has been investigated in high vacuum ( $\sim 10^{-5}$  –  $10^{-4}$  mbar). The evaluation shows that it is possible to clean the components enough to achieve degassing below detection level. Even though the adjustment screws are de-greased, they are still operational during the course of the evaluation.

### Keywords

Vacuum, optical components, outgassing, cleaning, optical interferometry, Fabry-Perót, optomechanics, off-the-shelf components

### 2 Introduction

Using components that are not adapted for use in high vacuum in a vacuum system may give some concern. Mechanical components, such as alignment mounts, are generally greased to enable smooth motion with minimum wear of threads etc. The degassing from the grease in the components, as well as any surface treatment, can increase the pressure in the application and even contaminate the system with undesired substances. However, cleaning the components can cause mechanical wear of the moving parts of the components, and in worst case cause metal chips to shear off from the components and fall into the turbo pump, which would be very detrimental.

Two mirror mounts from a major international supplier [1] have been purchased, disassembled, cleaned and reassembled for usage in vacuum. The disassembled mounts are shown in figure 1.



Figure 1: Photo of disassembled mirror mounts.

The purpose of the components is to create an optical Fabry-Perót cavity to be used in the measurement of variations in gas density [2]. The vacuum chamber, with unmounted flanges, is shown in figure 2.

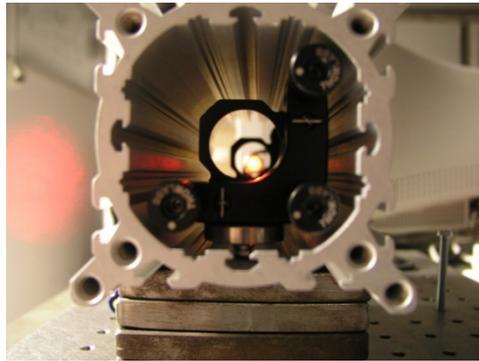


Figure 2: Photo of mirror mounts in vacuum chamber

### 3 Cleaning and Results

With a Residual Gas Analyzer (RGA) the degassing from the components was measured before and after cleaning of the components, to evaluate the improvement in vacuum performance. Each mirror mount contains three adjustment screws, and the tip of every screw is a crater where a steel ball (ball bearing) is mounted. Before cleaning, the space between the screw and the ball is filled with grease, as well as the threads. According to the manufacturer it is Apiezon-M grease (high-vacuum grease) mixed with sticky grease (not for use in vacuum). The grease was removed and all the parts of the mirror mounts was cleaned in an ultrasonically bath, first with isopropanol, then with ethanol. Finally they were flushed in distilled water, and dried.

After cleaning the components, the opto-mechanical components were inserted in a vacuum chamber and the pressure decreased to below  $10^{-5}$  mbar. During this operation, no specific outgassing could be detected by the RGA. Furthermore, optical mirrors were inserted in the mounts, and their angles were optimized through controlling the adjustment screws. No degradation in the smoothness of the adjustment motion could be noticed after removing the grease.

#### **4 Conclusion**

The evaluation of the vacuum usability of standard opto-mechanical components has been very successful. After appropriate cleaning, no degassing of any undesirable substances could be detected. Furthermore, the motion of the mirror mounts still works fine. In conclusion, standard off-the-shelf opto-mechanical components can be cleaned and used in high-vacuum, even though the life expectancy of the devices may be decreased.

#### **5 References:**

- [1] Thorlabs catalog V19 p 142
- [2] Viktoria Jonasson, Fredrik Arrhén, Måns Ackerholm, Per Olof Hedekvist, Leslie Pendrill "OPTICAL METHOD FOR DENSITY DETERMINATION IN VACUUM" presented at EVC-10, sept 22-26 2008.

**Appendix 3:**  
**GAS DENSITY DETERMINATION WITH HETERODYNE  
REFRACTOMETRY FOR LOW FLOW MEASUREMENTS**

Nikolaj Henriksson  
Master's Thesis in Engineering Physics, Mars 15, 2010  
Umeå universitet

# Gas density determination with heterodyne refractometry for low flow measurements

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Mars 15, 2010

Master's Thesis in Engineering Physics, 30 ECTS

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## Abstract

In many industrial and scientific applications it is necessary to determine small leaks with high precision. In doing this it is of high importance that the reference leaks used are calibrated with sufficient uncertainty. An integral part for measuring leak rates is the flow meter, which delivers a well determined gas flow into a low-pressure system. With a well determined flow into the low-pressure system, and by knowing the properties of the system, it is further possible to calculate the pressure inside the low-pressure chamber. A method for measuring the molar flow based on heterodyne refractometry is proposed in this thesis.

The proposed flow meter is setup as a resonant cavity, which works as an optical filter. By locking a laser to a transmission peak of the cavity, the density of the gas inside the cavity can be determined, from theoretical calculations. Letting the gas out of the cavity changes the density, and therefore, the transmission frequency. The frequency of the transmission peak must be measured with high accuracy. This can be achieved by mixing the output of the cavity with a reference laser originating from an optical frequency comb. By using a frequency counter on the mixed signal, it is possible to indirectly determine the density change, and in turn the molar flow out of the flow meter.

A number of factors can affect the measurements, or even render them useless, if not taken care of properly. Minimal necessary requirements for a given density resolution of the flow meter are examined, in terms of material choices, acceptable changes in thermal conditions, pressure ranges, etcetera. An overview is made on the necessary basic requirements of the laser source, and possible candidates for the setup are proposed. The resonant cavity, being the most important part for the measurement, is examined in detail.

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# 1

## Introduction

The aim of the project is to do a theoretical analysis and propose a flowmeter, or more correctly, a flow generator, based on heterodyne refractometry. The project is part of a larger low-flow determination project undertaken by SP Technical Research Institute of Sweden, which aims at developing a new method for determination of flows in the ultra high vacuum regime.

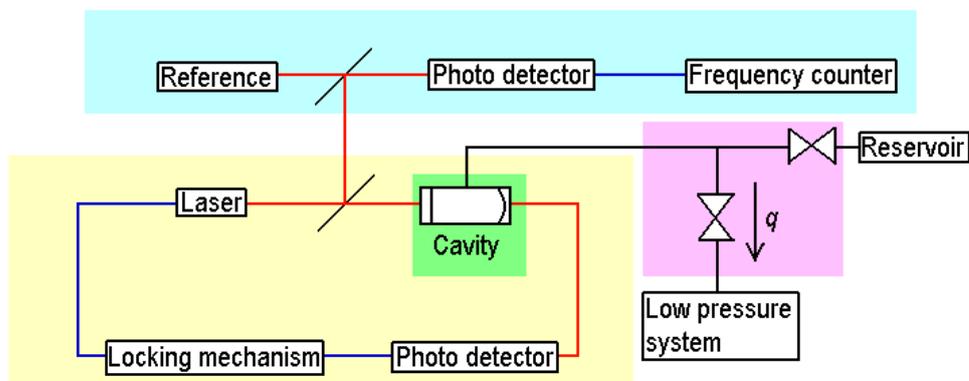
Historically, flowmeters have been based either on a constant volume approach, a constant pressure approach, or a combination of both. The main limitation of these systems is the that the temperature must be held very near constant, for reliable results. Additional error sources are introduced by the necessary moving parts, especially when a flow is generated by changing the volume. In the proposed flowmeter setup, these obstacles are in part avoided by instead measuring the density, which is the same irrespective of temperature.

The proposed flowmeter can be divided into four more or less interchangeable blocks, as is shown in Fig. 1.1. The blocks are not fully independent of each other, but with the correct treatment, it is possible to do this division. Since the theoretical background of the blocks differ, it is beneficial to handle each one separately. Accordingly, it is possible to isolate in which block main weaknesses or sources of uncertainty exist, and thus know where on the force of resolution should be directed.

Heterodyne refractometry refers to the method used for measuring the density of the gas inside the flow meter. The basic idea is to use a resonant cavity as a refractometer and relate the density inside it with the frequency of a transmission peak of the cavity. The theory relating density to frequency is handled in Sec. 2.1. The mechanical setup

## 1. INTRODUCTION

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**Figure 1.1:** Simplified schematic overview of the complete system, where each block has its own background color. The pink block corresponds to the gas chamber, with connections to an external gas reservoir and a low pressure system. The green block corresponds to the resonant cavity. The beige block corresponds to the laser system, along with locking mechanism. The blue block corresponds to the heterodyne measurement system. For clarity, the lines have been colored according to what they represent. Black lines carry gas, red lines carry light, and blue lines carry electricity.

of the resonant cavity, from a theoretical standpoint, and the theory of frequency measurements are handled in Sec. 2.3. An overview of sources generating the laser beam, together with the necessary laser locking mechanism, is given in Sec. 2.4. The heterodyne term in the title refers to the method of comparing the obtained frequency with a well-known stable reference frequency, making the measurement accurate. More on this topic can be found in Sec. 2.5.

All this come together in Chapter 3, where the complete system is put together, including numerical recommendations for the ingoing components.

Throughout the text efforts are made to find and quantify the main sources errors and uncertainties, along with limitations, of the proposed system. Likewise, ideas are given on how to resolve certain obstacles. Conclusions and a future outlook of the system is finally given in Chapter 4.

