Khalil Raisi

Determination of Susceptibility and Magnetization of Mass Standards

Thesis for the Degree of Master of Science, April 2004

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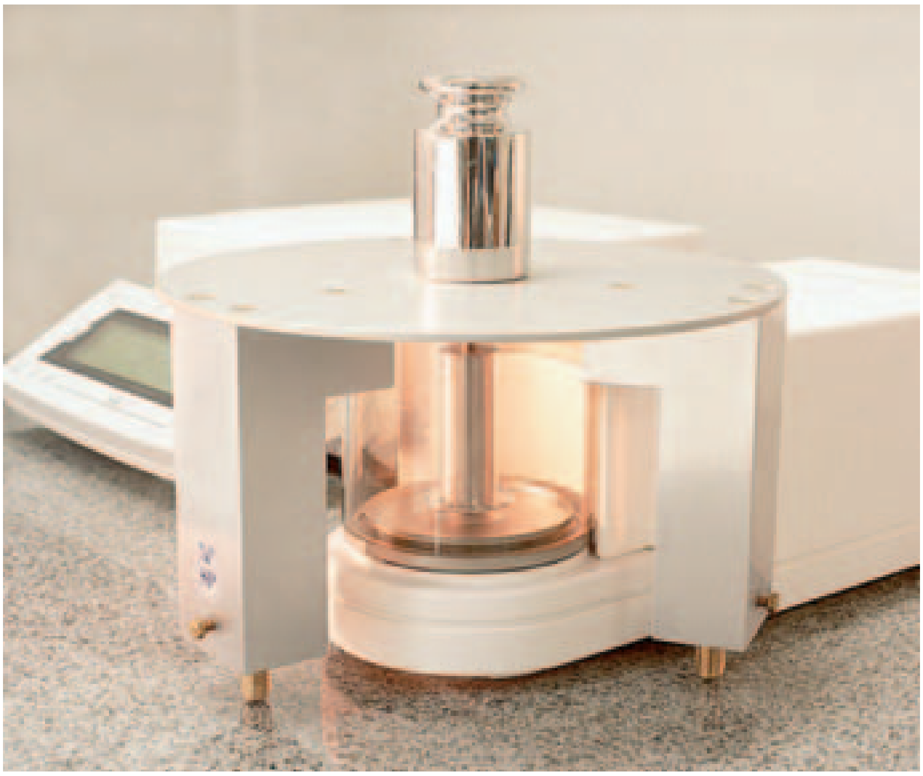

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Khalil Raisi

Determination of Susceptibility and Magnetization of Mass Standards

Abstract

In this report the magnetic properties of mass standards are determined using a susceptometer. Most models used for estimation of magnetic susceptibility and permanent magnetization, with the BIPM susceptometer, are based on the assumption of a magnetic field intensity from a magnetic dipole. This field model has been compared to magnetic field intensities derived by using the Biot-Savart Law. The main conclusion is that there are no significant differences between the different models for determining the magnetic properties of the mass standards when the measurements are performed at distances larger than 17 mm from the susceptometer magnet. It is adequate to use the dipole approximation to model the field of a cylindrical magnet. Furthermore the measured results of the magnetic properties of mass standards depend on the magnitude of the magnetic field intensity.

SP has during this project, participated in an international comparison for determination of the magnetic properties of mass standards. Calculations of the permanent magnetization and susceptibility agree with results obtained by other participants in the international comparison.

Key words: magnetization, permeability, susceptibility, mass standards, susceptometer.

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SP Swedish National Testing and Research Institute
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Postal address:
Box 857,
SE-501 15 BORÅS, Sweden
Telephone: +46 33 16 50 00
Telex: 36252 Testing S
Telefax: +46 33 13 55 02
E-mail: info@sp.se

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Preface

This report is a diploma work for the degree of Master of Science in Engineering Physics at Chalmers University of Technology, Gothenburg, Sweden. The examiner has been Prof. Mikael Persson, Department of Electromagnetics in the School of Electrical and Computer Engineering, Chalmers. The work has been performed at SP Swedish National Testing and Research Institute during the period November 2003 to April 2004 with Ulf Jacobsson as supervisor.

Sammanfattning

I denna rapport bestäms vikters magnetiska egenskaper med hjälp av en susceptometer. De flesta modeller som används för att bestämma susceptibilitet och magnetisering (med BIPM susceptometern) är baserade på ett magnetiskt fält från en magnetisk dipol. Denna modell har jämförts med fält som har härletts genom att använda Biot-Savarts lag. Huvudslutsatsen är att det inte finns signifikanta skillnader mellan de olika modellerna för att bestämma de magnetiska egenskaperna hos vikterna om mätningarna utförs på ett tillräckligt långt avstånd (större än 17 mm) från susceptometer magneten. Det går därför bra att använda dipol-approximationen för att modellera fältet från en cylindrisk magnet. Dessutom beror de uppmätta resultaten av vikters magnetiska egenskaper på styrkan hos det magnetiska fältet.

SP har under detta projekt deltagit i en internationell jämförelse för att bestämma vikters magnetiska egenskaper. Beräkningar av den permanenta magnetiseringen och susceptibiliteten överensstämmer med resultaten från andra deltagare i den internationella jämförelsen.

1 Introduction

In this project the magnetic properties of mass standards using a susceptometer developed at the BIPM are determined [1, 16]. We are particularly interested in the permanent magnetization and volume susceptibility of mass standards.

The permanent magnetization is the magnetization present in a body even in the absence of external magnetic fields. The magnetic volume susceptibility of a body is a property of the material which makes it possible for the body to become magnetized when it is placed in an external magnetic field. The magnetization due to the susceptibility is called induced magnetization. When a homogenous magnetic field is applied to a magnetized solid body, the field within the body is not the same as the applied field. For example the field within a paramagnetic body is stronger than the applied external field because paramagnetic materials attract the external field. For a diamagnetic body the external magnetic fields would have been repelled [8, 10].

There is a magnetic interaction between mass standards and modern balances with electromagnetic force compensation. The magnetic interactions give rise to forces which adversely affect the weighing results. It is important to estimate the magnetic properties of the mass standards in order to avoid erroneous mass readings. Weights having susceptibilities and permanent magnetizations exceeding certain values given in OIML R 111 (International Organization of Legal Metrology) should not be used for calibration of balances because these would lead to incorrect mass measurements. The maximum allowed deviation e.g. for a 1 kg mass standard of the finest class (E_1) is 0.05 mg due to magnetic effects [15].

Mass standards often have cylindrical shapes and are used in calibration of balances and also as reference standards. At SP there are weights of low susceptibilities. These are usually made of stainless steel or brass, and are slightly magnetized. The purpose of the project is to check how good the models used for calculating the permanent magnetization and susceptibility for weakly magnetized mass standards are, when using a susceptometer.

1.1 Static magnetic fields

Maxwell's equations states that the divergence of the magnetic flux density **B** (T) vanishes and that the curl of the magnetic field intensity **H** (A/m) is equal to the free current density **J** (A/m^2):

$$\nabla \cdot \mathbf{B} = \mathbf{0} \qquad (1.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (1.2)$$

The magnetization **M** (A/m) is related to the magnetic flux density and the magnetic field intensity in the following way:

$$\mathbf{B} = \mu_0(\mathbf{M} + \mathbf{H}) \qquad (1.3)$$

In a linear, isotropic homogenous medium the magnetization is a linear function of the magnetic field intensity:

$$\mathbf{M} = \chi\mathbf{H} \qquad (1.4)$$

where χ is the susceptibility of the medium.
By using equation (1.4) we can rewrite equation (1.3):

$$\mathbf{B} = \mu_0(1+\chi)\mathbf{H} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H} \tag{1.5}$$

where μ_0 ($VsA^{-1}m^{-1}$) is the permeability in vacuum, μ_r is called the relative permeability and μ is the absolute permeability [8, 14].

