Jan Hjelmgren

Dynamic Measurement of Force – A Literature Survey
Jan Hjelmgren

Dynamic Measurement of Force - A Literature Survey

SP Report 2002:27
Measurement Technology, MTm
Borås 2002
Abstract

Mechanical force is a quantity that needs to be measured in a lot of industrial applications. In some cases the force to be measured is static, or varying very slowly in time, and in some cases it is truly dynamic. Some examples of dynamic applications are: material testing, modal testing, crash testing, dynamic weighing, production processes such as grinding or drilling, robotics, aerodynamics and vehicle dynamics. Different applications have different typical force ranges, varying from subnewtons to several hundred kilonewtons, and different typical timescales varying from microseconds to several seconds.

The measurement engineer is not satisfied with the mere collection of measurement data. Data must also be evaluated and reported with an adjoining statement of measurement traceability and uncertainty. This uncertainty statement is a quantity describing the quality of measurement data obtained by involved operators using stated methods and equipment. Without reliable information about the quality of the measurement data the assessment of the impact of data on product quality is a risky business.

There is a big difference between performing traceability and uncertainty analyses for the cases of static and dynamic force measurements. This difference stems from the fact that force transducers are traceably calibrated only for static forces. When force transducers are used for dynamic measurements the measurement engineer is faced with a difficult question: how should the difference between the static calibration and the actual dynamic use be handled in the uncertainty budget? If there were methods available for dynamic calibration of force transducers it would be much easier to calculate uncertainties for dynamic force measurements.

The prime purpose of the present report is to survey the area of dynamic calibration of force transducers. Also some methods that do not fulfil the criterion of being traceable to the definitions of the SI units are described. To help the reader to understand the importance and the involved techniques of dynamic calibrations, introductory chapters are devoted to methods of force measurements, various application areas and measurement uncertainty. Finally areas in which further work is needed are pointed out.

Key words: force transducers, calibration, dynamic force
1 Introduction

Dynamic forces are measured in a more or less routine manner in a lot of applications today. The associated measurement uncertainty is, however not, estimated in a similar routine fashion. One principal reason for this is that there exist no commercially available methods for dynamic calibration of force transducers. Therefore, it is very easy to obtain measurement data but it is very difficult to evaluate them and produce a realistic estimation of measurement traceability and uncertainty. In many cases the engineer is faced with the question whether he should incorporate a large uncertainty component to accommodate for the fact that there is a big difference between the calibration and application circumstances or whether he should completely avoid the difficulties by ignoring them.

Traceable calibration is an essential part of quality assurance as stipulated in modern quality standards such as ISO 9000, QS 9000 or ISO 17 025. Of course a traditional static calibration fulfils this requirement but when using the transducer dynamically the uncertainties mentioned above have to be taken care of. A better way would be if traceable dynamic calibrations facilities were publicly available. In this way the formal requirements of quality assurance would be obeyed but more importantly the work of the measurement engineer would be much simpler when producing uncertainty budgets.

A recent survey1 of the area of dynamic force measurement in the UK concluded that about 35% of all force measurements performed were dynamic. The survey concluded, in agreement with the present paper, that the lack of traceability for dynamic measurements leads to increased cost in terms of reduced quality, increased scrapping and reduced competitiveness. Improvements in dynamic force measurement on the order of 1% could lead to savings within the UK economy of between £1 billion and £2 billion per year according to Ref. 1.

The present report is primarily a literature survey concerned with methods for traceable dynamic calibrations of force transducers. Before embarking on the main topic, force measuring principles and some of the applications in need of the dynamic calibration methods are discussed. The encountered force amplitudes vary from approximately 0,1 N to 1 MN and the frequency content being sought for extends from 0 Hz to approximately 100 kHz. After a discussion about applications different published methods are surveyed together with other techniques of reducing the measurement uncertainty. Finally, some work that should be performed by the research community in order to facilitate for the measurement engineer in his strive for lower and accurately quantified measurement uncertainty is outlined.
2 Measuring force

This chapter starts with a short summary of different mechanical force measurement principles. At least an acquired basic understanding of these principles is necessary to perform reliable dynamic measurements. A discussion of the differences between static and dynamic measurements follows. Some of the more important applications in which dynamic force measurement is a vital ingredient are reviewed. Concluding the chapter is a section about measurement uncertainty and general approaches to reduce it.

2.1 Measuring principles

2.1.1 The force measurement system

A force measurement system may consist of one or several force transducers, power supply, connecting cables, amplifiers, indicators, devices for data storage and a controlling unit (i.e. a PC). One schematic system is shown in Figure 1. When evaluating the data obtained by the system one has to acknowledge the fact that dynamic and static properties of all components may affect the quality of measurement data. In most cases, however, the differences between static and dynamic measurements can be traced to dynamic properties of the transducers and in some cases of the amplifiers.

The dynamic interaction between the transducer and the structure it is installed into may lead to unexpected results. One example of this is the possible significant lowering of the lowest natural frequency containing non-zero elastic energy for the transducer. This is discussed more in the chapter about further work that has to be performed.