## 2

# The suggested technique

The proposed flow meter measures flow indirectly through a measurement of the frequency of narrow-line width laser locked to an optical cavity. The basic theoretical chain consists of four steps. First, the change in frequency of light inside the chamber is measured. Second, the frequency change is related to the index of refraction of the medium in question. The third, and from a theoretical standpoint, last non-trivial step, is to relate the change in refractive index to a density change. The fourth step is merely a mathematical conversion from density change to molar flow, adding time dependence to the equations. It is instructive to note, however, that this last step, while simple in terms of mathematics, still requires all ingoing variables to be well-defined. In Sec. 2.1 the details of this somewhat intricate procedure will be given.

It is also important to note that the actual realization of the technique given above depends on the gas chosen for the flow meter. An elaboration on the available selections will also be found further down in the same section.

The practical construction of the flow meter and its connections with other instruments is discussed in Sec. 2.2, which also includes some comments about the possible working pressures.

### 2.1 The principles of flow measurement

A general theoretical overview of flow measurements will be given in this section. Fundamental equations concerning both the operation and the obtainable accuracy of the

## 2. THE SUGGESTED TECHNIQUE

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system as a whole will be established. For the sake of comparison, the main equations governing a traditional flow meter will also be derived.

### 2.1.1 Molar flow

It has been suggested (3) that leak rates should be specified in terms of amount of substance  $N$  and time  $t$  as a molar flow

$$q = \frac{\Delta N}{\Delta t}, \quad (2.1)$$

with gas species and temperature separately stated. For consistency and unambiguity reasons, this definition of molar flow will be used throughout this work.

The ideal gas law,  $PV = NRT$ , where  $R$  is the fundamental gas constant, can be used to relate the amount of substance  $N$ , pressure  $P$ , volume  $V$ , and temperature  $T$  with molar flow. Rearranging the gas law and explicitly stating the time  $t$  dependence on all variables yields

$$N(t, P(t), V(t), T(t)) = \frac{1}{R} \frac{PV}{T} \quad (2.2)$$

for the amount of substance. Performing a total derivation on Eq. 2.2 yields the full equation for the change in amount of substance

$$\begin{aligned} q\Delta t = \Delta N &= \frac{1}{R} \left[ \frac{V}{T} \Delta P + \frac{P}{T} \Delta V - \frac{PV}{T^2} \Delta T \right] \\ &= \frac{PV}{RT} \left[ \frac{1}{P} \Delta P + \frac{1}{V} \Delta V - \frac{1}{T} \Delta T \right], \end{aligned} \quad (2.3)$$

where  $\Delta t$  has been moved to the left hand side, for the sake of clarity. The molar flow can still be recovered by moving  $\Delta t$  back to the right hand side. Each term inside the square brackets must now be examined in detail.

Assuming that the temperature is held, more or less, constant, there are two basic modes of operation to be considered. Either volume or pressure can be chosen to be variable. For the measurement of molar flow as such, this choice does not matter. Irrespective of which mode is chosen, the last term in square brackets of Eq. 2.3 can be estimated to be of order

$$\frac{1}{T} \approx 3 \cdot 10^{-3} \text{ /K}, \quad (2.4)$$

## 2.1 The principles of flow measurement

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at  $T = 300$  K. Assuming that it is pressure that is measured, the second term in square brackets of Eq. 2.3 can be analyzed by realizing that the contribution from it actually corresponds to the thermal expansion of the working substance, according to

$$\frac{1}{V}\Delta V = \frac{1}{V}\frac{\Delta V}{\Delta T}\Delta T = \beta\Delta T, \quad (2.5)$$

where  $\beta$  is the coefficient of volumetric thermal expansion. The process is basically governed by the material of the surrounding chamber. The thermal expansion coefficient of various materials can be found at various places (13), and can be used as an estimate the second term. For many standard metals this value is of order  $10^{-5}$ .

An examination of the constant pressure mode will not be carried out here, but the results are similar.

Another possibility is to measure the the density change  $\Delta\rho$  instead. The density is related to the other quantities, using the ideal gas law, through

$$\frac{P}{TR} = \frac{N}{V} = \rho, \quad (2.6)$$

implying that any change in  $P$  or  $T$ , intended or not, will be detected only if it generates a molar flow out of the chamber. Redoing the differentiation in the same way as above yields

$$q\Delta t = \Delta N = V\rho \left[ \frac{1}{\rho}\Delta\rho + \frac{1}{V}\Delta V \right]. \quad (2.7)$$

The third term of Eq 2.3 is not present anymore, but has been absorbed into the  $\Delta\rho$  term, which is possible to measure. Thus, both pressure and temperature can be removed as error sources, and instead be used as means of controlling the change in substance (or molar flow). Thus, in a leap, the error contributions has lessen by an order of  $10^2$ . A great advantage, indeed. It is important to note that the above ideas rely on the fact that the volume is well known. As in the previous case, the thermal volume dependence is described by Eq 2.5, and the same discussion as before applies here too.

In this project a method for measuring density changes will be proposed, with the volume held constant. The constant volume method has the added benefit of avoiding moving parts, being in-line with project goals.

## 2. THE SUGGESTED TECHNIQUE

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### 2.1.2 Relating refractive index to molar flow

Parts of the procedure outlined in this section are based on earlier works (5; 6). The main additions are the determination of a frequency shift introduced due to distance changes, and a brief discussion on how to refine the measurements through the inclusion of virial coefficients.

The refractive index can be related to the density of a gas through the Lorentz-Lorenz equation (15), which reads

$$\frac{n^2 - 1}{n^2 + 2} = \rho (A_R + B_R \rho + \dots), \quad (2.8)$$

where  $A_R$  and  $B_R$  are virial coefficients accounting for molar polarizability and two-body interaction effects, respectively. The density can in turn be related to quantities such as absolute pressure through the equation of state

$$\frac{P}{RT} = \rho (1 + B(T)\rho + C(T)\rho^2 + \dots), \quad (2.9)$$

where virial coefficients for the non-ideal compressibility behavior are included. Combining these equations makes it possible to relate and calculate the absolute pressure, in terms of  $A_R$ ,  $B_R$ ,  $B(T)$ , and  $C(T)$ , based on the observed refractive index of a gas. In the present situation, in which the main interest is to determine the molecular flow, the later two coefficients are not needed.

As a note, Eq. 2.9, including all virial coefficients, could obviously be used in place of 2.2 for more accurate results.

The molar flow can be calculated by expanding Eq. 2.1, yielding

$$q = \frac{\Delta N}{\Delta t} = V \frac{\Delta \rho}{\Delta t}, \quad (2.10)$$

where  $V$  has been assumed constant, as stated earlier in Sec. 2.1.1. By removing the time dependence and rearranging, Eq. 2.10 simplifies to

$$\frac{\Delta N}{V} = \Delta \rho, \quad (2.11)$$

with density being the quantity measured.

Differentiating Eq. 2.8 with respect to  $n$  yields part of the solution. Since the virial coefficient  $B_R$  is non-linear in  $\rho$ , it can be omitted for the sake of simplicity. Further, the approximation

$$\frac{n^2 - 1}{n^2 + 2} \approx \frac{2}{3} (n - 1), \quad (2.12)$$

## 2.1 The principles of flow measurement

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can be assumed acceptable, as  $n$  is close to 1, and even more so at low pressures. With the introduced simplifications the differentiation of Eq. 2.8 simplifies to

$$\Delta\rho = \frac{1}{A_R} \frac{2}{3} \Delta n. \quad (2.13)$$

For more accurate results, the above simplifications should not be used. The calculations are straight-forward, but the expressions become cumbersome.

The change in refractive index  $n$  can be determined by using a resonant cavity as an optical filter and measuring the frequency  $\nu$  of the transmitted light.

Only frequencies with certain frequencies can exist in a resonant cavity. The frequencies that can exist are given by

$$\nu = q \frac{c}{2nd}, \quad (2.14)$$

where  $d$  is the length of the cavity,  $c$  the speed of light in vacuum, and  $q$  an integer. This implies that the index of refraction is given by

$$n = \frac{qc}{2\nu d}, \quad (2.15)$$

A more thoughtful treatment of resonant cavities, the ingoing variables, and related quantities can be found in Sec. 2.3, but for now it is sufficient to assume that Eq. 2.15 is valid. Thus, by differentiation, Eq. 2.15 can be rewritten as

$$\Delta n = -\frac{n}{\nu} \Delta \nu, \quad (2.16)$$

which then forms the second part of Eq. 2.10. Combining this result with Eq. 2.13 leads to the complete equation

$$\Delta\rho = -\frac{1}{A_R} \frac{2}{3} \frac{n}{\nu} \Delta \nu, \quad (2.17)$$

which can be used to determine the density change in terms of the changes of  $\nu$ . This equation is theoretically idealized, however, omitting the fact that  $\Delta \nu$  of Eq. 2.15 depends not only on the refractivity  $n$ , but also on the distance  $d$  of the cavity. Adding this contribution to Eq. 2.17 yields

$$\Delta\rho = -\frac{1}{A_R} \frac{2}{3} n \left[ \frac{\Delta \nu}{\nu} - \frac{\Delta d}{d} \right]. \quad (2.18)$$

This equation determines the performance requirements of the measurement system, for a given resolution in density. The second term inside the square brackets represents the

## 2. THE SUGGESTED TECHNIQUE

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frequency shift due to distance changes between the mirrors. At working frequencies in the visible or near infrared ranges, and necessary measurable frequency changes being far less than gigahertz, this clearly puts high demands on the setup. Making  $d$  near constant, or otherwise knowing how it changes in the working density range, is essential for the success of the experiment.