Within a solid the magnetic field is given by a sum of the local magnetic field intensity ($\mathbf{H}_l$) and a demagnetization magnetic field intensity ($\mathbf{H}_d$):

$$\mathbf{H} = \mathbf{H}_l + \mathbf{H}_d \tag{1.6}$$

Consider a cylinder with homogenous magnetization **M**, having the principal axis in the direction of $\mathbf{H}_l$. $\mathbf{H}_d$ arises because there are different magnetic polarities at each end of a cylinder. These are due to the magnetization of the cylinder. $\mathbf{H}_d$ is called a demagnetization field because it is essentially directed in the opposite direction to $\mathbf{H}_l$. In fact, ellipsoids with their principal axes aligned with $\mathbf{H}_l$ have a demagnetization field given by:

$$\mathbf{H}_d = -N\mathbf{M} \tag{1.7}$$

where N is a demagnetization factor (a positive number between 0 and 1) [2, 17]. Combining (1.4), (1.6) and (1.7) leads to:

$$\mathbf{M} = \frac{\chi\mathbf{H}_l}{1+N\chi} \tag{1.8}$$

Approximate values of N for cylinders can be found in [4]. The demagnetization factor for cylinders depends on the aspect ratio (the ratio of height and diameter) of the cylinder. If the cylinder's height is much smaller than the diameter, the demagnetization factor is close to one. On the other hand, an aspect ratio much larger than one lead to a small demagnetization factor, $N \ll 1$, since the poles of the cylinder are the source of $\mathbf{H}_d$ and these are located far from each other [2].

2 Materials

2.1 Diamagnetism and paramagnetism

Diamagnetic substances become magnetized when an external magnetic field (**H**) is applied. The magnetization (**M**) of the substance is proportional to the **H**-field, but it is directed in the opposite direction. When the field is removed the magnetization vanishes. Since **M** points in the opposite direction to **H** a diamagnetic material is weakly repelled by a bar magnet. Diamagnetic materials (such as copper, silver, gold) have very small negative susceptibilities ($|\chi| << 1$), on the order of -10^{-5} [8, 10].

Paramagnetic materials on the other hand are magnetized in the same direction as the applied **H** field, therefore a paramagnetic substance is attracted to a bar magnet. The magnetization is proportional to the magnetic field intensity. Figure 1 is a schematic diagram, showing how the magnetization varies with the external magnetic field intensity. Atoms having closed electron shell structure are purely diamagnetic, but atoms with incomplete electron shell structure have dipole moments and are paramagnetic [10]. Paramagnetic substances (such as platinum-iridium alloy, aluminium and chromium) have very small positive susceptibilities ($\chi << 1$), on the order of 10^{-4} [2, 8]. Susceptibilities of both paramagnetic and diamagnetic materials are independent of the applied external magnetic field intensity. Furthermore the susceptibilities for diamagnetic materials are also independent of temperature.

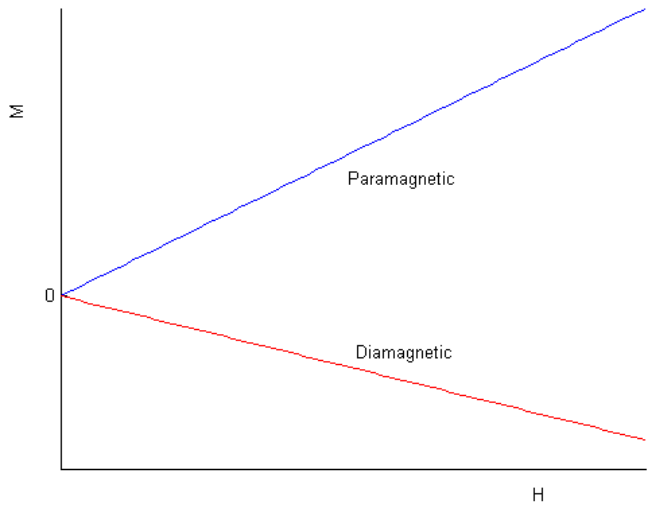


Figure 1. Magnetization due to external *H*-field for diamagnetic and paramagnetic materials.

2.2 Ferromagnetism

A ferromagnetic material is spontaneously magnetized, i.e. magnetized even though it is not placed in an external magnetic field. Examples of ferromagnetic substances are iron, cobalt and nickel; these are ferromagnetic at room temperature. Ferromagnetic materials have very high susceptibilities. A ferromagnetic substance can be split into many small magnetic domains, each domain containing about 10^{15} atoms [14]. When there is no external field present the directions of the magnetic moments of the separate domains are randomly orientated resulting in a weak net magnetization. When a ferromagnetic material is exposed to an external magnetic field the volume of the domains, which have magnetic moments aligned with the applied field, will grow. Consequently this will lead to an increased magnetization. There are ferromagnetic materials, which have large magnetization although no external magnetic field is present. If a weak field is applied the domain volume changes are reversible, that is the magnetization returns to its original value. On the other hand, if a stronger field is applied the volume changes are no longer reversible [11, 14]. Figure 2 shows how the magnetization varies with the external magnetic field intensity. Even if the magnetic field is increased (beyond the point *a* in figure 2) the magnetization does not increase, i.e. the magnetization is saturated. When the saturation point has been reached, the magnetization does not vanish when the external field is removed. Instead there is a remanent magnetization M_r. Nevertheless a body can be demagnetized when placed in a coercive field (force) H_c in the opposite direction. The curves in figure 2 are hysteresis curves for soft and hard alloys. Soft alloys (Mu metal and transformer steel) have small coercive forces, which are typical for materials used in electrical machines and transformers. Hard alloys (carbon steel and $Nd_2Fe_{14}B$) on the contrary, have high coercive forces and these are usually used in permanent magnets [5, 8]. The area in the hysteresis loop corresponds to energy losses caused by friction in domain movements.

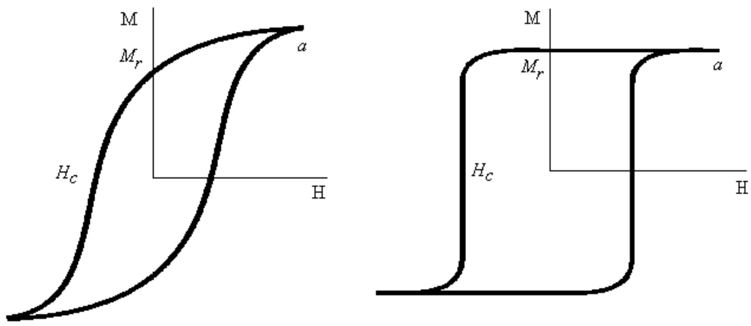


Figure 2. Hysteresis curves for soft alloys (left side) and for hard alloys (right side).

3 Experimental Setup

3.1 Balances with electromagnetic force compensation

Figure 3 shows the features of a balance with electromagnetic force compensation. An object can be placed at the pan (1), which is connected to two parallel guides (2, 3). The parallel guides are used to make it less sensitive to pendulum movements and vibrations. Current passes through a coil (4), located in the gap of a magnet (5), and through a resistance (6). A voltage, which is a function of the mass of the sample is obtained. The signal is transmitted to an analogue-digital converter (7), filtered by a microprocessor (8) and then shown on a display (9). The current through the coil is adjusted to the load on the pan in order to keep the pan at a fixed position.

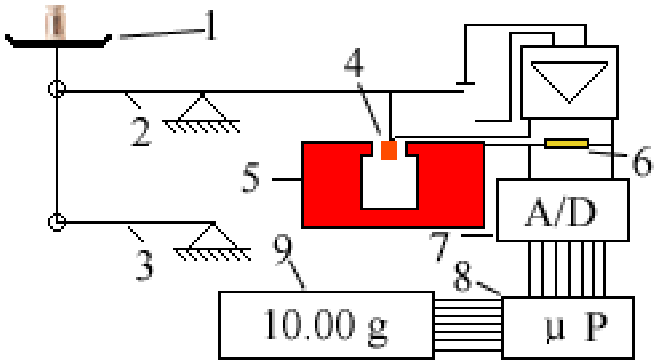


Figure 3. A schematic diagram of a laboratory balance with electromagnetic force compensation.

3.2 Susceptometer

Figure 4 gives a schematic picture of an apparatus used for measuring susceptibility and magnetization of weights [1]. The distance from the centre of the magnet to the top surface of the bridge (z_0) is determined with a micrometer screw.

The magnet currently used is made of the rare earth metal neodymium and have remanence 1.1 T and coercivity 740 kA/m. The height and diameter are equal to 5 mm. According to [7] a magnet with height diameter ratio 0.87 approximates closely the field of a magnetic dipole. The ratio 1 is also an adequate approximation. The magnetic moment of the magnet has been estimated by using two additional magnets of the same dimensions. By using the susceptometer to estimate the forces between the different magnets it is possible to get an estimation of the magnetic moment for each magnet [1, 7]. Another way of determining the magnetic moment of the magnet is to use a reference standard of known susceptibility, again by using the susceptometer it possible to determine the magnetic moment [2]. The two methods used give the same value for the magnetic moment m = 0.084 Am^2 with an uncertainty of 0.002 Am^2.