![Figure 1. Schematic picture of a force measuring system](image)

2.1.2 The elastic transducer

Most force transducers rely on the principle that when subjected to a force, the force transducer deforms in a manner proportional to the magnitude of force. The main part of such a force transducer is an elastic body, as in Figure 2.
Several physical principles\textsuperscript{3,4} are possible to use in order to sense the deformation of the elastic element, i.e. resistive, piezoelectric, capacitive, inductive, magnetoelastic and optical principles.

### 2.1.3 Resistive sensing

The most common method to measure force relies on resistive sensing. This means that resistive strain gages are attached to an elastic body, Figure 3. When subjected to load the strain gages deform and the resistance is changed in proportion to the applied load. Several (usually at least four) strain gages are electrically connected in a so-called Wheatstone bridge circuit to produce a voltage proportional to the change in resistance. The output signal is low (typically a few mV/V) and amplification is necessary.
The strain gage bridge is supplied either by a DC-system (typically 5-10 VDC) or by a so-called AC carrier frequency system. For static measurements the carrier frequency system can be shown to have some definite advantages, such as higher immunity to thermoelectrical noise. For dynamical measurements, however, the AC-system may be disadvantageous due to inferior high-frequency properties.

Force transducers based on strain-gage technology must have sufficient strain when the rated load is applied to produce measurable output signals. This fact in combination with the fact that traditional strain gages (foil type) have a spatial distribution of several millimetres mean that the force transducer cannot be arbitrarily small. In order words, the force transducer will have a relatively low stiffness as well as some mass. This can be translated to a lowest natural frequency that has to be considered when performing dynamic measurements (and estimating the uncertainty of the measurements). Additional care must be taken since the natural frequency stated by the transducer manufacturer is the natural frequency for the transducer with idealised boundary conditions. The lowest natural frequency for the transducer installed in the structure where the forces should be measured may deviate a lot from the natural frequency given in the data sheet.

Sometimes semiconductor strain gages are used instead of metal foil strain gages. These gages, also called piezoresistive gages, are characterized by a much higher gage factor, i.e. higher output signal as a function of a given sensed strain. This means that they can be much stiffer, compared to transducers based on foil gages, resulting in a higher natural frequency. This is advantageous in dynamic measurements.

2.1.4 Piezoelectric sensing

Piezoelectric sensors are quite different from sensors based on strain gages. The piezoelectric effect means that certain crystalline materials deposit an electrical charge on attached metal plates when subjected to changes in applied force. Very small deformations are needed which means that the sensors can be made very stiff resulting in high natural frequencies. This makes them suitable for dynamic measurements.

Since the charge inevitably leaks out due to finite resistance and capacitance, the sensor is not suited for truly static measurements. The measuring system is characterized by a so-called discharge time that describes the time-rate of charge leakage. Discharge time depends on not only the transducer itself but also on used cables and charge amplifiers.

The induced charge is not easily measured. Generally, the high-impedance charge signal is converted by a so-called charge amplifier to a low-impedance voltage signal that can be measured (and displayed) with standard instruments. Charge amplification can be performed either by electronics internal to the transducer or by external electronics.
2.1.5 Sensing by LVDT

An LVDT (linear variable differential transformer) is an inductive position sensor. By connecting it to an elastic force-sensing element a force transducer is obtained. The LVDT contains one primary and two secondary coils, see Figure 5. When the core is moved an AC-signal is produced that can be used to determine amplitude and direction of the core movement. The resulting transducer can be made comparatively stiff and with low mass that makes it suitable for dynamic measurements.

![Figure 5. LVDT position sensor](image)

2.1.6 Sensing by capacitive methods

In the simplest case the principle used in a capacitive sensor is that the capacitance between two metal plates is changed when the distance between the plates is altered. Force can be measured if the metal plates are connected to an elastic element. This is used in some miniaturized force transducers as the one in Figure 6, below.

![Figure 6. Example application of a capacitive force transducer](image)

2.1.7 Sensing by optical methods

A very active research area is the area of fibre-optical sensors. Of special interest here are extrinsic fibre Fabry-Pérot interferometer (EFPI) strain sensors and fibre Bragg grating (FBG) strain sensors. The working principle of the EFPI-sensor is that when subject to a force the fibre deforms and the distance between the fibre end-faces changes which is detected by optical interferometer techniques. In the FBG-sensor the Bragg grating serves as an optical strain gauge, reflecting incident light with a wavelength corresponding to the wavelength of the Bragg grating. With appropriate mechanical transduction monitoring the shift in wavelength of the reflected light results in a precise and stable force transducer. Optical sensors are interesting for dynamic force measurement since they
can be made small and do not require much deformation to produce sufficient output. This means that they can be made very stiff with resulting high natural frequencies.

2.2 Static vs. dynamic measurements

To understand some of the differences between static and dynamic measurements, a simple model of a force transducer has to be studied. Obviously, every force transducer is a distributed-parameter system that should be modelled by a single, or a system of, partial differential equations of motion. However, it is well known that it is possible to find a reduced-order discrete-parameter model that arbitrarily well describes the vibration behaviour of the distributed-parameter system in a selected finite number of eigenmodes. If we assume a linear and time-invariant behaviour, and if we are satisfied with describing the internal behaviour of the force transducer in its lowest eigenmode (this is often sufficient since one should in most cases not expect to be able to measure above the lowest eigenfrequency, we can choose the model in Figure 7.