### 2.1.3 Working substance

One important part of the flowmeter that has not been discussed in detail, so far, is the working gas. It enters the mathematics basically through Eq. 2.8 and Eq. 2.9, in the form of virial coefficients  $A_R$ ,  $B_R$ ,  $B(T)$ , and  $C(T)$ . As stated earlier, the later two of these are not used when dealing with molar flow.

$A_R$  is proportional to the atomic and molecular polarizability  $\alpha$  through

$$A_R = \frac{4\pi}{3} N_A \alpha, \quad (2.19)$$

and  $B_R$  is considered negligible in the present case, as discussed before. Thus, the ideal choice of working substance would be one where  $\alpha$  can be determined with a high degree of accuracy.

In principle it is possible to use any gas in the flowmeter, as long as its properties are well known at the specific temperature and pressure considered. Although there exist data on several different substances, helium proves to be the most suitable choice (15), as it has well known refractive index properties. The reason for this is mainly because of the relative simplicity of the molecule, which makes it possible to accurately determine microscopic properties such as polarizability from initial theoretical calculations. These calculations can then be transferred to the macroscopic case through Eq. 2.19.

The polarizability for helium has been determined by (2; 10; 18) as

$$A_R = \left[ 0.15725407 + \frac{1197.5410}{\lambda^2} + \frac{3.290677 \cdot 10^6}{\lambda^4} + \frac{9.80084 \cdot 10^9}{\lambda^6} \right] 10^{-6}, \quad (2.20)$$

expressed in units of  $\text{m}^3/\text{mol}$ . Inserting the wavelength  $\lambda = 1530$  nm into this equation yields  $A_R = 0.5177662 \cdot 10^{-6} \text{ m}^3/\text{mol}$ , which is used in this text. The main uncertainty in this value comes from the uncertainty in the Avogadro constant. It should also be noted that (2) used wavelengths lower than  $\lambda = 1000$  nm, when making comparisons with other sources. This implies that a more detailed treatment of  $A_R$  is necessary to assure accurateness when working at  $\lambda = 1530$  nm.

Helium can further be used to estimate the change of length of the cavity in the considered density range, Eq. 2.18, as was done by (18). The method relies on the theoretical calculations of the refractive index of helium.

## 2.2 Flowmeter construction

The suggested flowmeter consists of a sealed chamber, wherein the optical cavity is placed. The sealed chamber is connected through a valve to some form of external low-pressure system, the simplest case being just a vacuum pump. A more sophisticated possibility would be to connect it to a volume expansion system (8; 11; 12), enabling measurements at lower pressures than otherwise possible.

If the flowmeter is to be run by changing its working volume, some form of variable volume needs to be connected, as well. This mode of operation is, however, somewhat questionable in the present case, as the goal of the experiment is in part to remove moving parts susceptible to wear. Detailed analysis and experiments on these ideas and the external vacuum system have been carried out (7).

The flowmeter must, at least initially or at some stage, be filled up with a clean working substance. This can in practice be done either through the same valve as mentioned above, or through a separate valve. In either case, care must be taken that the working substance is not contaminated, preferably by pumping down the system as much as possible before filling.

The optical cavity itself can be placed in the chamber in a few different ways. Depending on how the source laser beam enters the resonant cavity, it might be necessary to try to reduce vibrations of the system. One possible way would be to actually use the cavity itself as a chamber. There are certain difficulties with this setup, and many details rely heavily on the resonant cavity itself. Therefore a detailed discussion on this topic is postponed to Sec. 3.4.

### 2.2.1 Pressure range

The theoretical pressure range of the system is determined by the applicable regime of the fundamental equations used. In this text the ideal gas law has been used and the molar flow has been assumed viscous, thus setting a lower limit in pressure somewhere

## 2. THE SUGGESTED TECHNIQUE

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of the order  $10^2$  Pa. It is important to note, however, that since the flow meter measures the density, and if substance is lost from the flow meter, over time, it will be detected.

### 2.3 Cavity

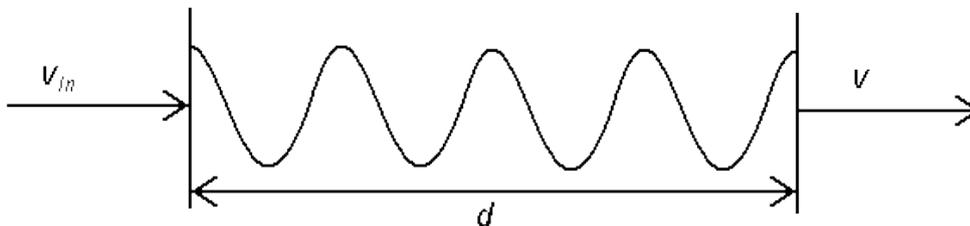
There is a correspondence, albeit small, between the refractive index of a medium and the density of it, as described earlier in Sec. 2. When the gas density approaches zero, or vacuum, the refractive index approaches unity, which is not a big surprise. As was mentioned above, one way to track the refractive index change is by using a resonant cavity as an optical filter. By locking a laser to a transmissive peak of the cavity, which depends on the refractive index, it is possible to obtain the density indirectly by measuring the frequency of the locked laser. By choosing the ingoing components of the cavity, such as mirrors and lenses, in an appropriate way, it is possible to track extremely small changes in frequency.

In the coming subsections the operation of a cavity will be described, as well as some related equations that will prove useful. The conditions for making the cavity stable, and equations for determining the necessary geometrical requirements of the system at hand will also be defined. The necessary stability of the setup and alignment sensitivity of the cavity will be handled at the end of the section.

The success of the method depends, in part, on how well it is possible to actually measure the frequency of the laser used. Possible usable laser sources will be discussed in Sec. 2.4 and ideas on how to accurately measure the laser frequency can be found in Sec. 2.5.

#### 2.3.1 Resonant cavities

The frequency selectivity properties of a cavity are based upon on the fact that a cavity is made resonant. As will be shown below the basic conditions for this are surprisingly simple. Based on these results the frequency selectivity properties will follow. However, it is important to remember that these simple results are based on some idealized assumptions. They will do fine as a connection between the transmissive frequency of the cavity and the refractive index of the substance therein, but to actually construct the cavity, some refinements will be necessary.



**Figure 2.1:** The incoming laser beam  $\nu_{In}$  is assumed broad in terms of frequencies. Only the frequencies that constructively interfere inside the cavity, will pass through.

### 2.3.1.1 Resonance condition

To be able to observe the frequency selectivity capabilities of a cavity, it needs to be in resonance. A simplified, yet useful way of determining the necessary conditions for resonance is by following one wave as it bounces back and forth between the mirrors. The incoming light  $\nu_{In}$  is assumed to be coherent, with a planar wave front, and of limited transverse extent. See Fig. 2.1 for the simplified cavity setup.

It is reasonable, for the matter of simplicity, to assume that there is an initial field near the input mirror. This field travels to the output mirror, reflects, and then travels back to the input mirror. During its propagation the field will experience an amplitude change, corresponding to the reflectivities of the mirrors. If the field returning from the output mirror is in phase with the initial field near the input mirror, constructive interference will occur. The cavity is thus resonant when the round trip phase shift is a multiple of  $2\pi$ . Taking into account the distance  $d$  between the mirrors, the resonance condition can be written as

$$2kd = q2\pi, \quad (2.21)$$

where  $q$  is the number of half wavelengths and  $k = 2\pi n\nu/c$ . Using the last relation leads to the equivalent resonance condition

$$d = \frac{qc}{2n\nu}, \quad (2.22)$$

but now in terms of the specific frequency  $\nu$  of the light source and the refractive index  $n$  of the medium inside the cavity. This is the key equation for measuring changes in refractive index.

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The transmittance through a cavity without losses can be shown (19) to be

$$T(\theta) = \frac{(1 - R_1)(1 - R_2)}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} \sin^2 \theta}, \quad (2.23)$$

where  $R_{1,2}$  are the reflectivities of each mirror and  $\theta = 2\pi d\nu/c$  is the optical length of the cavity.

Clearly, the cavity will transmit most light when the denominator in Eq. 2.23 is at minimum. This happens when  $\theta$  is an integer multiple of  $\pi$ , as expected by the resonance condition. If two identical mirrors are chosen,  $T(q\pi) = 1$  independent of  $R$ , which might be somewhat less expected. This is explained by the fact that Eq. 2.23 is derived with the assumption that the light source is coherent. Further, this demonstrates the energy storing capabilities of a resonant cavity.

### 2.3.1.2 Frequency selectivity

There are quite a few related important quantities that largely describe a cavity and its properties. These include the free spectral range (FSR), the peak full width at the half maximum (FWHM), the finesse ( $F$ ), and the photon lifetime ( $\tau_p$ ).