The magnet is placed on a pedestal (made of aluminium, which has very small susceptibility). The pedestal is put on the load receptor of the balance. The balance used is a Mettler-Toledo MT5, with a capacity of 5.1 g and readability of 1 µg. Because of magnetic interaction between the weight and magnet there is an attracting force, which can be measured by the laboratory balance. A balance reading of –1 mg corresponds to 9.817 µN.

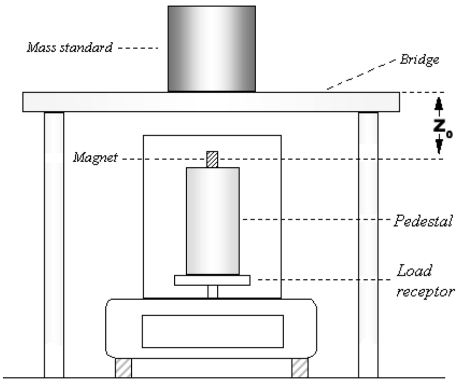


Figure 4. Design of a susceptometer.

3.3 Hall Sensors

A Hall sensor was used to measure earth's magnetic induction $\mathbf{B}_e$. Figure 5 is a schematic diagram showing the principle of a Hall sensor. It measures $\mathbf{B}_e$ perpendicular to the plate. A current i passes through the plate a voltage V_H, called the Hall voltage, is obtained. V_H is proportional to both i and $\mathbf{B}_e$. By holding i constant it is possible to determine the magnetic field intensity in a direction perpendicular to the plate. A probe is connected to the Hall sensor, the active area (1 mm^2) of the sensor is at the tip of the probe. The Hall sensor has a readability of 1 μT.

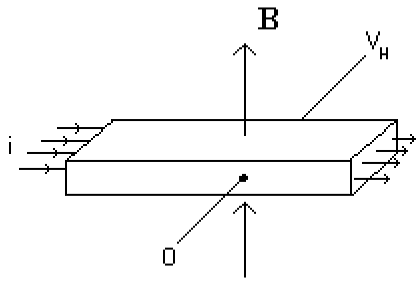


Figure 5. Hall sensor.

3.4 The attraction method

A body with higher susceptibility is more attracted to a magnet than a body with lower susceptibility. This principle is used in one type of permeability indicator, called the Low Mu Permeability Indicator. Figure 6 shows the design of it. A bar magnet is placed at one end of a lever and a counterweight is placed at the other end. The counterweight is used to counteract the effect of gravity. There are references of known relative permeability, μ_r = 1.01, 1.02,…, 1.05, 1.10, 1.20, 1.40,…, 2.20 and 2.50. The relationship to the susceptibility is $\mu_r = 1+\chi$. If the mass standard is attracted to the magnet of the permeability indicator then it must have a permeability (susceptibility) larger than the reference material. If on the other hand, the magnet is attracted to the reference material then the mass standard must have a smaller permeability than the reference. By testing with different references an interval of the permeability (susceptibility) of the mass standard can be estimated.

A big disadvantage of this instrument is that weakly magnetized mass standard can be magnetized by the magnet of the permeability indicator and the reference materials used are difficult to calibrate.

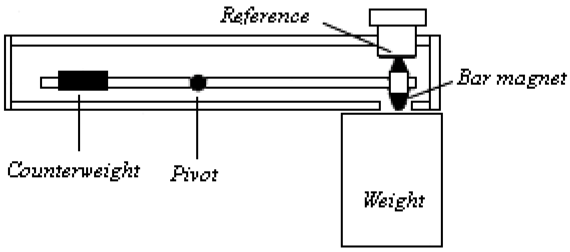


Figure 6. The design of a permeability indicator.

4 Modelling

4.1 Determination of susceptibility and magnetization

One way to measure magnetic susceptibility and the permanent magnetization of weights is to use a susceptometer (see section 3.2). This device measures forces caused by the interaction between the magnet in the susceptometer and the weight placed at a distance z_0 above the magnet. The susceptibility and the magnetization are connected to the measured force (in the vertical direction) by the following relationship:

$$F_z = -\frac{\mu_0}{2}\int_V \frac{\partial}{\partial z}\left(\tfrac{\chi}{1+N\chi}\mathbf{H}\cdot\mathbf{H}\right)dV \quad - \quad \mu_0\int_V \frac{\partial}{\partial z}(\mathbf{H}\cdot\mathbf{M})dV \qquad (4.1)$$

where **H** [A/m] is the local magnetic field intensity, **M** [A/m] is the permanent magnetization of the object, χ (dimensionless) is the effective volume susceptibility, N is a demagnetization factor (for a derivation of the force equation see Appendix A) [2, 6]. Values of N can be found in [4]. The integration is over the volume of the object. For $\chi << 1$ the factor $\chi/(1+N\chi)$ can be replaced by χ. The effective volume susceptibility is a sum of the susceptibility of air and the susceptibility of the mass standard: $\chi = \chi_a+\chi_m$. Since $\chi_a = 3.5\cdot10^{-7} << 1$, the susceptibility of the mass standard can be approximated by the effective volume susceptibility $\chi_m \approx \chi$ [2, 3].

In order to determine the susceptibility and magnetization of an object with the BIPM susceptometer, two force measurements are made. In the first measurement the south pole of the magnet is placed on the balance pan, exposing the object under test to the north pole. In the second measurement the magnet is turned upside down. Denote the force in the first measurement by F_a and the force in the second measurement by F_b. By taking the sum of the two forces the second term in equation (4.1) disappears. On the other hand, by taking the difference between F_a and F_b, the first term in equation (4.1) disappears. Hence the total force measured is given as a sum of two forces:

$$F_z = F_\chi + F_M \qquad (4.2)$$

F_χ is due to the susceptibility of the mass standard and F_M is due to the permanent magnetization of the mass standard:

$$\begin{cases} F_\chi = \dfrac{F_a + F_b}{2} \\ F_M = \dfrac{F_a - F_b}{2} \end{cases} \qquad (4.3)$$

In order to calculate the susceptibility and permanent magnetization, assume that χ has a constant value over the entire mass standard. That is we have a linear isotropic and homogenous medium. Furthermore assume that the permanent magnetization **M** is also homogenous and directed in the vertical direction.

The geometry in cylindrical coordinates is given in figure 7. The mass standards are cylindrical and it is possible to simplify equation (4.1) by writing it as a difference with respect to z instead of taking the derivative and integrating it with respect to z (furthermore $\mathbf{H}(z, \rho)$ does not depend on the angle φ):

$$F_\chi = -\frac{\mu_0 \chi \pi}{(1+N\chi)} \int_0^{R_1} \left(\mathbf{H}(z_T,\rho)\cdot \mathbf{H}(z_T,\rho) - \mathbf{H}(z_B,\rho)\cdot \mathbf{H}(z_B,\rho) \right) \rho d\rho \quad (4.4)$$

$$F_M = -2\pi\mu_0 \int_0^{R_1} \left(\mathbf{H}(z_T,\rho) - \mathbf{H}(z_B,\rho) \right) \cdot \mathbf{M} \rho d\rho \quad (4.5)$$

where R_1 is the radius of the mass standard, z_T and z_B indicate the bottom and top surface of the mass standard (the integrations can be evaluated numerically, see Appendix B). Later on calculations in Cartesian coordinates will be performed. The integrals in Cartesian coordinates are over the top and bottom surface (A) of the mass standards:

$$F_\chi = -\frac{\mu_0 \chi}{2(1+N\chi)} \iint_A \left(\mathbf{H}(x,y,z_T)\cdot \mathbf{H}(x,y,z_T) - \mathbf{H}(x,y,z_B)\cdot \mathbf{H}(x,y,z_B) \right) dxdy \quad (4.6)$$

$$F_M = -\mu_0 \iint_A \left(\mathbf{H}(x,y,z_T) - \mathbf{H}(x,y,z_B) \right) \cdot \mathbf{M} dxdy \quad (4.7)$$

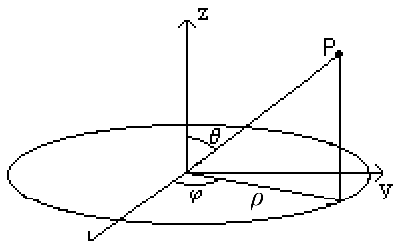


Figure 7. Cylindrical coordinate system.