![Figure 7. Simple and often sufficient model of a force transducer](image)

The equations of motion can be written as

\[
\begin{align*}
\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) &= F_1 \\
\ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) &= F_2
\end{align*}
\]

(1a)

in which \( m \) is mass, \( c \) is viscous damping, \( k \) denotes stiffness and \( F \) is external load. If we consider the case of a strain gage force transducer the bridge circuit delivers a signal that is proportional to the elastic deformation. This means that the signal from the transducer \( U \) is given by

\[
U = S \cdot k(x_1 - x_2) = S \cdot F_{el}
\]

(2)

Here \( S \) is a sensitivity factor. If we now assume that \( F_1 \) is the force that we want to measure and that mass \( m_1 \) is connected to a rigid structure, that is \( x_2 = 0 \), we have the situation in which we want to measure one external force but have only information about one internal force, namely the elastic force.
The equation of motion can be rewritten in the customary way

\[ \ddot{x}_i + 2\xi\omega_e \dot{x}_i + \omega_e^2 x_i = \frac{F_i}{m_i} \]  

(3)

Here \( \omega_e \) is the undamped natural frequency and \( \xi \) is the viscous damping factor. They are given by

\[ \omega_e = \sqrt{\frac{k}{m}}, \quad \xi = \frac{c}{2m_i \omega_e} \] 

(4a, b)

For us it is interesting to look at the quotient between elastic force (which we can measure) and the external force (which we want to measure)

\[ \frac{F_{el}}{F_i} = \frac{kx_i}{m_i \dot{x}_i + c\ddot{x}_i + kx_i} \] 

(5)

From this equation we can see that for the static case, when all time derivatives are equal to zero, the elastic force is identical to the external force (which is obvious since static equilibrium prevails). However, in the dynamic case the difference between the elastic force and the external force may be arbitrarily large. The equations above will be of further use in the chapter about other methods to reduce measurement uncertainties.

### 2.3 Dynamic applications

The following subsections describe applications in which dynamic force measurements are routinely used. For every application some references are given. The references in these sections are by no means meant to be exhaustive. An (near) exhaustive list of references is given only in the section about methods to dynamically calibrate force transducers.

#### 2.3.1 Material testing

In fatigue testing of components material testing machines are used to generate dynamic loads. Since the time taken (assuming that to a certain extent damage is not dependent on the loading frequency) for a test is proportional to the frequency of the applied load there is a definite demand for high-frequency testing. For high-cycle fatigue testing the loading frequency may reach approximately 1000 Hz.

In most cases the assumption is made that since the eigenfrequency of the force transducer itself is of several kHz the static calibration of the force transducer is sufficient to secure high-quality measurement results. However, the installation of the force transducer in the machine, together with a test specimen may significantly alter the lowest eigenfrequency which contains elastic energy of the force transducer. This means that one may operate close to the lowest eigenfrequency of the system if not to the lowest eigenfrequency of the isolated transducer. This points to the fact that to reduce measurement uncertainty a dynamic calibration should be carried out or a well-founded computational model should be used to investigate the true dynamic behaviour of the system.
A more simple thing that should be considered is that what one wants to measure is the load applied to the test specimen. Since there is a finite inertia that has to be accelerated between the test specimen and the force transducer there will always be a difference in the force recorded by the force transducer and the force experienced by the test specimen. This could be accounted for by measurement of acceleration and estimation of the inertia or by filtering compensation techniques. Obviously this will reduce the measurement uncertainty but to what extent is largely unknown. Comparison could be made, under dynamic conditions, to a reference specimen but to ensure traceability the (dynamic) calibration of this should be traceable to the basic SI units. This is, however, not generally the case.

Another case of material testing that is closely related to the dynamic measurement of force is the area of impact testing. In this case a lot of energy is imparted to the structure in a short time compared to traditional fatigue testing. A typical test set-up is shown in Figure 9. Used transducers are calibrated under static conditions. By arranging transducers in series (one reference and one to be verified) a comparison calibration can be carried out in some cases. One way to calibrate the reference transducer was proposed using energy considerations.

The area of material testing is characterized by the fact that the boundary conditions for the installed instrumentation are well defined and do not change considerably from time to time. This makes it plausible that if an efficient method for dynamic calibration of reference transducers was at hand and if this was used, with another dynamic calibration
with the reference transducer installed in the actual machine, a coherent traceable calibration would be possible to achieve. Variations in boundary conditions due to specimen stiffness and inertia could possibly be estimated in the uncertainty budget by a combination of testing and calculation.

2.3.2 Modal testing

Modal testing\(^{21}\) aims at finding (or improving existing) mathematical models, best fitted to experimental data, of the vibration behaviour of mechanical structures. Vibration theory is used to obtain parameterised models and measured data is used together with numerical techniques to obtain a best-fit of data to assumed models.