The FSR is defined as

$$\Delta\nu_{FSR} = \frac{c}{2nd} \quad (2.24)$$

and can be deduced from Eq 2.22. It describes the distance between the resonant transmissive peaks of a cavity, in terms of frequency  $\nu$ . The FWHM is a measure of the width of a transmissive peak at resonance. An approximate expression for the FWHM, which is fully acceptable when only the approximate shape of the peak is of interest, is

$$\Delta\nu_{1/2} = \frac{c}{2nd} \frac{1 - \sqrt{R_1 R_2}}{\pi (R_1 R_2)^{1/4}}. \quad (2.25)$$

It can be seen by application of Eq. 2.25 that higher values of the reflectivities yield sharper peak transmittance of the cavity. Another useful term for measuring the filtering properties of the cavity is the finesse, defined as

$$F = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}} = \frac{\pi (R_1 R_2)^{1/4}}{1 - \sqrt{R_1 R_2}}. \quad (2.26)$$

By inspection of Eq. 2.26 it can be seen that  $F$  gets higher if the distance between the peaks gets larger or if the peaks themselves get narrower. The main reason for using

the finesse as a measure of the cavity properties is that it depends solely on the mirror reflectivities, and thus is the same irrespective of the distance between the mirrors.

Note that all the above definitions are valid only for the special case of a simple two-mirror cavity system. When dealing with more general cavities, equations corresponding to Eqs. 2.25 - 2.26 can be somewhat hard to obtain. A simpler approach is then to calculate, or measure, the photon lifetime of the cavity.

Without going into details, the photon lifetime is defined as

$$\tau_p = \frac{\tau_{RT}}{1 - S}, \quad (2.27)$$

where  $S$  is the survival factor of the cavity and  $\tau_{RT}$  is the time of one round trip. The survival factor includes all the elements affecting the number of photons surviving one round trip. In the simple two-mirror cavity discussed above  $S = R_1 R_2$  and  $\tau_{RT} = 2nd/c$ . This implies that the cavity mode width and the finesse are related to the cavity linewidth by

$$\Delta\nu_{1/2} = \frac{1}{2\pi\tau_p} \quad (2.28)$$

and

$$F = \frac{2\pi}{1 - S}. \quad (2.29)$$

The photon lifetime concept can prove useful for determining the actual reflectivity of the mirrors used in the cavity, instead of relying on the data from the manufacturer. Further, if it at some stage becomes necessary to insert any optical components inside the cavity, the photon lifetime can be used to obtain the above mentioned information.

### 2.3.2 Gaussian beams

Until now uniform planar wave fronts with limited transverse extent have been assumed. While this simplifies the discussion, it does not necessary lead to the correct results. Lifting these restrictions on the wave propagation leads to the more correct description incorporating Gaussian-Hermite modes (19). Most equations introduced in Sec. 2.3.1 are still applicable, but some need to be refined. Specifically, while the round trip phase shift still must equal  $2\pi$  in accordance with the resonance condition, Eq. 2.21, Gaussian beams introduce a much more complicated phase description.

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### 2.3.2.1 Stability condition

A resonant cavity is considered stable if the beam inside it reproduces itself infinitely many times, or at least enough many times (marginally stable). The condition for this to happen can be defined as

$$0 \leq g_1 g_2 \leq 1, \quad (2.30)$$

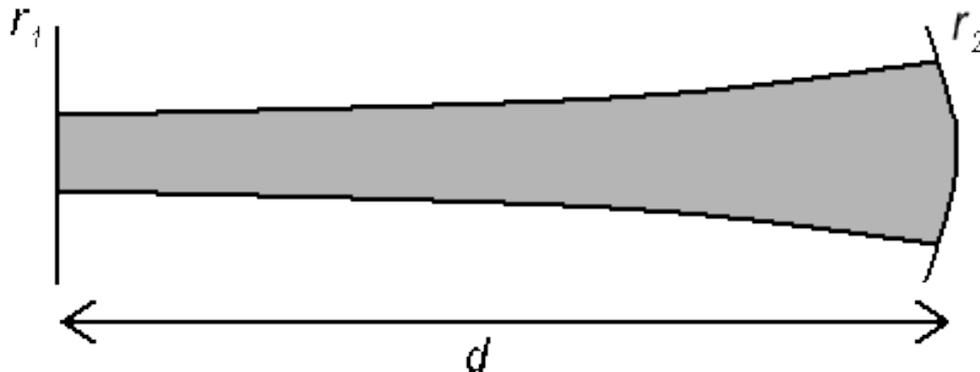
where  $g_{1,2} = 1 - d/R_{1,2}$ . Here,  $R_{1,2}$  refers to the radius of curvature of each mirror. Eq. 2.30 is based on the ray tracing model and hence it does not take into account the properties of the, in most cases, more correct gaussian beam shapes. A gaussian beam will not maintain an initially planar wavefront, but rather it will change to have a spherical wavefront (actually there is even more to it). Therefore, a cavity with planar mirrors will never be able to sustain a gaussian beam, at least not by itself, leading to the conclusion that in most cases a simple planar cavity is not useful. Still, Eq. 2.30 makes perfect sense as a rough estimation of the necessary cavity geometry.

Clearly, planar mirrors only are not a good choice when constructing a resonant cavity. A more suitable arrangement of mirrors would be either both mirrors spherical or one spherical and one flat mirror. The later, so-called, hemispherical arrangement has the advantage of simplifying Eq 2.47, but more importantly it places the beam waist of a gaussian beam right at the flat mirror. This greatly simplifies spatial mode matching, as will be seen later in 2.3.2.4. Using two spherical mirrors, one simple arrangement (confocal) could be such that the beam waist gets located in between the mirrors. However, this can result in a degenerate cavity, which is not very appropriate in the present case. Mode matching would also become a bit harder, though in no way impossible.

In this text a hemispherical cavity will be considered, as shown in Fig. 2.2. However, the arrangement with two spherical mirrors might have its advantages and should not be ruled out in future work.

### 2.3.2.2 Cavity with gaussian beams

A more exact description of resonant cavities, taking Gaussian beams into account, can be accomplished by ray transfer matrix analysis. A detailed treatment of the analysis method will not take place in this text, only the main procedure will be outlined. A much more complete walkthrough can be found in other publications (19).



**Figure 2.2:** The cavity setup, showing a propagating gaussian beam. The input window is planar, thus  $r_1 = \infty$ .

A gaussian beam can be described by the complex beam parameter

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda_0}{\pi n w^2(z)}, \quad (2.31)$$

where  $R(z)$  is the radius of curvature of the beam wavefront, the second term is the beam waist, and  $z$  is the distance from the minimum spot size along the axis of propagation. The spot size is defined as  $z_0 = n\pi w_0^2/\lambda_0$ , from which it is possible to rewrite Eq. 2.31. The full expression for  $R(z)$  is

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right] = z \left[ 1 + \left( \frac{\pi n w_0^2}{\lambda_0} \right)^2 \right], \quad (2.32)$$

which can be used to determine the curvature of the wavefront at an arbitrary point  $z$ . The full expression for the beam waist is

$$w^2(z) = \frac{\lambda_0 z_0}{\pi n} \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right] = w_0^2 \left[ 1 + \left( \frac{\lambda_0 z}{\pi n w_0^2} \right)^2 \right]. \quad (2.33)$$

A Gaussian beam traveling through a waveguide of optical components can be transferred through

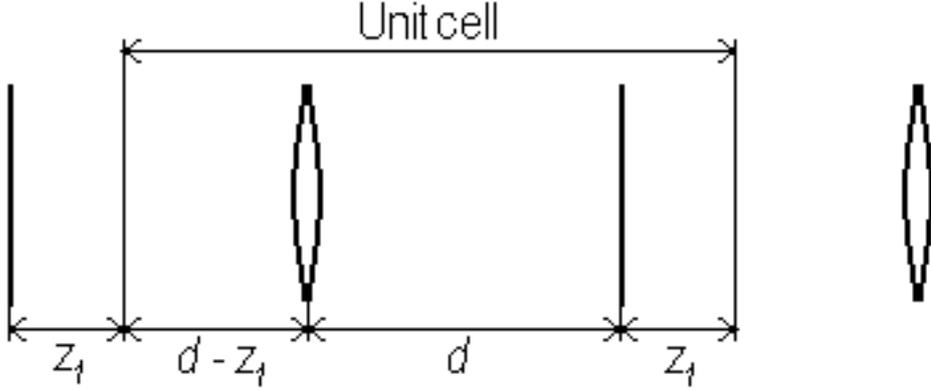
$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}. \quad (2.34)$$

$A$ ,  $B$ ,  $C$ , and  $D$  can be found through transmission matrices on the form

$$\mathbf{T} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \quad (2.35)$$

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**Figure 2.3:** The unit cell is shown in the waveguide that corresponds to the resonant cavity. The bars represent the flat mirror, while the lenses represent the curved mirror.

where the form of the matrix depend on the optical component. The total transmission through a waveguide can be obtained by multiplying the matrices of all components in inverse order, i.e.

$$\mathbf{T}_{Total} = \mathbf{T}_n \cdots \mathbf{T}_1. \quad (2.36)$$

The outlined procedure can also be used to determine the parameters of a stable resonant cavity. By forcing the Gaussian beam to transform into itself after it has traveled one full round-trip through the cavity, an equivalent waveguide can be formed.