4.2 Modelling magnetic fields

The local magnetic field intensity is basically a sum of the magnetic field intensity from the susceptometer magnet ($\mathbf{H}_b$) and the magnetic field intensity of earth ($\mathbf{H}_e$):

$$\mathbf{H} = \mathbf{H}_b + \mathbf{H}_e \quad (4.8)$$

There are different ways of modelling $\mathbf{H}_b$. One way is to model the magnetic field intensity from the cylindrical magnet placed on the balance pan (see figure 4) as the magnetic field intensity from a magnetic dipole [8, 14]. The magnetic field intensity in spherical coordinates from a magnetic dipole is given by:

$$\mathbf{H}_{DS}(r,\theta) = \frac{m}{4\pi r^3}(2\cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}}) \quad (4.9)$$

and it can be transformed into cylindrical and Cartesian coordinates:

$$\mathbf{H}_D(\rho,z) = \frac{m}{4\pi(\rho^2+z^2)^{2.5}} \left((2z^2-\rho^2)\hat{\mathbf{z}} + 3z\rho\hat{\boldsymbol{\rho}} \right) \quad (4.10)$$

$$\mathbf{H}_{DC}(x,y,z) = \frac{m}{4\pi(x^2+y^2+z^2)^{2.5}} \left(3xz\hat{\mathbf{x}} + 3yz\hat{\mathbf{y}} + (2z^2-x^2-y^2)\hat{\mathbf{z}} \right) \quad (4.11)$$

where m [Am^2] is the magnetic moment of the magnet.

Other expressions for the magnetic field intensity can be derived by using the Biot-Savart Law [8]. Biot-Savart Law can be used to find the magnetic field intensity from a current in a closed circuit. Assume that the field generated by the cylindrical magnet can be approximated by a single circular current loop with a current I and radius R. Then by applying Biot-Savart Law the magnetic field intensity in Cartesian and cylindrical coordinates can be expressed as:

$$\mathbf{H}_{SC}(x,y,z)=\frac{IR}{4\pi}\int\frac{z\cos\varphi_s\,\hat{\mathbf{x}}+z\sin\varphi_s\,\hat{\mathbf{y}}+(R-x\cos\varphi_s-y\sin\varphi_s)\,\hat{\mathbf{z}}}{\left(R^2+x^2+y^2+z^2-2xR\cos\varphi_s-2yR\sin\varphi_s\right)^{1.5}}d\varphi_s \qquad (4.12)$$

$$\mathbf{H}_{S}(\rho,z)=\frac{IR}{4\pi}\int\frac{z\sin\varphi_s\,\hat{\boldsymbol{\rho}}+(R-\rho\sin\varphi_s)\,\hat{\mathbf{z}}}{\left(R^2+\rho^2+z^2-2\rho R\sin\varphi_s\right)^{1.5}}d\varphi_s \qquad (4.13)$$

The dipole model $\mathbf{H}_D$ is an approximation of $\mathbf{H}_S$ and it is valid at large distances from the magnet [9]. Consider a cylindrical magnet having a uniform permanent magnetization along the z-axis, M_m. Biot-Savart Law applied on a homogenous, magnetized and cylindrical magnet leads to the following formula for the magnetic field intensity:

$$\mathbf{H}_{Db}(\rho,z)=\frac{M_mR}{4\pi}\iint\frac{(z-z_s)\sin\varphi_s\,\hat{\boldsymbol{\rho}}+(R-\rho\sin\varphi_s)\,\hat{\mathbf{z}}}{\left(R^2+\rho^2+(z-z_s)^2-2\rho R\sin\varphi_s\right)^{1.5}}dz_s\,d\varphi_s \qquad (4.14)$$

$$\mathbf{H}_{DbC}(x,y,z)=\frac{M_mR}{4\pi}\iint\frac{(z-z_s)\cos\varphi_s\,\hat{\mathbf{x}}+(z-z_s)\sin\varphi_s\,\hat{\mathbf{y}}+(R-x\cos\varphi_s-y\sin\varphi_s)\,\hat{\mathbf{z}}}{\left(R^2+x^2+y^2+(z-z_s)^2-2xR\cos\varphi_s-2yR\sin\varphi_s\right)^{1.5}}dxdy$$

(4.15)
(for numerical evaluation of the integrals see Appendix B).

Figure 8 and figure 9 display the magnitudes of the magnetic field intensities as expressed in equations (4.10), (4.13) and (4.14) as a function of z at fixed ρ (see figure 7). At $\rho = 1$ mm, $z = 15$ mm the maximum difference between the magnetic field intensities is less than 6 % and clearly the differences decline as the distance to the magnet is increased.

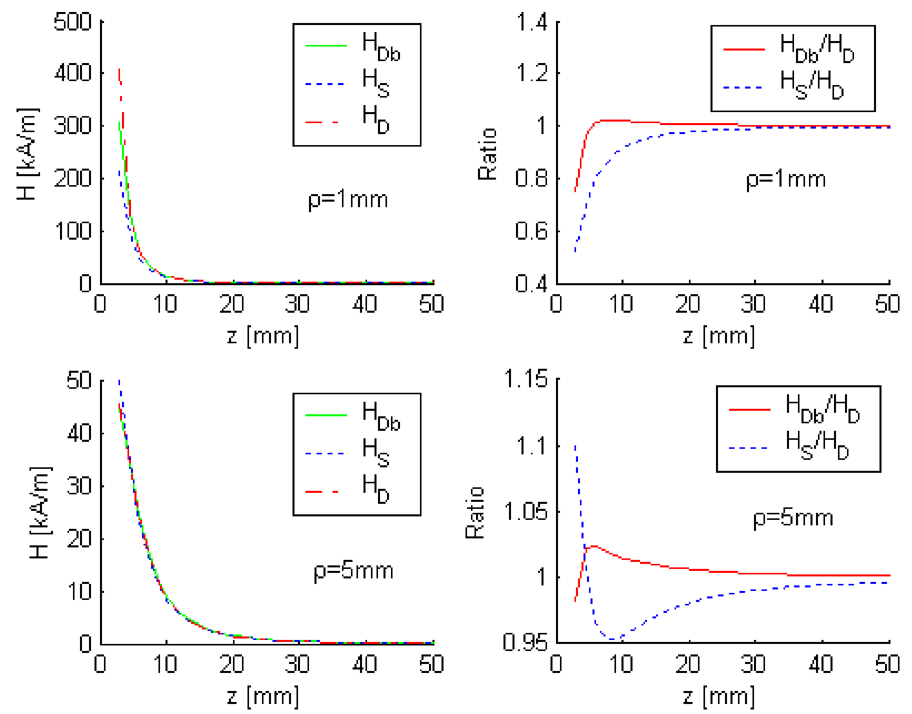


Figure 8. The magnitudes of magnetic field intensities calculated at $\rho = 1$ mm and $\rho = 5$ mm.

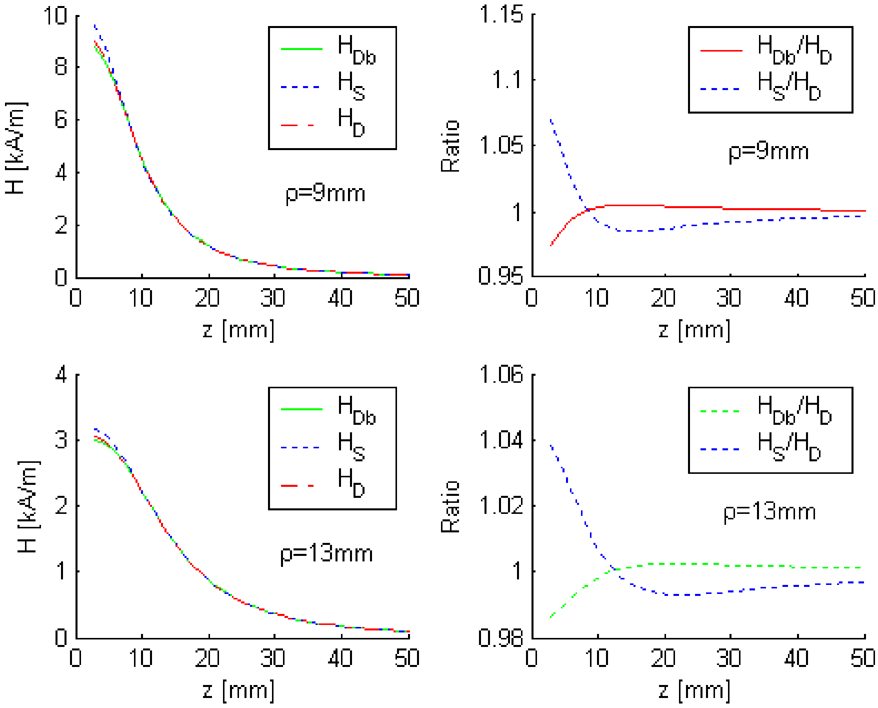


Figure 9. The magnitudes of magnetic field intensities calculated at ρ=9 mm and ρ=13 mm.