The structure subject to test is excited in some way and the response due to the excitation is measured. Knowing the excitation and the response, the system parameters can be calculated. Commonly, the system parameters sought for are a number of eigenfrequencies, damping factors and eigenmodes.

\[
\text{Response} = \text{Properties} \times \text{Excitation}
\]

*Figure 10. Relation between input and output for an arbitrary system*

In modal testing the true exciting forces have to be measured together with the structural response. Forces are most often measured with piezoelectric force transducers and the response by piezoelectric accelerometers. The exciting force may be harmonic (stationary or quasi-stationary), impulsive or some kind of random excitation. In all cases accurate measurements of forces are necessary to obtain good mathematical models of the structure subject to test. Many of the parameter-fitting methods in use are quite sensitive to small changes in measured data, something that even more underlines the importance of trustworthy measurement data.

To the present author’s knowledge no comprehensive study of the traceability and uncertainty pertaining to the resulting mathematical model has been published in the literature. Even the more basic question about the uncertainty of the measured forces is very rarely discussed in the modal testing community.

Since the transfer function from force to acceleration is commonly sought for the calibration of individual force transducers and accelerometers is not always necessary. Instead the measuring chain from force to acceleration can be characterized\(^{27}\). In this way the problem with realizing a traceable dynamic force is circumvented.
2.3.3 Crash testing

In a majority of cases, crash testing is synonymous with crash testing of vehicles (mostly automobiles). Measurement results from crash tests are used as a vital instrument to provide safer vehicles. Nevertheless, calibrations of the used transducers are performed in the traditional static way. A way that is known to lead to significant errors.

Figure 12. Crash test of a complete truck

2.3.4 Dynamic weighing

Weighing implies measurement of force, namely the gravitational force acting on the object being subject to weighing. Since the gravitational force acts in a well-defined direction and since the problems with boundary conditions, so evident in other areas of force measurement, are more or less non-existing the uncertainties obtainable in static weighing applications can be very small. Dynamic weighing on the other hand implies that one tries to measure the gravitational force on an object that is moving in an inertial frame of reference. In many cases the object moves in a direction that is more or less orthogonal to the direction of the gravitational force (one example is a vehicle travelling on a straight and smooth road, see Figure 13) but in some case the object has an acceleration component in the same direction as the gravitational force (one example of
this is weighing of garbage while lifting the garbage). In some cases the object moves over and in some cases together with the weighing system (onboard weighing).

Figure 13. Low-speed weigh-in-motion (LS-WIM)$^{24}$

The area of dynamic weighing is a vast area that is surveyed in another research project at SP. In the present paper it is only noted that dynamic weighing involves dynamic force measurement so a deeper understanding of the performance of dynamic weighing systems necessitates knowledge about dynamic force measurements in general$^{25}$. One peculiar thing about dynamic weighing is that in most cases one wants to extract a time-invariant quantity (namely the mass) from a time-series of measurement data$^{26}$. This extraction process is not directly related to the measurement of force except that the probability to succeed with the extraction increases (presumably nonlinearly) with increasing quality of force measurement data.

When calibrating a dynamic weighing system the situation is quite simple if the static weight is the chosen measurand (instead of for instance dynamic tire forces) since a well-defined static weight is easily obtained by weighing (with object stationary) on a traditional static scale. This means that there is no need for realizing dynamic calibration forces even though the dynamic weighing system measures such forces.

2.3.5 Production processes

Dynamic force measurement is important in various production processes. One important example is the measurement of cutting forces when turning, milling, grinding or drilling$^{27}$. High-speed machining is desired since this increases the number of objects produced in a given time. This leads to high demands on the ability of the tool dynamometer to measure dynamic forces. Different compensation methods$^{28}$ to improve and extend the dynamic area have been advanced for this area similar to those used for material testing purposes. No investigation of the uncertainty possible to achieve with these approaches seems to have been published and neither has any investigation about how to calibrate the systems been described except in some few cases by comparison with something that is presumably better$^{29}$. Tool dynamometers have some additional (as compared to material testing load cells) difficulties since they generally measure three orthogonal forces (in some cases also three moments).
2.3.6 Robotics

In robotics force transducers are used to measure and control the dynamic forces during robot manoeuvres. The quality of control is dependent on the ability of the force measurement devices to accurately measure the dynamic forces. The maximum frequency in robotics is probably not as high as in tool dynamics for instance. But on the other hand when a closed-loop system is using the force measurement data, instability may occur if the measurement data is not accurate enough. The uncertainty of closed-loop performance is a complicated nonlinear function of the uncertainty of transducer measurements. Techniques of robust control could handle uncertainties in measurement data but if the uncertainties in measurement data could be reduced, the control system could be designed for better performance instead of to a great extent coping with uncertainties in measurement data.

2.3.7 Aerodynamics

Aerodynamic forces on objects are measured in wind tunnels. Forces in several directions (and sometimes also moments) can be obtained with special types of balances. Since testing is expensive and usually performed on scaled models, the ability to link the experiments to numerical calculations is vital to improve and gain confidence in the computational models. These computational models can then be used as a supplement to traditional testing. The dynamic performance of the balances must be investigated.
before too much trust can be put on the measurement data. Usually balances are calibrated statically only.

Figure 16. Aerodynamic forces being measured in a wind tunnel experiment

2.3.8 Vehicle dynamics

Dynamic forces are measured in many applications of vehicle engineering such as tire/road-forces, suspension forces and structural forces. Even though the automotive industry since long has been leading in implementing quality systems, the concept of measurement traceability and uncertainty as it pertains to dynamic force measurement seems to be lacking. By referring to simplistic statements of the kind that my equipment is traceably calibrated one chooses to forget the in many cases significant difference between static calibration and actual dynamic use.