For the purposes in this experiment, a hemispherical cavity will be used with one planar and one spherical mirror, as shown in Fig. 2.2. Assuming that the cavity is stable, a corresponding waveguide can be constructed, as shown in Fig. 2.3. Applying the condition that the Gaussian beam transforms into itself after each round-trip, Eq. 2.34 transfers to

$$q = \frac{Aq + B}{Cq + D}. \quad (2.37)$$

Using the fact (19) that  $AD - BC = 1$ , Eq. 2.37 can be rewritten as

$$\frac{1}{q(z)} = -\frac{A - D}{2B} - j \frac{\sqrt{1 - \left(\frac{A+D}{2}\right)^2}}{B}, \quad (2.38)$$

thus conforming to the form of the equation describing a complex beam, Eq. 2.31. The matrix components can be determined by considering the unit cell in the waveguide

shown in Fig. 2.3. The curved mirror of the cavity has been replaced by a thin lens in the waveguide. This replacement is motivated by the fact that a curved mirror mathematically corresponds to a thin lens through  $f = R/2$ , where  $f$  is the focal length of the lens and  $R$  is the radius of curvature of the mirror.

The transmission matrix for free space and a thin lens are

$$\mathbf{T}_{Space} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}, \quad (2.39)$$

and

$$\mathbf{T}_{Lens} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad (2.40)$$

respectively, where  $d$  is the distance between the two mirrors. Combining these in accordance with Eq. 2.36, yields the total transmission matrix of one round-trip of the cavity,

$$\mathbf{T} = \mathbf{T}_{Space} \mathbf{T}_{Lens} \mathbf{T}_{Space} = \begin{pmatrix} \frac{R+2z_1-2d}{R} & \frac{2dR+2z_1^2-2d^2}{R} \\ -\frac{2}{R} & \frac{R-2z_1-2d}{R} \end{pmatrix}, \quad (2.41)$$

where  $f = R/2$  has been used and where  $z_1$  is the distance from the flat mirror to the starting point of the unit cell. By inspection of components  $A$  and  $D$  of Eq. 2.41, it is possible to see that choosing  $z_1 = 0$ , i.e. starting the unit cell at the flat mirror, leads to  $A = D$ . According to Eq. 2.38 this corresponds to the beam having a planar phase front, since the real part of the equation vanishes, and thus the spot size has been found. Inserting the components of Eq. 2.41 into Eq. 2.38 and performing simplifications, yields

$$\begin{aligned} \frac{1}{q(z)} &= \frac{1}{R(z)} - j \frac{\lambda_0}{\pi n w^2(z)} \\ &= -\frac{A-D}{2B} - j \frac{\sqrt{1 - \left(\frac{A+D}{2}\right)^2}}{B} \\ &= 0 - j \frac{1}{\sqrt{d(R-d)}}. \end{aligned} \quad (2.42)$$

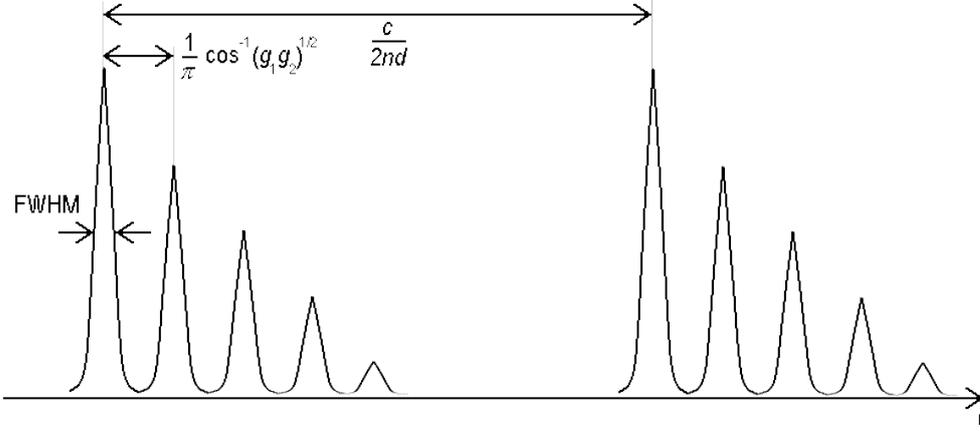
Thus, the last term on the bottom line of Eq. 2.42 is the spot size

$$z_0 = \frac{\pi w_0^2}{\lambda_0} = \sqrt{d(R-d)}. \quad (2.43)$$

By inserting the spot size  $z_0$  into Eq. 2.31, through application of Eqs. 2.32 - 2.33, it is possible to find the radius of the beam at any point along its propagation. This in turn makes it possible to find the correct radius of the spherical mirror, for the cavity to be resonant.

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**Figure 2.4:** A generic frequency spectrum showing the mathematical relations between the peaks. To get high transmission of the fundamental mode, it might be necessary to actively suppress the higher order modes. The cavity should be properly mode matched to the the fundamental mode, if high transmission of this mode is necessary.

### 2.3.2.3 TEM<sub>*m,p,q*</sub> modes in resonant cavity

Without going into the mathematical maze of Gaussian beams, the resonant TEM<sub>*m,p,q*</sub> mode frequencies of a simple resonant cavity with two spherical mirrors are given by (19)

$$\nu_{m,p,q} = \frac{c}{2nd} \left[ q + \frac{1+m+p}{\pi} \cos^{-1} \sqrt{g_1 g_2} \right], \quad (2.44)$$

where  $g_{1,2} = 1 - d/R_{1,2}$ , as before. Thus, for each longitudinal mode  $q_0$  there is a whole spectrum of transverse TEM<sub>*m,p,q\_0*</sub> modes at different frequencies. A generic frequency spectrum is printed in Fig. 2.4, showing how the different modes are related. If the frequency of a transverse mode coincides with that of a longitudinal mode, such that

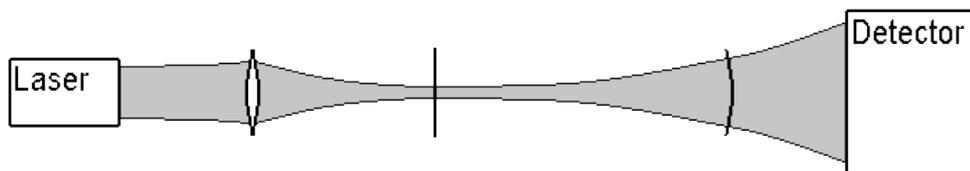
$$\nu_{m,p,q} = \nu_{0,0,q+\Delta q}, \quad (2.45)$$

the cavity is said to be degenerate. Combining Eq. 2.44 and Eq. 2.45 yields the condition for a mode degenerate cavity as

$$\frac{m+p}{\pi} \cos^{-1} \sqrt{g_1 g_2} = \Delta q. \quad (2.46)$$

Solving further for  $g_1 g_2$ , yields

$$g_1 g_2 = \cos^2 \left( \frac{\Delta q \pi}{m+p} \right) \quad (2.47)$$



**Figure 2.5:** A highly exaggerated picture showing a beam propagating through a waveguide and how mode matching can be performed. The lens on the left side of the cavity might need to be replaced by a doublet to reduce aberrations on the wave front.

from which the corresponding conditions on the length of the cavity and the radius of curvature of each mirror can be determined (recalling  $g_{1,2} = 1 - d/R_{1,2}$ ). A degenerate cavity should be avoided, however, if the interest lies in obtaining precise frequency selectivity, since it leads to broadening in the peaks.

#### 2.3.2.4 Spatial mode matching

Spatial mode matching of the cavity can be accomplished through the use of ray transfer matrix analysis, much the same way as was done in Sec. 2.3.2.2. In short, a lens, or lens train, needs to be placed in front of the input window of the cavity, as is shown in Fig. 2.5, in such a way that the output parameter  $q_2$  of the lens wave guide equals Eq. 2.43 (which is purely imaginary since the phase front must be planar at the input mirror). If the used laser outputs a clean Gaussian beam of first order, with a planar phase front, this calculation is straight forward.

Depending on what type of detector is used on the output side of the cavity, a similar analysis as above might be necessary. This would be especially important if the output is to be collimated into a fiber. On the other hand, assuming that the detector is a large intensity measuring device, or in short, a photo detector, it would be sufficient to just place it directly after the cavity.

#### 2.3.2.5 Wavefront

The fundamental requirement for obtaining resonance is constructive interference. This implies that the wavefront of the beam inside the cavity must remain in phase, which in turn requires the surfaces of the mirrors to be smooth to within parts of wavelength. For cavities with spherical mirrors some sources (17) suggest that a flatness of  $\lambda/10$

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might be sufficient, at least for low finesse cavities, i.e.  $F < 100$ . Higher finesse cavities require smoother mirrors, and generally mirrors should be chosen as smooth as possible to minimize the risk with disturbed wavefronts. Especially when considering the labor required to construct the cavity in the first place, this trivial source of error should be avoided, whatever the finesse of the cavity is.