5 Results

5.1 Magnetic intercomparison

SP has participated in an international comparison for determining the susceptibility and magnetization of three 1kg mass standards (“OIML”, “C&CA” and “[52]”). The OIML and C&CA weights are not, entirely, cylindrical. The susceptibility and magnetization can be calculated for “outer” and “inner” cylinders according to figure 10 [1]. An outer cylinder surrounds the entire volume of the mass standard, while an inner cylinder is enclosed within the volume of the mass standard. The susceptibility and magnetization will in the first case be underestimated and in the second case overestimated. A reasonable estimate is an average of the outer and inner cylinders. According to [2, 4] a demagnetization factor $N = 0.5$ can in this particular case be chosen. By using a Hall sensor the earth’s magnetic field intensity in the horizontal and vertical direction has been measured to $H_{\mathrm{eh}} = 13$ A/m and $H_{\mathrm{ez}} = 28$ A/m respectively. The distance between the centre of the magnet to the bottom of the mass standard, z_0, has been estimated to 17.4 mm. By using (4.4) and (4.5) it is possible to calculate the susceptibility. The susceptibilities χ_D, χ_{Db}, χ_S, and magnetizations M_D, M_{Db} and M_S are calculated by modelling the magnetic field intensity as $\boldsymbol{H}_D$, $\boldsymbol{H}_{Db}$ and $\boldsymbol{H}_S$ according to (4.10), (4.13) and (4.14). It is obvious from figure 11 that the maximum ratio for the susceptibility (χ_S/χ_D) is smaller than 1.04 for the different models. The corresponding value for the relative magnetization (M_S/M_D) is smaller than 1.002, see figure 12.

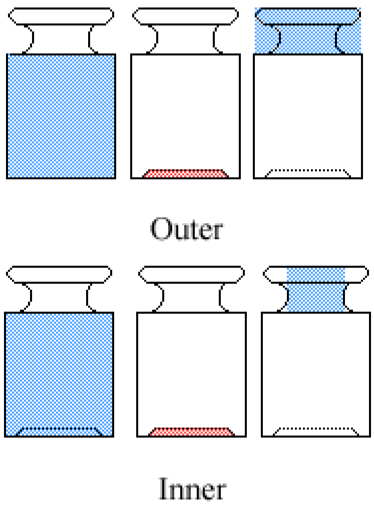


Figure 10. Outer and inner cylinders of OIML weights.

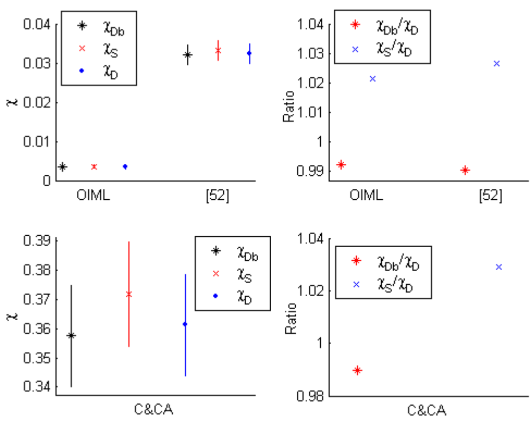


Figure 11. Susceptibility calculated for 1 kg mass standards.

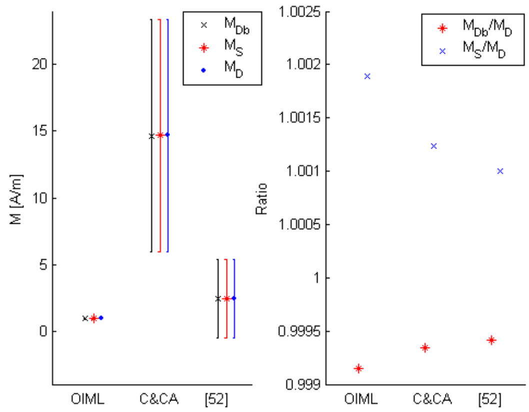


Figure 12. Permanent magnetization calculated for 1kg mass standards.

5.1.1 Uncertainty analysis

There are errors associated to various measurements. There are random errors and systematic errors. Random errors have different values for each measurement while systematic errors are on average constant. Examples of random errors are those arising from electrical noise and 50 Hz-disturbance. Model- and adjustment errors are examples of systematic errors.

There are uncertainties due to determination of z_0 (Δz_0), force measurements (ΔF), shape correction, reproducibility and in the magnetic moment m (Δm). The uncertainty components have been estimated by using standard deviations. According to [13] the uncertainties of the susceptibility can be estimated by:

$$\frac{\partial \chi}{\partial z_0}\Delta z_0, \quad \frac{\partial \chi}{\partial F}\Delta F, \quad \frac{\partial \chi}{\partial m}\Delta m, \quad \frac{\chi_I - \chi_O}{2}, \quad 0.004\chi \qquad (5.1)$$

where χ_I, χ_O are inner and outer susceptibilities respectively, χ is the mean value and 0.004 is the reproducibility factor (see Appendix B). Uncertainties of the permanent magnetization can be estimated in the same way (replace χ with M in equation (5.1)).

The total uncertainty is the root square sum of each uncertainty component in (5.1). The uncertainty components of the susceptibility and permanent magnetization and their contribution to the total uncertainty are given in table 1. The largest uncertainty components are due to uncertainty in magnetic moment and force measurements.

Table 1. Below is presented how much each uncertainty component contributes to the total uncertainty of the susceptibility and magnetization for the three 1kg mass standards.

Uncertainty component	OIML (χ)	C&CA (χ)	[52] (χ)	OIML (M)	C&CA (M)	[52] (M)
z_0	0.0496	0.1165	0.0659	0.0430	0.0025	0.0012
Force measurements	0.0210	0.3091	0.5905	0.2426	0.9505	0.9766
Shape	0.0455	0.0078	0	0.0201	0.0023	0
Reproducibility	0.0290	0.0512	0.0344	0.0996	0.0064	0.0032
Magnetic moment	0.8550	0.5154	0.3093	0.5948	0.0383	0.0190

5.2 Varying the distance

In SP's susceptometer $z_0 = 17.2$ mm (z_0 is the distance between centre of the magnet and the bottom of the weight) is the default value but measurements have also been performed at $z_0 = 10.5$, $z_0 = 19.2$ and $z_0 = 22.2$ mm. The corresponding susceptibilities and magnetizations are denoted by χ_{10}, χ_{17}, χ_{19}, χ_{22} and M_{10}, M_{17}, M_{19} and M_{22} respectively.

Figure 13 gives the susceptibilities and permanent magnetizations calculated at different z_0 for four weights 100 g OIML, 500 g OIML, 500 g special and 1 kg cylindrical (comparisons of the different models used for calculating the magnetic properties are given in Appendix C). Obviously the estimated susceptibility of a weight varies with z_0, also the measured magnetization varies with z_0. In table 2 the ratios of the susceptibilities and magnetizations are given at different heights (these values have been calculated by modelling the magnetic field intensity as $\mathbf{H}_S$, see section 4.2). The susceptibilities of the mass standards have also been estimated by using a permeability indicator. This instrument indicate that the susceptibility of the 1 kg cylindrical weight is smaller than 0.01, and the susceptibilities of the other weights are between 0.01 and 0.02. The permanent magnetization has been estimated to M < 8 A/m by a Hall sensor for all the weights.

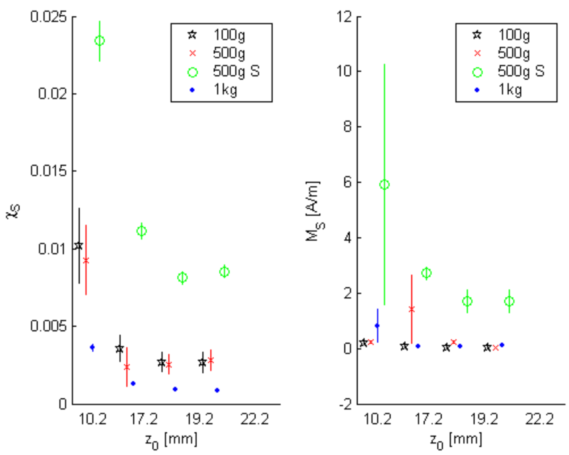


Figure 13. Susceptibility and permanent magnetization calculated at different distances z_0.

Table 2. Ratio of susceptibilities and magnetizations at various z_0.