Figure 17. Characterization of suspension properties by measurement of dynamic forces

2.4 Measurement uncertainty

Assuming that not every reader is familiar with the concept of measurement uncertainty a short summary of the theory is given below. For a more complete treatment the reader should consult the internationally agreed Guide to the expression of Uncertainty in Measurement (GUM). Only reporting the values obtained during a measurement is not sufficient. Since the measurement data in many cases is used to judge the quality of a product, or as a basis for changes being made during a development phase, measurement data must be adjoined by
a quality label. This quality label is the so-called measurement uncertainty. A complete report from a measurement of a quantity \( Y \) (which in our case is a time series) reads

\[
y \pm U
\]

in which \( y \) is the best estimate (using all available information) of \( Y \) and the interval \([y-U, y+U]\) is designed in such a way that it with a given probability (typically 95%) covers the true value of the measured quantity. The quantity \( U \) is called the expanded (measurement) uncertainty. The measurement uncertainty represents an indication of the quality (and usefulness) of the measurement. As a simple example, to report that the measurement result is \( 10 \pm 0.1 \) N is far more valuable than reporting the result \( 10 \pm 5 \) N.

### 2.4.1 Obtaining the measurement uncertainty

The general procedure is started by modelling the measurement (and evaluation) process by a functional relationship \( f \) defined by

\[
Y = f(X_1, X_2, \ldots, X_N)
\]

in which \( X_i \) is a set of input quantities on which the output quantity \( Y \) depends. After all measurement data has been collected and used together with information from calibration certificates, experience, data sheets and known literature estimations, the input quantities are calculated. These estimations denoted by \( x_i \) are used to obtain an estimate \( y \) of the output quantity \( Y \) using the equation

\[
y = f(x_1, x_2, \ldots, x_N)
\]

It is assumed that all estimations are the most reliable ones, corrected for all significant (known) effects. If this is not the case correction factors may be treated as separate input quantities. To obtain a measurement uncertainty for the output quantity the measurement uncertainty for all input quantities must be obtained. These uncertainties are expressed as standard deviations (standard uncertainty) of the estimated input values and are denoted by \( u(x_i) \).

Standard uncertainties are evaluated in two different ways. A type A evaluation of uncertainties is performed by statistical methods on repeated measurement data. Type B uncertainties are evaluated by any other method than statistical analysis of repeated measurements. Typically this involves previous experience, computational models, data sheets and information obtained in the literature. In this case to obtain the standard uncertainty a probability distribution must be assumed (typically rectangular or triangular).

Using the standard uncertainties of the input quantities the combined standard uncertainty of the output quantity can be calculated

\[
u^2(y) = \sum_{i=1}^{N} c_i^2 u_i^2(x_i)
\]

in which the so-called sensitivity coefficients \( c_i \) are given by the partial derivatives.
\[ c_j = \frac{\partial f}{\partial x_j} = \frac{\partial f}{\partial X} \bigg|_{X_1=x_1,...,X_n=x_n} \] (10)

This assumes that the first-order terms of the Taylor expansion are sufficient and that the input quantities are uncorrelated. If the model function \( f \) is highly nonlinear, higher order terms must be appended and if input quantities are correlated cross-terms must be used.\(^3^8\)

We now have the standard uncertainty of the estimate \( y \) of the measurand \( Y \). It remains to design an interval that with a given probability covers the true value. The expanded uncertainty \( U \) serves this purpose. It is given by

\[ U = k \cdot u(y) \] (11)

The determination of the coverage factor \( k \) can be both simple but also quite involved. In the simplest case when a Normal distribution can be assumed and sufficient information has been collected to estimate its mean, the coverage factor corresponding to a confidence level of 95% will be 1.96 (given by the values of the Normal distribution). If a Normal distribution can be assumed (for instance in the case of any at least three input quantities having standard uncertainties of approximately the same magnitude) but too few measurements (<10) have been collected to reliably estimate the mean, the coverage factor (for 95% confidence level) will have values ranging from 2 to 3. In the third case when a Normal distribution can not be assumed, the actual distribution must be used to calculate the coverage factor which in this case may take on values typically ranging from 1.5 to 3. For further details the reader is referred to the GUM.\(^3^8\)

### 2.4.2 Uncertainty in force measurements

Here a simple example is given to illustrate some of the uncertainties that may be encountered in dynamic force measurements. Assume that one wants to measure the maximum value of the dynamic force that occurs when producing a certain part in aluminium using the set-up given in Figure 18.

![Figure 18. Measurement of force during production](image)
The measured forces have the general appearance given in Figure 19.