### 2.4 Laser

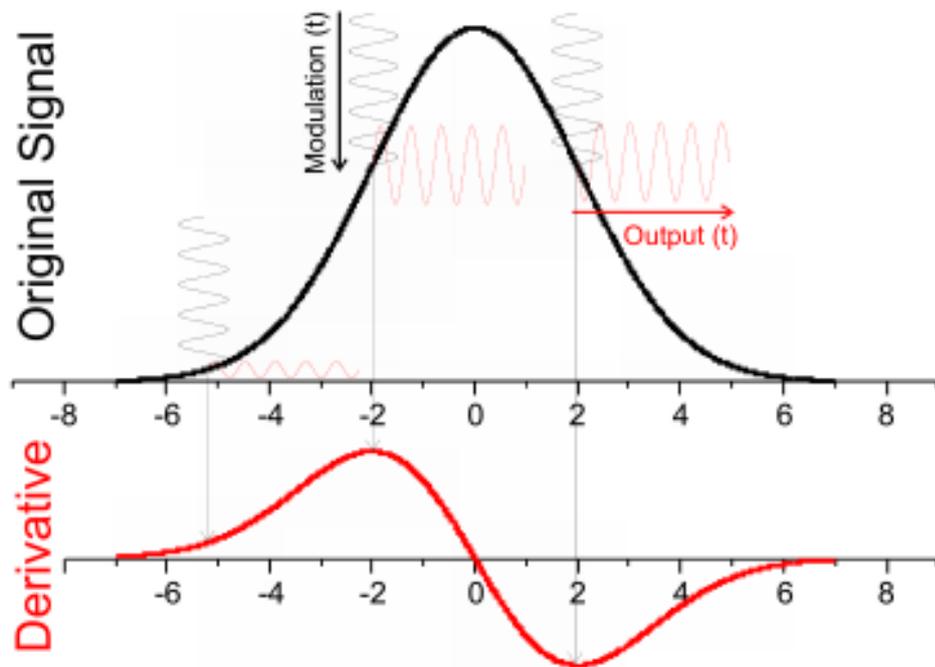
The most important requirements on the laser source are the beam shape and the tunability possibilities. The resonant cavity needs to be mode matched properly, according to Sec. 2.3.2.4. The beam shape of the laser must be Gaussian for this to work efficiently, at least without introducing complex mathematics. The requirement that the laser needs to be tunable comes from the fact that the laser needs to be locked onto a transmission peak that changes with gas density. The linewidth of the laser should obviously not be larger than the frequency that needs to be measured. Based on how accurately, and in which range, frequency changes need to be measured, a laser source can be selected.

#### 2.4.1 Laser selection

It is out of scope of this text to choose a suitable laser source, but a very thorough treatment of a similar system has been done (4). The requirements on the frequency of the laser, in terms density change sensitivity, are primarily ruled by Eq. 2.18. It can be tempting to conclude that making the frequency higher, would yield better results. However, this rises the requirements on the  $\Delta d/d$  term, just as much. Therefore, the main concern when choosing laser should not be its frequency, but rather its stability and linewidth.

#### 2.4.2 Frequency locking

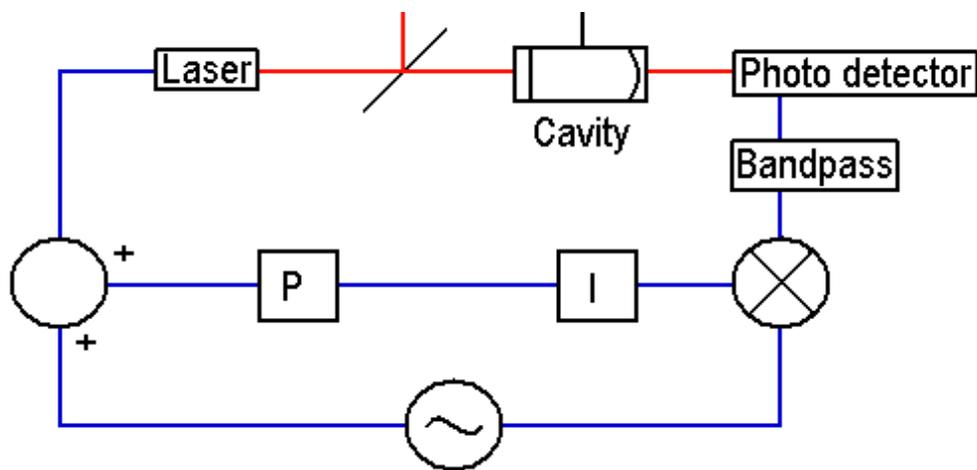
There are various established methods for performing laser frequency locking. For the purposes of this project it is beneficial to lock onto the top of the transmission peak. The simplest locking (although not that trivial in reality) can be performed by using a PID-regulator together with an electrical mixer and a frequency generator for modulating the laser. This generates an error signal that is equal to the derivative of



**Figure 2.6:** The derivative signal from frequency modulated laser locking. Image courtesy to (14).

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**Figure 2.7:** A lock-in amplifier with one proportional and one integrating block. Blue lines are electrical, while red lines represent laser light. The laser is frequency modulated at a specific frequency by a wave generator. The photo detector detects the signal when the cavity is on a transmission peak. The bandpass filter filters the signal so that only the specific frequency component is left. The filtered signal is mixed with the signal from the wave generator, which results in a signal containing the derivative of the position of the transmission peak. The proportional and integrating block perform the servo steering and generate an error signal. Finally, the loop is closed when the error signal is added to the modulation frequency signal, and the laser adjusts according to the error signal.

the transmission peak. The mathematical details are a bit involved, and will not be treated here, but the principle is shown in Fig. 2.6. A lock-in amplifier based on a PI-regulator, adapted for the present setup, is shown in Fig. 2.7.

A similar technique is the Pound-Drever-Hall scheme, which make use of side peaks to make a tighter lock to the main peak. The details for laser locking will no be given here, but can be found in, for instance (1; 14).

## 2.5 Frequency reference

The frequencies need to be detected with an accuracy in the MHz range, while visible and near infrared light lie in the upper THz range. No ordinal optical spectrum analyzer can resolve these frequencies with the required accuracy. By using optical heterodyne detection, it is possible to overcome this limitation by measuring the difference between the unknown frequency and the frequency of an accurate fixed-frequency laser.

It follows that the accuracy of the method is primarily limited by the frequency stability of the reference laser. Possible reference laser candidates are proposed later in this section.

Heterodyne detection is based on the multiplication of two periodic signals, which results in one difference signal and one sum signal. The mathematical principle of heterodyne detection, mixing two periodic waves, follows

$$\sin(2\pi\nu_1 t) \sin(2\pi\nu_2 t) = \frac{1}{2} \cos(2\pi(\nu_1 - \nu_2)t) - \frac{1}{2} \cos(2\pi(\nu_1 + \nu_2)t). \quad (2.48)$$

Since a standard photo detector detects energy present upon it, and energy detection inherently is proportional to the square of the incoming intensity, the output current will contain any mixed frequencies (19). By directing the laser beam together with the reference beam at the detector, it is possible to automatically perform the multiplication and thus heterodyne detection.

The output difference signal can be observed by an electrical spectrum analyzer, or for more precise results, it may be fed into a frequency counter. The important condition is that the difference between the unknown frequency and the reference frequency is in the working range of the counter. Frequency counters up to a few GHz are readily available.

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### 2.5.1 Optical frequency comb

A quite recent progress in laser science is the development of highly stable optical frequency combs (16). There are systems that can output a comb of frequencies spaced only 250 MHz apart, throughout the whole visible spectrum and into the near infrared range (16). The wide selection of available frequencies simplifies the selection of laser source, described in Sec. 2.4. Optical heterodyne detection with such a system requires the use of optical bandpass filters, primarily to reduce the intensity of the beam hitting the photo detector, and also to reduce the present frequencies.

### 2.5.2 Molecular transition locked lasers

An iodine stabilized He-Ne laser could be used as reference. A disadvantage, although not very severe, is that source laser must operate in the vicinity of 633 nm. Another possibility could be to use saturated absorption spectroscopy of atomic rubidium, as have been done in other heterodyne applications (9), with the frequency  $\nu = 384227981.877$  MHz (20).

### 2.5.3 Other possible laser references

It is out of scope of this text to list all possibilities. It is probably enough to state that the single most important parameter is that the reference laser fulfills the frequency stability requirements of the flowmeter as a whole.

## 3

# Proposed experimental setup

A number of factors need to be taken into account when constructing the flowmeter, according to the principles given above. The obtainable density resolution of the system highly depends on the choice of components, and the possibilities to control the surrounding environment. High accuracy measurements cannot be expected unless every detail, from source laser to external vacuum system, are well set up and properly characterized.

The system proposed in this paper should be reasonably easy to build, but nevertheless, it should yield results in accordance with theoretical estimations. Because of this, it is most important that fundamental parts of the setup are chosen in a way consistent with theoretical calculations and that uncertainties are found and pointed out (characterized as far as possible). With this in mind, it should be quite possible to find parts of the setup where refinements can readily be made, and thus achieve better results.

The overall system design follows the schematic shown in Fig. 1.1, while more detailed layouts are presented, when appropriate, in the subsections below.

### 3.1 Laser source

Preliminary tests performed with a 196 THz laser show that it is very reasonable to believe that an accuracy in frequency  $\Delta\nu = 15$  MHz is obtainable, in terms of possible laser locking width. The laser locking should be performed according to the ideas of Sec. 2.4. In the initial setup, the simpler PID-regulator based locking method should

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be sufficient, but this can always be changed relatively easy to a more sophisticated method later relatively easy (not taking into account the trimming in of the regulator).