	100 g OIML	500 g OIML	500 g special	1 kg cylindrical
χ_{10}/χ_{17}	2.86	3.89	2.10	2.91
χ_{19}/χ_{17}	0.75	1.07	0.73	0.75
χ_{22}/χ_{17}	0.75	1.18	0.77	0.65
χ_{22}/χ_{19}	0.99	1.11	1.05	0.87
M_{10}/M_{17}	2.39	0.15	2.18	14.04
M_{19}/M_{17}	0.23	0.16	0.63	1.18
M_{22}/M_{17}	0.37	0.022	0.63	2.14
M_{22}/M_{19}	1.65	0.14	0.996	1.82

5.3 Determination of active volume

In this section the magnetic field intensity from the balance has been modelled as $\boldsymbol{H}_S$ given in equation (4.13).

For large mass standards mainly the volume at the bottom of each weight has a large effect on the measured results of the susceptibility and magnetization. This is called the active volume. The diameter and height of the 1 kg cylindrical mass standard are $D = H = 52$ mm. Using these dimensions the susceptibility and magnetization can be estimated to $\chi = 0.0013$ and $M = 0.056$ A/m by using equations (4.4) and (4.5). By holding the radius fixed at 26 mm, the permanent magnetization and the susceptibility of the weight have been evaluated at different heights (see figure 14). In the same way by fixing the height of the weight at 52 mm χ and M have been estimated at various radii (see figure 15).

The susceptibility converges faster than the magnetization to the ”real” values mentioned above ($\chi = 0.0013$ and $M = 0.056$ A/m) as the height is increased (see table 3). This is not true as the radius is increased (see table 4).

Table 3. The susceptibilities and magnetizations calculated at height h are compared to corresponding values at the height $H = 52$ mm of the 1 kg cylindrical weight.

Height h in mm	10	15	20	35
χ_h/χ_H	1.17	1.08	1.04	1.01
M_h/M_H	2.14	1.61	1.37	1.09

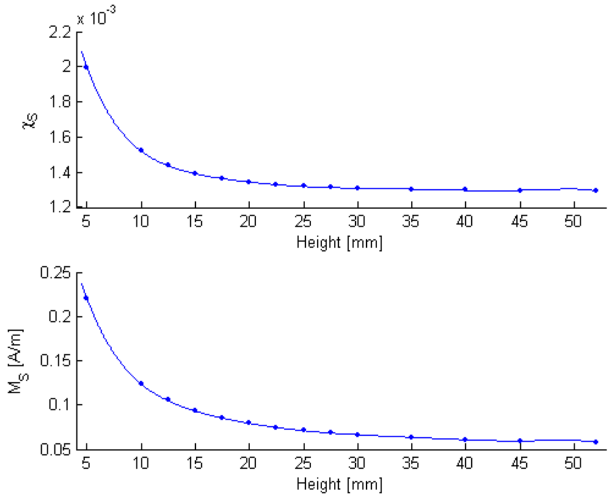


Figure 14. Susceptibility and permanent magnetization of 1 kg cylindrical mass standard calculated at different heights (dots).

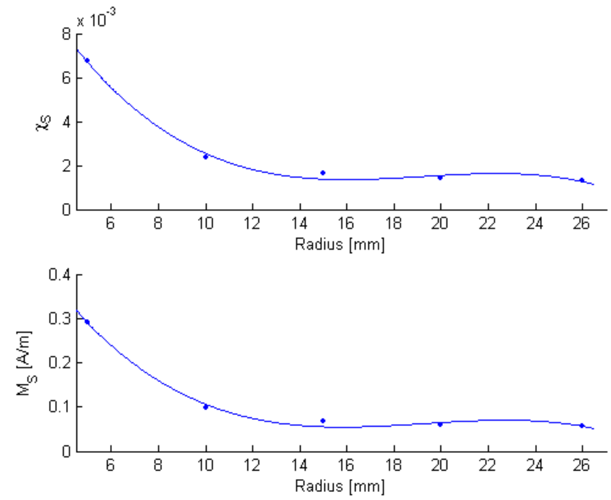


Figure 15. Susceptibility and permanent magnetization calculated at various radii (dots).

Table 4. The susceptibilities and magnetizations calculated at various radii *r* are compared to corresponding values at the radius *R* = 26 mm of the 1 kg cylindrical weight.

Radius *r* in mm	5	10	15	20
χ_r/χ_R	5.19	1.84	1.26	1.08
M_r/M_R	5.04	1.71	1.15	1.002

5.4 Envelope surface

The magnetization and susceptibility of mass standards have been calculated by placing the weights as shown in figure 16. The axis of the magnet is perpendicular to the axis of the weight. In order to determine the magnetic properties of the mass standards, outer and inner cuboids analogous to the inner and outer cylinders are used. Assume that we have a linear, isotropic and homogenous material, which have a constant magnetization along the y-axis. Figure 17 gives the susceptibilities χ_D, χ_{Db}, χ_S and figure 18 gives the permanent magnetization M_D, M_{Db} and M_S for three mass standards. The susceptibilities and magnetizations are calculated by modelling the magnetic field intensity as $\boldsymbol{H}_{DC}$, $\boldsymbol{H}_{SC}$ and $\mathbf{H}_{DbC}$ according to (4.11), (4.12) and (4.15). The susceptibility of the mass standards have also been estimated by the attraction method to $0.01 < \chi < 0.02$. The magnetization have been measured to $M < 8$ A/m by a Hall sensor.

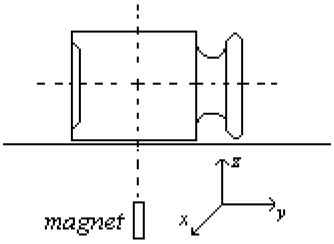


Figure 16. The axis is of the magnet is perpendicular to the principal axis of the mass standard.

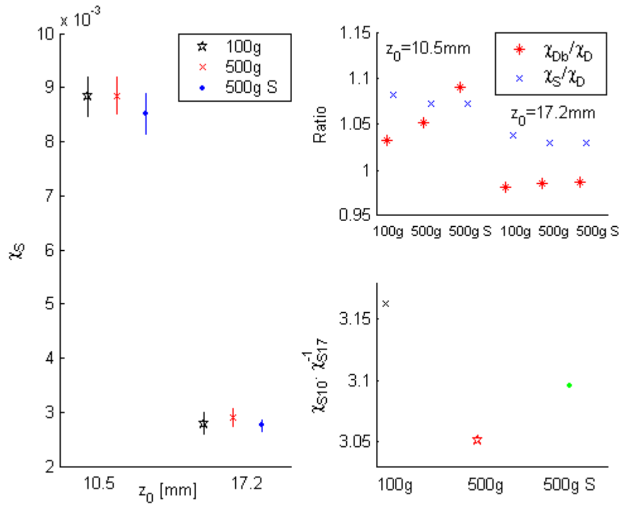
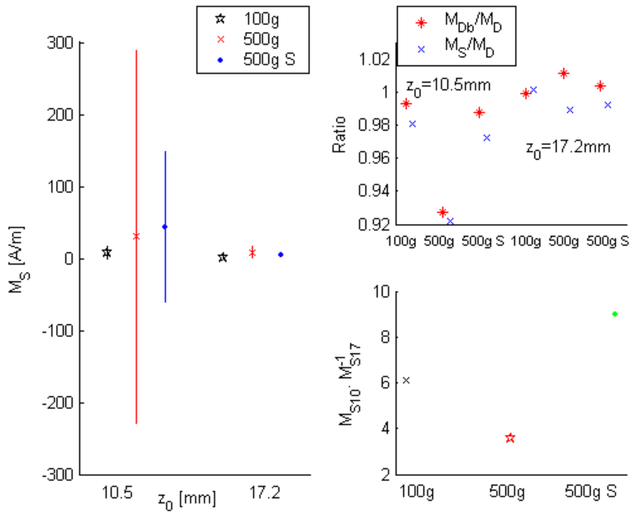


Figure 17. Susceptibilities calculated at z_0 = 10.5 mm and z_0 = 17.2 mm and ratio of susceptibilities.



0Figure 18. Permanent magnetizations calculated at z_0 = 10.5 mm and z_0 = 17.2 mm and ratio of magnetizations.

6 Conclusions and Discussion

The susceptibility and permanent magnetization of weights have been determined when using a susceptometer. Comparison of the magnetic field intensity from a magnetic dipole and magnetic field intensities derived using Biot-Savart Law has been made. Since the magnitude of the magnetic field intensity declines fast ($1/r^3$), essentially only the bottom of the weights have impact on the measured results of the magnetic properties (e.g. for a slightly magnetized 1 kg weight the knob have a negligible effect on the result for the susceptibility and magnetization).