![Figure 19. Measured force during production of part](image)

The model function in this case can be written as (observe that additional terms could be added but are left out for the sake of simplicity)

\[
F_{\text{max}} = \bar{F}_{\text{max}} + \Delta_{I,c} + \Delta_{\text{temp}} + \Delta_{\text{sid}} + \Delta_{\text{dyn}} + \Delta_{\text{drift}} + \Delta_{\text{repr}}
\]  

(12)

in which the following notations have been used:

- \(\bar{F}_{\text{max}}\): measured mean value of maximum forces
- \(\Delta_{I,c}\): error of force transducer and additional instrumentation (from calibration certificates)
- \(\Delta_{\text{temp}}\): error due to difference in temperature compared to calibration event
- \(\Delta_{\text{sid}}\): error due to differences in side-force distribution between actual measurements and calibration
- \(\Delta_{\text{dyn}}\): error due to the fact that dynamic measurements are performed while the calibration was performed with static forces
- \(\Delta_{\text{drift}}\): error due to drift of force transducer
- \(\Delta_{\text{repr}}\): error due to lack of ability to reproduce measurement conditions

Using the available measurements and all other knowledge the maximum value of the dynamic force during production is reported as (at 95% confidence level)

\[
F_{\text{max}} = 950 \pm 120 \text{ N}
\]  

(13)

Different uncertainties contributed as shown in the uncertainty budget in Table 1, below.
Table 1. Uncertainty budget for the example of Figure 18

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimate (N)</th>
<th>Distribution</th>
<th>Standard uncertainty (N)</th>
<th>Sensitivity</th>
<th>Contribution to standard uncertainty</th>
<th>Variance (N^2)</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{max}}$</td>
<td>949.8</td>
<td>normal</td>
<td>14.2</td>
<td>1</td>
<td>14.2</td>
<td>201.64</td>
<td>8</td>
</tr>
<tr>
<td>$\Delta_{c}$</td>
<td>0.000</td>
<td>normal</td>
<td>2.5</td>
<td>1</td>
<td>2.5</td>
<td>6.25</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Delta_{\text{temp}}$</td>
<td>0.000</td>
<td>rect.</td>
<td>4.16</td>
<td>1</td>
<td>4.16</td>
<td>17.28</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Delta_{\text{sid}}$</td>
<td>0.000</td>
<td>rect.</td>
<td>5.77</td>
<td>1</td>
<td>5.77</td>
<td>33.33</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Delta_{\text{dyn}}$</td>
<td>0.000</td>
<td>rect.</td>
<td>57.7</td>
<td>1</td>
<td>57.7</td>
<td>3333</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Delta_{\text{drift}}$</td>
<td>0.000</td>
<td>triang.</td>
<td>2.04</td>
<td>1</td>
<td>2.04</td>
<td>4.17</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Delta_{\text{repr}}$</td>
<td>0.000</td>
<td>rect.</td>
<td>11.5</td>
<td>1</td>
<td>11.5</td>
<td>133.3</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$F_{\text{max}}$</td>
<td>949.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3729</td>
<td>2781</td>
</tr>
</tbody>
</table>

2.4.3 Ways to reduce uncertainty

A general approach to reduce uncertainty is to calibrate the used measurement system. There will always be differences between the calibration situation and the situation at which the actual measurements are performed. In the example given above, large uncertainties resulted since there was a considerable difference between calibration and actual use, namely the measurement system was calibrated statically but used dynamically.

The best way of reducing uncertainty in this case is to reduce the differences (not possible to completely eliminate them) between calibration and actual use. Some different approaches seem possible:

1. Dynamic calibration closely matching the actual use
2. Idealised dynamic calibration

The second case means that methods must be available to transfer the results to the actual measurement situation. This will involve testing to obtain information about dynamic properties of the structure in which the force transducers will be installed as well as computational methods to link this testing to the dynamic idealised calibration. Unfortunately, for the measurement engineer, neither item 1 nor 2 above are commercially available and at present only item 2 is available to a very limited extent at some research labs.
3 Calibration of force transducers

Before discussing various methods of dynamic calibration, the internationally agreed definition of calibration will be given since the meaning of calibration is often misunderstood. The full-length definition is a “set of operations that establish, under specified conditions, the relationship between values of quantities indicated by a measuring instrument or measuring system, or values represented by a material measure or a reference material, and the corresponding values realized by standards”. A simpler definition is to “determine by how much the instrument reading is in error by checking it against a measurement standard of known error”. This reveals that calibration does not imply any adjusting to decrease the instrument error, only a determination of the errors. An instrument is therefore not better nor worse after a calibration although the calibration results can be used to correct for the identified errors, thereby reducing the errors.

3.1 The concept of traceability

Various modern quality standards such as ISO-9000, QS-9000 and ISO 17 025 require measuring instruments to be traceably calibrated. The term traceability is defined as a “property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties”. The importance of traceability stems from the fact that measurement results are frequently compared between different company departments, companies and countries. This comparison can only be performed if they can be traced back (through several steps all having associated uncertainties) to realizations of the basic SI units.

3.2 Static calibration

Static calibration of force transducers is well established since long. Calibrations are performed either by primary or secondary standards. A primary standard is defined as a “standard that is designated or widely acknowledged as having the highest metrological qualities and whose value is accepted without reference to other standards of the same quantity”. A secondary standard is defined as a “standard whose value is assigned by comparison with a primary standard of the same quantity”.