The figure  $\Delta\nu = 15$  MHz corresponds to an accuracy in density  $\Delta\rho \approx 10^{-1}$  mol/m<sup>3</sup>, according to Eq. 2.17. Since, in the end, it is the flow rate that is the quantity of interest, a conversion according to Eq. 2.10 must be carried out. Assuming the chamber volume of the flowmeter to be  $V = 10^{-3}$  m<sup>3</sup> = 1 dm<sup>3</sup>, the molar flow can be determined to be

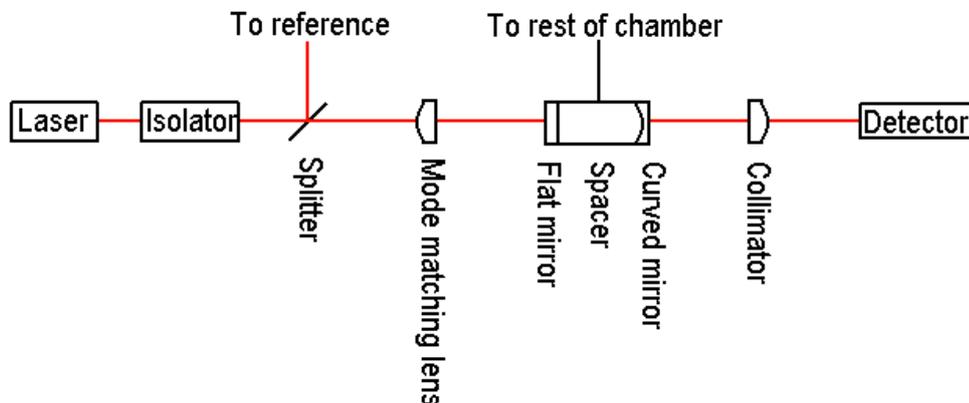
$$q = V \frac{\Delta\rho}{\Delta t} = \frac{10^{-4}}{\Delta t} \text{ mol/s}, \quad (3.1)$$

where  $\Delta t$  can be interpreted as the reading rate. For instance, requiring a reading only once every ten seconds would allow a flow rate  $q = 10^{-5}$  mol/s. Eq. 3.1 clearly shows that simply reducing the volume of the flowmeter is an obvious way to obtain lower flow rates. These numbers for the flow rate should be used as an indication only, though, as there are uncertainties related to variations of the chamber geometry, as discussed below in Sec. 3.4.

### 3.2 Cavity geometry and dimensions

There are various parameters to consider when constructing the resonant cavity. From a molar flow detectability point of view, it is advantageous to have the resonant cavity, or rather the whole flow meter chamber, as small as possible. The reason for this can be tracked down to the basic fact that less molecules need to leave the chamber for a given density change, which is the detection entity. On the other hand it might be difficult to construct a very short cavity. The strictly hemispherical arrangement requires the curved mirror to have a correspondingly short radius of curvature, in order to maintain focus at the flat mirror. Such a mirror might be hard to find. A more serious problem, however, would be to do spatial mode matching on such cavity, since using a simple lens would introduce aberrations on the wave front.

A more suitable setup is to use a hemispherical cavity where the radius of the curved mirror is much larger than the distance between the mirrors. This setup introduces a spectrum of transverse TEM<sub>*m,p,q*</sub> modes at different frequencies, according to Sec. 2.3.2.3. According to Eq. 2.44, a larger ratio between distance and curvature of the curved mirror, yields closer spacing between the modes. With a clever choice



**Figure 3.1:** The proposed arrangement of the optical part of the setup.

of distance between the mirrors, however, it should not be too hard to find the fundamental frequency, and lock onto it. Of course, the difference in frequency between the modes must be large enough in comparison with the FWHM of the fundamental mode. There are advantages to generate these  $TEM_{m,p,q}$  modes also. Broadening of the fundamental frequency is avoided, by shifting the other modes out of the fundamental mode. The frequency difference between the modes could also be used to determine either the exact distance between the mirrors, or the exact curvature of the spherical mirror. Most important, this setup makes spatial mode matching considerably easier to do, as the lens used should have a focal length of the same order as the curved mirror.

The necessary components, from laser source to photo detector used for locking, are described in detail below. A schematic is shown in Fig. 3.1.

### 3.2.1 Mirrors

The requirements on the cavity can be determined by calculating the free spectral range, Eq. 2.24, and the FWHM, Eq. 2.25. Setting the length of the cavity and the reflectivities of the mirrors, somewhat arbitrary, to  $d = 1$  cm and  $R_1 = R_2 = 97\%$ , respectively, yield the FSR

$$\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{c}{2 \cdot 10^{-2}} \approx 15 \cdot 10^9 \text{ Hz}, \quad (3.2)$$

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and the FWHM

$$\Delta\nu_{1/2} = \frac{c}{2nd} \frac{1 - \sqrt{R_1 R_2}}{\pi (R_1 R_2)^{1/4}} = \frac{c}{2 \cdot 10^{-2}} \frac{1 - 0.97}{\pi (0.97)^{1/2}} \approx 145 \cdot 10^6 \text{ Hz.} \quad (3.3)$$

Assuming it reasonable to lock onto less than 10 % of the transmission peak (5),  $\Delta\nu_{1/2}$ , yields the desired resolution  $\Delta\nu = 15 \text{ MHz}$ , as stated above. An easy way to improve this figure, at least from a theoretical viewpoint, is to use mirrors with higher reflectivity. The moderate reflectivity choice in this case simplifies the lining up of the cavity, and there is no reason to construct a cavity with a linewidth that is smaller than that of the laser used. The finesse of this cavity is  $F \approx 100$  according to Eq. 2.26, which allows for not easy, but possible, lining up by hand (17), while higher finesse cavities require more sophisticated methods (4). Basically, part of this sophistication must be included anyway, as will be seen when considering stability requirements in the next section, thus  $F \approx 100$  should be possible to attain.

#### 3.2.2 Spacer

The space between the mirrors must be analyzed in detail, regarding both the connection to the rest of the chamber and the stability of the resonant cavity itself. Eq. 2.18 of Sec. 2.1.2,

$$\Delta\rho = -\frac{1}{A_R} \frac{2}{3} n \left[ \frac{\Delta\nu}{\nu} - \frac{\Delta d}{d} \right], \quad (3.4)$$

describes the frequency shift introduced when the distance between the mirrors changes. Preliminary tests at some points in this text have been made with a laser in the vicinity of  $\nu = 196 \text{ THz} = 196 \cdot 10^{12} \text{ Hz}$ , so this value will be used here as well. Together with  $\Delta\nu \approx 10^7 \text{ Hz}$  (according to the earlier mentioned  $\Delta\nu \approx 15 \text{ MHz}$ ) yields

$$\frac{\Delta\nu}{\nu} = \frac{10^7}{196 \cdot 10^{12}} \approx 5 \cdot 10^{-8}, \quad (3.5)$$

settling the requirement on the stability in distance between the mirrors. It is not controversial to directly conclude that a simple air-spaced arrangement will not do in this case. Rather, the mirrors need to be lined up by the use of one or more spacers, preferably made out a material with a low thermal expansion coefficient. The term  $\Delta d/d$  can be treated as the thermal expansion coefficient of the spacer. It is obvious, by looking at the numerical value of Eq. 3.5, that the selection of available suitable

materials is not very wide (13). Further, this implies that any temperature gradients inside the cavity must be measured and, possibly, compensated for. As such, long-term stability in temperature is not strictly required, as long as any changes can be measured with an accuracy not lower than that corresponding to  $\Delta\nu/\nu$ . The short-time stability in temperature is primarily determined by the locking system of the laser. A possible solution is to use zerodur, which is claimed to have a thermal expansion coefficient  $\beta < 5 \cdot 10^{-8} / \text{K}$ .

Exact numbers for allowed temperature fluctuations are not given here, as they depend heavily on the material choice, but they should be simple to estimate. Clearly, temperature sensors must be attached to the cavity setup at various strategic points, irrespective of material selection. Note also that the discussion about temperature here only takes into account the mirror setup itself, not the flowmeter as a whole.

### 3.2.3 Lenses (Spatial mode matching)

Depending on the profile of the source laser beam, an appropriate lens (or lens train), must be inserted before the input mirror of the cavity, in order to make it behave in an optimal way. The shape of the lens is calculated according to Sec. 2.3.2.4.

### 3.2.4 External optical components

The external optical components in part depend on the choice of laser source and laser reference.

A beam splitter, placed either before or after the cavity, is used to split out a measurement signal for the heterodyne measurement. Preliminary tests suggest that the laser beam taken out of the system should be on par with the reference signal, in terms of irradiance. This condition, however, can be relaxed quite a bit. If placed in front of the cavity, it is far more important that the beam splitter retains the shape of the wave front, than anything else.

The light needs to enter the cavity in some way. If a free-space approach is used, it is most important that the entrance window of the chamber does not distort the wave front of the beam. A fiber solution might be harder to setup, but it has advantages. Lenses coupling the beam out of the fibers must be used. If the resonant cavity is aligned up properly and integrated with the collimating fiber lenses and the beam splitter, it can be made very stable.

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Possibly an iris needs to be placed somewhere in the system to reduce the width of the laser beam and suppress higher-order  $\text{TEM}_{m,p,q}$  modes.

A photo detector, used for feedback to the locking system of the source system, is placed as the last component of the system. The basic requirement of the detector is that it is fast enough for the locking mechanism. Obviously, it must operate in the same frequency range as the source laser.