When performing measurements at distances z_0 (the distance from the centre of the magnet to the bottom of the mass standard) larger than 17 mm the different expressions for the magnetic field intensities lead to deviations of the susceptibility and permanent magnetization. The deviations are smaller than 5 %. These deviations are, in normal circumstances, only small perturbations compared to the uncertainties in measurements of forces, magnetic moment of the magnet and the distance between the magnet and the mass standards.

We have assumed that the permanent magnetization is homogenous and directed in the vertical direction. This assumption is unwarranted, but in mass metrology we are interested of a conventional value of the magnetization, in order to determine whether the permanent magnetization causes a problem for weighing or not. The intention is not to give a complete description of the permanent magnetization. Nevertheless the magnetization can be estimated at various spots of the mass standard by using a Hall sensor. This is a more suitable way of determining the permanent magnetization.

By changing the distance z_0 different values of the permanent magnetization and susceptibility are obtained. Mass standard being placed close to the susceptometer magnet are exposed to higher magnetic fields than mass standards placed further away. Therefore the calculations of the susceptibility and permanent magnetization of weights indicate that the measured magnetic properties can be treated as a function of the magnitude of the external magnetic field intensity. Similar results have also been obtained in [2]. The susceptibilities obtained by the susceptometer method at $z_0 = 10.5$ mm agree best with values obtained by the attracting method. The problem is however, that the mass standards may become magnetized at distances z_0 smaller than 17 mm. For example the magnetic field intensity at a distance of $z_0 = 10.5$ mm is about 16 kA/m and a mass standard of the finest class (OIML E_1) can be magnetized at 2 kA/m [15]. There is a risk though, of underestimating the susceptibility if the measurements are carried out at a distances $z_0 > 17$ mm.

SP has participated in an international comparison for estimating the magnetic properties of mass standards during this project. The measurements in the international comparison have been performed at various distances z_0 by the different participants. The measurements should have been performed at a specified distance z_0 (consequently, also the magnets being used should have the same strength), since the estimated magnetic properties of the mass standards can depend strongly on z_0 and the magnitude of the magnetic field intensity. The values (of the permanent magnetization and susceptibility) obtained at SP have though been in accordance with results (not shown) obtained by other participants despite different measurement setups.

The susceptometer method is a good method for estimating the susceptibility and magnetization of weakly magnetized objects having small susceptibilities (susceptibilities close to one). For susceptibilities much larger than one the material may be inhomogeneous and nonlinear. Then the assumptions used for calculating the susceptibility and magnetization are no longer valid.

In conclusion the dipole model is adequate to use for modelling the magnetic field intensity from a cylindrical magnet at distances z_0 larger than 17 mm and it is important to specify z_0 because the measurements of the magnetic properties can strongly depend on z_0 and the magnitude of the magnetic field intensity.

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Appendix A

Derivation of force equation

Assume that we have an infinitesimal magnetic moment **dm** placed in an external magnetic induction **B**. Then there will be a potential energy:

$$U = -\mathbf{dm} \cdot \mathbf{B} \tag{A.1}$$

The force in the vertical direction is obtained by taking the derivative of the potential energy with respect to *z*:

$$\frac{\partial U}{\partial z} = -\mathbf{dm} \cdot \frac{\partial \mathbf{B}}{\partial z} \tag{A.2}$$

By applying (A.2) for a body of finite volume *V* leads to:

$$F_z = -\int_V \mathbf{M} \cdot \frac{\partial \mathbf{B}}{\partial z} dV \tag{A.3}$$

In free space the magnetic field intensity is given by **H**= μ_0**B**. The magnetization (**M)** is a sum of the permanent ($\mathbf{M}_p$) and induced magnetization ($\mathbf{M}_{ind}$). Ellipsoids with their principal axis aligned with the external magnetic field intensity have an induced magnetization given by (see section 1.1):

$$\mathbf{M}_{ind} = \frac{\chi \mathbf{H}}{1 + N\chi} \tag{A.4}$$

We can rewrite (A.3) by separating the permanent and induced magnetization to obtain the force equation:

$$F_z = -\frac{\mu_0}{2} \int_V \frac{\partial}{\partial z} \left(\tfrac{\chi}{1+N\chi} \mathbf{H} \cdot \mathbf{H} \right) dV \quad - \quad \mu_0 \int_V \frac{\partial}{\partial z} \left(\mathbf{H} \cdot \mathbf{M}_p \right) dV \tag{A.5}$$

Appendix B

Numerical calculations

The magnetic field intensities have been calculated in Matlab. There are algorithms available for performing integrations in one, two and three dimensions.

When modelling the field of the susceptometer magnet as the field from a magnetic dipole there exist analytical solutions of the force equations (4.4) and (4.5). These solutions can be obtained by using symbolic software e.g. Mathematica or Mathcad. For the magnetic field intensities derived by using Biot-Savart Law there are only numerical solutions of the force equations (4.4), (4.5), (4.6) and (4.7). The integrations have been performed in Matlab by using the composite midpoint rule:

$$\int_a^b f(\rho)d\rho \approx \sum_{i=1}^{n} (\rho_i - \rho_{i-1}) f\left(\tfrac{\rho_i + \rho_{i-1}}{2}\right) \qquad \text{(B.1)}$$

where $a = \rho_0 < \rho_1 < \ldots < \rho_n = b$ and the number of steps have been chosen to $20 < n < 50$ in order to obtain small calculation errors compared to other errors. The midpoint rule have also been used in two dimensions:

$$\int_a^b \int_c^d f(x,y)dxdy \approx \sum_{i,j=1}^{n} (x_i - x_{i-1})(y_j - y_{j-1}) f\left(\tfrac{x_i + x_{i-1}}{2}, \tfrac{y_j + y_{j-1}}{2}\right) \qquad \text{(B.2)}$$

where $a = x_0 < x_1 < \ldots < x_n = b$, $c = y_0 < y_1 < \ldots < y_n = d$ and $10 < n < 20$.

It is also possible to perform the calculations by using Monte Carlo simulations [12, 18, 19]. A disadvantage of Monte Carlo is that it converges slowly compared to the midpoint rule described above.

The uncertainties have been estimated by using (5.1). The partial derivatives have been approximated by the centred difference formula:

$$\frac{\partial f}{\partial z} \approx \frac{f(z+h) - f(z-h)}{2h} \qquad \text{(B.3)}$$

Appendix C

Deviations of susceptibility and magnetization

Deviations of susceptibilities due to the different models used for the magnetic field intensities are given in table C.1 (see section 5.2). The corresponding deviations for permanent magnetizations are given in table C.2. The susceptibilities χ_D , χ_S, χ_{Db} and permanent magnetizations M_D, M_S and M_{Db} are calculated by modelling the magnetic field intensity as $\mathbf{H}_D$, $\mathbf{H}_S$ and $\mathbf{H}_{Db}$ (field from a magnetic dipole and fields based on the Biot-Savart Law, see section 4.2). The distance from the centre of the susceptometer magnet to the surface of the bridge is z_0 (see figure 4).

Table C.1. Deviations of susceptibilities for four weights at various z_0.

	100 g OIML	500 g OIML	500 g special	1 kg cylindrical	z_0 (mm)
$(\chi_S - \chi_D)/\chi_D$	0.0767	0.0602	0.0690	0.0715	10.5
$(\chi_S - \chi_D)/\chi_D$	0.0393	0.0266	0.0265	0.0294	17.2
$(\chi_S - \chi_D)/\chi_D$	0.0336	0.0227	0.0216	0.0246	19.2
$(\chi_S - \chi_D)/\chi_D$	0.0270	0.0185	0.0167	0.0195	22.2
$(\chi_{Db} - \chi_D)/\chi_D$	-0.0234	-0.0204	-0.0242	-0.0231	10.5
$(\chi_{Db} - \chi_D)/\chi_D$	-0.0128	-0.0096	-0.0101	-0.0104	17.2
$(\chi_{Db} - \chi_D)/\chi_D$	-0.0110	-0.0083	-0.0084	-0.0088	19.2
$(\chi_{Db} - \chi_D)/\chi_D$	-0.0090	-0.0068	-0.0066	-0.0071	22.2

Table C.2. Deviations of permanent magnetizations for four weights at various z_0.