In the field of static force calibration the so-called deadweight machine is a primary standard. In the deadweight machine, forces are realized by letting the gravitational force of calibrated weights act on the calibration object.
In other cases static calibrations are performed by realizing forces and comparing the output from a reference transducer to the output from the transducer subject to calibration. Traceability is secured in this case if the reference transducer is calibrated using a primary standard.

Figure 21. Calibration of a force transducer using a (secondary) hydraulic calibration standard

### 3.3 Dynamic calibration

The dynamic or time-varying forces to be measured may be periodic (harmonic or of a more general kind) or transient. It is well known that a general periodic signal can be expressed as a sum (Fourier synthesis) of harmonic components and that a transient signal can be described in the frequency domain by use of Fourier integrals. Dynamic calibration methods working in the frequency domain may therefore be used to quantify the behaviour of the transducer when subjected to arbitrary forces. However, if it is known that the transducer will be used to measure short-term transient loads it may be an advantage to calibrate it using the same kind of loads thereby simplifying for the measurement engineer when using the result of the calibration.
Primary static force calibration is fairly simple since it is possible to realize traceable static forces using the gravitational force acting on masses. Controlled dynamic forces are more difficult to realize. Forces may be realized by use of electrodynamic shakers, impact apparatus or other machinery. Calibration implies that the output signal from the measuring system should be compared to the force actually applied to the system. The actual force may be determined using the general equation of motion (the second law of Newton)

\[ F_{\text{tot}} = m \ddot{x} \]  

in which it must be acknowledged that the resulting acting force, \( F_{\text{tot}} \), and the resulting acceleration, \( \ddot{x} \), are vectorial quantities. Measuring the acceleration and the mass, \( m \), the force applied may be determined. The problem with this is that the dynamic force calibration is limited by the uncertainty possible to achieve in acceleration measurements which is typically not much below 1 %.

Published methods concerning harmonic and impulse force calibrations are described below.

### 3.3.1 Harmonic methods

The general idea is to excite the transducer harmonically (by use of an electrodynamic shaker) and compare the signal from the transducer to the force actually applied. A similar set-up\(^40\) is used as when calibrating accelerometers, see Figure 22. Inertial forces are generated by loading the transducer with known masses and exciting the system by the shaker. Interferometrically calibrated accelerometers or direct interferometry\(^41\) (fringe counting method) are used to measure acceleration.

The force transducer is modelled by a simple discrete-parameter system shown in Figure 23. This model is sufficient to describe the dynamics below the first eigenfrequency under the condition that only motion (and force) in one direction needs to be modelled. The dynamic load acting on the measuring element of the force transducer is given by

\[ F = (m_{\text{end}} + m_{\text{load}}) \ddot{x}_{\text{end}} \]  

The so-called end mass of the transducer, which is the inertia situated between the measuring element of the transducer and the load mass, is unknown but it can be determined as follows. By exciting the system by a sweep in frequency and using the equation above as well as the output from the transducer, the dynamic sensitivity can be determined for the tested frequencies and at the same time the end mass can be determined by a least-square approach.
Drawbacks of the method as described are that one assumes that the masses behave as rigid bodies in the tested frequency range. This assumption includes also the connection between transducer and loading mass. Another problem is that excitation and motion will not be entirely in the direction indicated by Figure 23. Some transversal motion will always occur and the tested system may have transversal resonances that make the system susceptible to transverse parasitic excitations.

To reduce the influence of transversal motion, several accelerometers can be applied to the load mass and averaged\textsuperscript{42} or a guiding system\textsuperscript{43} using air bearings may be used. In the same way several accelerometers can be used to obtain better (when the assumption of rigid connection fails) estimations of the actual dynamic force acting on the measuring element of the transducer being calibrated.

The tested frequencies being reported range from 20 Hz to 1000 Hz with a maximum force of 1.5 kN. The estimated expanded measurement uncertainty for calibration of (lightweight) piezoelectric force transducers was stated\textsuperscript{41} as 0.3% to 0.6% depending on frequency and transducer characteristics.
3.3.2 Impulse methods

For transducers mainly used to measure impulsive forces the use of data from a harmonic dynamic calibration is not as straightforward as if data could be obtained from a calibration in which the loading more closely resembled the actual forces to be measured in the service life of the transducer. Recognising this some methods have been developed for transducers used in impact and crash testing.

By making a known mass collide via an elastic element with the tested force transducer and by using an optical interferometer to measure velocities immediately before and after impact, the impulse imparted to the tested transducer can be estimated.

\[ \int F dt = M(v_1 - v_2) \]  

in which \( M \) is the total mass of the impacting object and \( v_1 \) and \( v_2 \) are the velocities measured by the optical interferometer. The impulse given by the equation above is compared to the time-integration of the force transducer signal. The equipment used did not allow for accurate measurements of acceleration and therefore it was not possible to calibrate the ability of the force transducer to measure force values but only the integral of force values during the impact.

The range of forces was quite limited since the maximum force value reported was approximately 3 N and the impact did not result in much high-frequency dynamics (due to the elastic sponge in Figure 24). The pulse time was approximately 0.5 s. The relative standard uncertainty in the determination of impulse was estimated to be less than 10^-3.