#### 3.3 Laser reference

The basic requirement of the laser reference is that it operates in the same frequency region as the working laser of the flowmeter. With this basic requirement fulfilled, there should be no problem to carry out the actual heterodyne measurements. The next requirements that need to be considered, concern the stability and accuracy of the reference in terms of frequency. These parameters, or specifications, of the reference laser must be at par with the flowmeter, to not become a performance limiting factor. Considering the fact that measurements down to at least the kHz range have been made (20), there should be no problem to fulfill these requirements.

The optical frequency comb discussed in Sec. 2.5 seems to be the most promising option to use as a reference. It has a very wide working range in terms of frequencies, making it possible to concentrate on the characteristics of the working laser rather than its frequency, and has good performance characteristics. Another advantage is that it is already installed at the facility.

With the optical frequency comb spikes being spaced 250 MHz apart, a frequency counter of the same region is required, with a resolution of at least 15 MHz. The photo detector used should obviously have its working range in the same range as the source laser.

#### 3.4 Working chamber and connection to external low-pressure system

The resonant cavity setup should be aligned properly and then placed inside the working chamber.

### 3.4 Working chamber and connection to external low-pressure system

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Another possibility is to use the cavity itself as a chamber, but this requires it to be vacuum sealed, something that can be hard to achieve. A disadvantage with this method is that the cavity will be exposed to the atmosphere, making it sensitive to pressure and temperature changes. When changing the pressure inside the cavity, this could lead to the mirrors slightly changing geometry, jeopardizing the accuracy of the flow meter.

Irrespective of the method chosen, the molar flow, or rather change in amount of substance  $\Delta N$ , in the chamber, can be determined by using Eq. 2.7, here expanded to include the thermal expansion

$$\begin{aligned} q\Delta t = \Delta N &= V\rho \left[ \frac{1}{\rho}\Delta\rho + \frac{1}{V}\Delta V \right] \\ &= V\rho \left[ \frac{1}{\rho}\Delta\rho + \frac{1}{V}\frac{\Delta V}{\Delta T}\Delta T \right]. \end{aligned} \quad (3.6)$$

The volume is chosen to be held constant, since this permits the flowmeter to be built without any moving parts, other than the necessary connection valves. As noted in Sec. 2.1.1, only the second term in the square brackets contributes to the uncertainty of molar flow, or change in amount of substance, measurement.

The density of a gas at atmospheric pressure and room temperature is  $\rho \approx 41 \text{ mol/m}^3$ . A density change  $\Delta\rho \approx 10^{-1} \text{ mol/m}^3$ , as stated back in Sec. 3.1, corresponds to a change in the density term  $\Delta\rho/\rho \approx 2.4 \cdot 10^{-3}$ . These assumptions are mainly used to show the approximate requirements on the  $\Delta V/V$  term, and are not necessary to follow in the final setup. As stated earlier, the thermal expansion coefficient of many common metals (13) is of the order

$$\beta = \frac{1}{V} \frac{\Delta V}{\Delta T} \approx 10^{-5}/\text{K}, \quad (3.7)$$

being of order  $10^2$  less than the measurement quantity. Lowering the initial density  $\rho$  makes the contribution to the measurement accordingly smaller.

Obviously, building the chamber out of a material with a lower thermal expansion coefficient, would result in even better results. If the resonant cavity itself is used as a chamber, this will almost certainly be the case, since, considering the conclusions of Sec. 3.2.2, the cavity should be built of low thermal expansion materials. It can be concluded that measurement of the temperatures at various strategic points across the

### 3. PROPOSED EXPERIMENTAL SETUP

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chamber, is advantageous in order to obtain exact results, but is not necessary to a reasonable degree in accuracy.

Further, efforts should be made to make the chamber free of vibrations. In principle, the theoretical setup is resistant to vibrations, but possible changes in geometry, especially concerning the distance between the mirrors, cannot be rejected if the setup is exposed to vibrations. It is not very easy to quantify these contributions, therefore care should be taken when connecting the flow meter to the external low-pressure system. This especially concern any pumps that might be present in the low-pressure system.

## 4

# Conclusions

The fundamental theory for the operation of the proposed flowmeter has been laid out in this thesis. According to theoretical and mechanical assumptions it should be fairly easy to construct a flowmeter with an accuracy in density  $\Delta\rho \approx 10^{-1} \text{ mol/m}^3$ . This number corresponds to a frequency accuracy  $\Delta\nu = 15 \text{ MHz}$ .

### 4.1 Possible uses

The flowmeter could be used as a flow generator to generate a reference leak, with the above mentioned accuracy. Helium was found to be the best source as reference substance, since there is most known about it. Other substances should, however, not be ruled out in the future, as the fundamental theory is applicable regardless of this choice.

In principle, the flow meter could be fit for any working pressure, as long as practical limits are obeyed. However, presently only the classical regime has been considered, where a gas behaves more or less ideally and the gas flow is viscous. Therefore, no clues can be given in the current work on the possible performance characteristics outside these limits. It can be concluded that the working range of the flowmeter lies between a few hundred Pa up to atmospheric pressure. Nevertheless, with suitable modifications to the theory, it should be very possible to extend the pressure outside the classical regimes.

A correctly calibrated flowmeter could possibly be used as a gas detector. This of course requires better knowledge of the virial coefficients introduced back in Sec. 2.1.2.

## 4. CONCLUSIONS

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### 4.2 Limitations and improvements

The frequency accuracy  $\Delta\nu = 15$  MHz was found to be, when initial trials were performed, mainly limited by the linewidth of the unlocked source laser. It is not unreasonable to believe that a properly locked laser would enhance these numbers by several orders of magnitude.

The main theoretical weakness of the proposed flowmeter is the temperature dependence of the spacer between the mirrors. Therefore, this should be the first problem to resolve. As a first measure, the spacer must be made of a material with low thermal expansion and the temperature must be controlled to a high degree. There are further, more sophisticated steps that can be taken.

A very promising way to improve the theoretical resistance against temperature changes, is by constructing a dual laser system. Details cannot be given here, but the basic idea consists of locking two lasers at different frequencies to the resonant cavity, and following each of them. By observing the difference in frequency, when the density changes inside the cavity, it is possible to mathematically compensate for any temperature changes. It should not be impossible to route both laser beams through the same path in the cavity and then split them on the output side of the it. Obviously, appropriate optical filters and other components must be added to the setup for this to work. The advantage to use the same routing through the cavity is that this implies that the lasers measure the exact same properties of the gas inside the cavity. The optical frequency comb is very well suited for these kind of measurements, because of its simultaneous 250 MHz spacing between the spikes through the very wide working range.

When the obstacles concerning temperature have been removed and the locking mechanism has hit its limit, a possible next step to increase accuracy is to construct higher finesse cavity. It cannot be mentioned too many times, though, that the locking mechanism of the laser should be refined as much as possible first, before using a higher finesse cavity.

The lining up of the cavity, especially when using a high finesse cavity, might be simplified by using piezo actuators on the mirrors. If using a high finesse cavity, this simplification can actually become a requirement.

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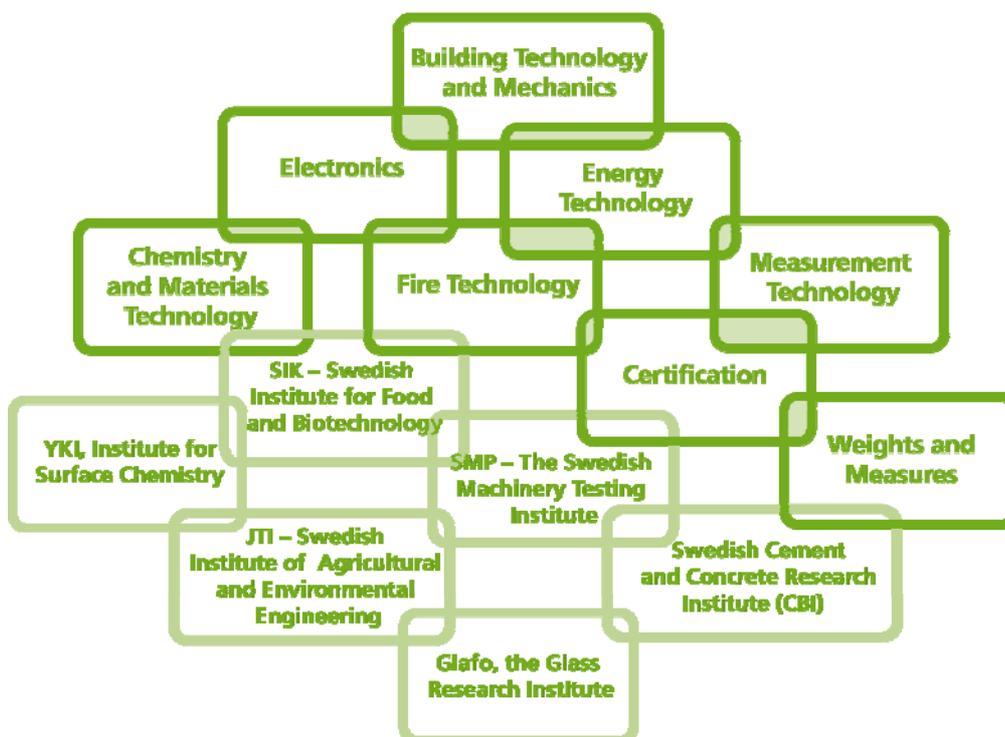
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