	100 g OIML	500 g OIML	500 g special	1 kg cylindrical	z_0 (mm)
$(M_S - M_D)/M_D$	0.0122	-0.0010	-0.0035	-0.0014	10.5
$(M_S - M_D)/M_D$	0.0143	0.0033	0.0004	0.0034	17.2
$(M_S - M_D)/M_D$	0.0134	0.0040	0.0010	0.0040	19.2
$(M_S - M_D)/M_D$	0.0119	0.0045	0.0017	0.0044	22.2
$(M_{Db} - M_D)/M_D$	0.0049	-0.0013	-0.0024	-0.0015	10.5
$(M_{Db} - M_D)/M_D$	0.0050	-0.0019	-0.0014	-0.0020	17.2
$(M_{Db} - M_D)/M_D$	0.0046	-0.0020	-0.0014	-0.0021	19.2
$(M_{Db} - M_D)/M_D$	0.0041	-0.0021	-0.0014	-0.0021	22.2

Appendix D

Force measurements

The figures below shows the measured forces that have been used to calculate the susceptibility and permanent magnetizations of various mass standards. The susceptibility give rise to the force F_χ. The force F_M is due to the permanent magnetization of the object.

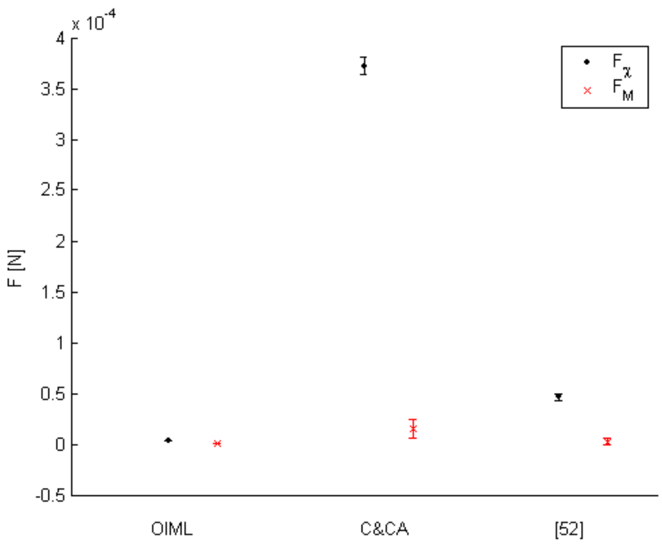

Figure D.1. Measured forces for the 1 kg mass standards used in the international comparison.

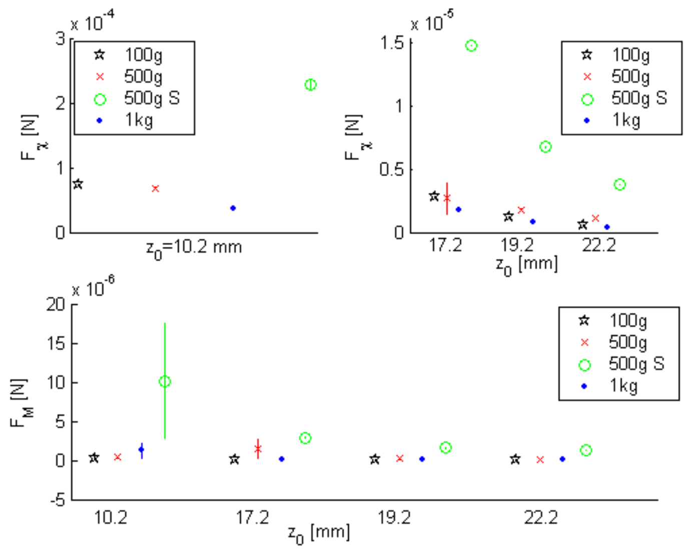

Figure D.2. Measured forces at various distances z_0.

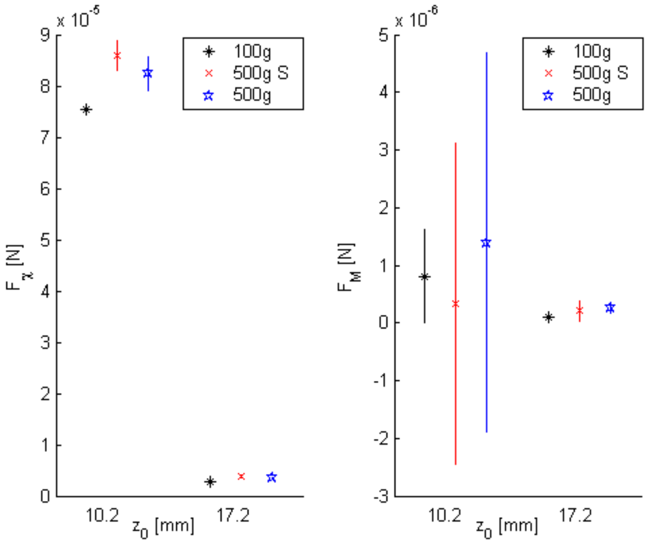

Figure D.3. Forces measured when the mass standards are placed with the envelope surface on the susceptometer bridge.

Appendix E

Characteristics of mass standards

Figure E.1 is a schematic diagram of an OIML mass standard. In table E.1 the dimension of mass standards used in this work are given in mm [15].

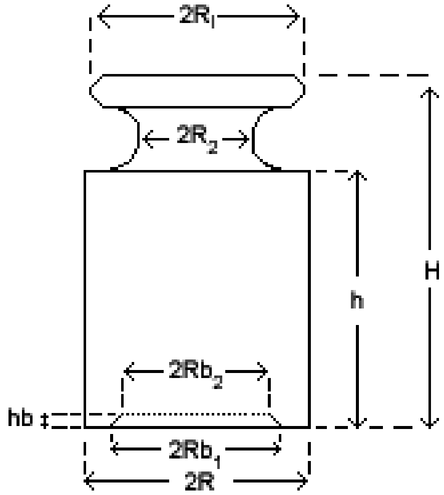

Figure E.1. The dimensions of an OIML mass standard.

Table E.1. Dimensions of mass standards given in mm.

Mass standards	R	R_1	R_2	Rb_1	Rb_2	H	hb	h
5g OIML	3.90	3.40	2.20	2.25	1.25	15.00	0.20	11.10
10g OIML	4.92	4.36	2.96	3.60	2.08	18.65	0.31	14.21
20g OIML	6.50	5.70	3.70	3.50	1.80	22.00	0.40	16.40
50g OIML	8.94	7.80	4.95	6.72	4.87	29.34	0.40	21.15
100g OIML	10.93	9.93	6.50	7.67	6.10	38.79	0.46	28.10
500g OIML	18.95	16.93	11.03	13.49	9.97	63.90	0.81	47.00
1kg OIML	23.93	21.40	13.50	17.07	15.23	81.60	0.38	60.00
500g Cylindrical	21.73	0.00	0.00	0.00	0.00	43.05	0.00	0.00
1kg Cylindrical	26.00	0.00	0.00	0.00	0.00	52.00	0.00	0.00
500g Special	21.40	10.50	4.65	0.00	0.00	63.90	0.00	35.50
1kg OIML	23.95	21.55	13.65	20.70	17.75	82.90	2.17	60.50
1kg C&CA	26.70	12.70	13.65	24.00	22.10	76.70	0.70	53.60
1kg [52]	26.95	0.00	0.00	0.00	0.00	55.10	0.00	0.00

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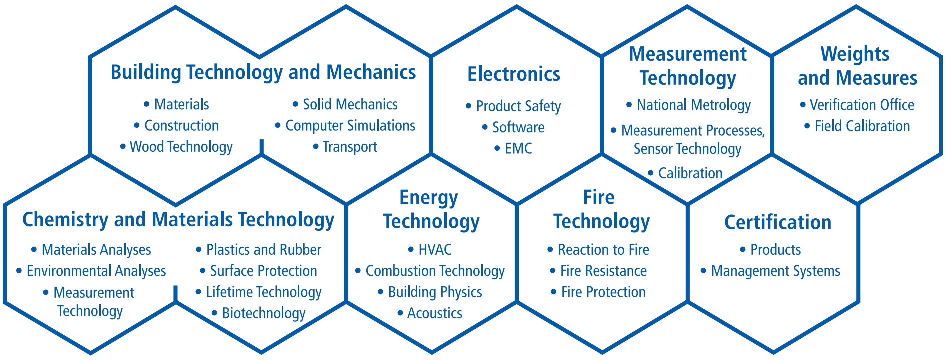

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SP Swedish National Testing and Research Institute
Box 857
SE-501 15 BORÅS, SWEDEN
Telephone: + 46 33 16 50 00, Telefax: +46 33 13 55 02
E-mail: info@sp.se, Internet: www.sp.se

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