The apparatus described above was later improved (see Figure 25) to make it possible to calibrate also the instantaneous values of force (by use of acceleration measurements provided by a laser Doppler interferometer). The maximum force measured was close to 90 N and the pulse time approximately 0.01 s. Quoted combined standard uncertainty in
measuring the inertial force (which is compared to the signal from the force transducer) was 0.9 N or approximately 1%.

Figure 25. Improved facility to calibrate instantaneous force values during impact

More experiments with a somewhat improved equipment was reported\(^4^6\). The maximum force was 230 N. The standard uncertainty of the inertial force was 0.9 N and was dominated by the uncertainty of the electrical counter measuring the instantaneous value of beat frequency.

A similar approach has been developed\(^4^7\) with the main difference being that the force transducer to be calibrated is mounted to the moveable body \(M_2\) in Figure 26. Velocities of the two colliding bodies (\(M_2\) is initially at rest while \(M_1\) is set in motion by the piston) are measured by laser Doppler interferometry (LDI) and exactly as described above the true force during impact is calculated by use of the calculated acceleration and the known masses. Linear air bearings are used.

Figure 26. The planned impulse calibration facility at PTB

As mentioned in the section about harmonic calibration, to obtain the true force acting on a body with finite dynamic stiffness it is not sufficient to measure only one acceleration but instead the acceleration distribution must be measured and the force calculated by

\[
F(t) = \int \rho(x) a(x,t) dV
\]  

(17)
in which $V$ is the volume of the body, $\rho(x)$ is the density distribution and $a(x,t)$ is the acceleration field. Obviously, the complete acceleration field can not be measured but instead one or more measurements must be used to estimate the field. With the LDI-technique measurements of surface velocities can be achieved. Finite element (FE) calculations were performed\textsuperscript{47} to find the correlation between surface measurements and distribution of acceleration inside the body during impact of bodies with the geometries shown in Figure 26. The goal was to find a correction factor leading from surface measurement to mean velocity of the complete body. No definite conclusion was reached and no experiments were reported\textsuperscript{47}. Later more calculations were reported\textsuperscript{48} that showed the quantitative correlation between surface measurements and mean volume data. The difference was shown to be of the order of 1%.

Development of the test rig shown in Figure 26 is under development with the goal to reach forces up to 20 kN and pulse times of the order of 1 ms. The predicted\textsuperscript{49} relative measurement uncertainty for the dynamic realization of the mentioned forces are in the order of 1%. The intended purpose is to calibrate special reference transducers in the facility and then to provide traceability for other transducers using secondary comparison methods. These secondary methods have not been developed and there are no reference transducers having high enough eigenfrequencies.

Preliminary tests of the method proposed\textsuperscript{47,48} were reported\textsuperscript{48} using the test rig shown in Figure 27. A piezoelectric force transducers was used and different elastic pads were used to achieve force pulses of the magnitude 1500 N and pulse times down to 0.3 ms. Good qualitative agreement between LDI-derived force data and the output from the piezoelectric transducer was achieved but no quantitative results concerning accuracy and uncertainty were given.

![Figure 27. Pendulum test rig for preliminary tests of impulse force calibrations at PTB](image)

In material testing it is well-known that many properties of the tested object can be deduced from the force-deformation diagram. A calibration method for the instrumented strikers used in pendulum and dropped weight testing has been presented\textsuperscript{50}. This method uses measurement of pendulum angles (or dropped weight velocities) before and after impact. By use of an assumption of a time-invariant (linear or nonlinear) relation between sensor output and force, a single dynamic sensitivity can be obtained through an optimisation procedure using repeated measurements. The method has the great advantage that it requires no other equipment than the equipment ordinarily used in the impact testing. It is shown that the difference between static and dynamic sensitivity can
be in the order of 10%. The method should also be useful for calibrating sensors (for instance piezoelectric) that are used in other applications than pendulum and dropped-weight testing. In this case a method must be developed to take care of the differences in boundary conditions during calibration (in the pendulum machine) and actual use. Also an uncertainty budget for the method remains to be developed and the feasibility of using a single mean dynamic sensitivity must be further investigated.

### 3.3.3 Negative step method

Another approach to generate transient loads for dynamic calibration is by use of the so-called negative step-force generator shown in Figure 28. Force is produced by a hydraulic press (item 10 in Figure 28) and is increased until the brittle rod (item 6 in Figure 28) ruptures at a fairly well-defined load, \( F_0 \). At the time of rupture the force transducer (item 3 in Figure 28) is unloaded or subjected to a negative step-load of \( -F_0 \). Similar set-ups have been produced for step-loads ranging from \( 10^{-2} \) N to 9.8 MN. By measuring the force transducer response to the negative step-force and by fitting this data to a mathematical model, the transducer response to other types of loads can be calculated. It must be observed that the obtained model actually represents the force transducer only as installed in Figure 28. Further investigations must be performed to use this model for the transducer with other boundary conditions. No estimation of measurement (and adjoining model) uncertainty was produced. Traditional static calibrations of force transducers provide a measure of the transducer sensitivity (that is the relation between electrical output and force) but the negative-step method provides a dynamic mathematical model of the transducer. Using this mathematical model the frequency-dependent sensitivity can be calculated if wanted. This means that if there was an accompanying statement of traceability and uncertainty this method could be a useful method of dynamic calibration.

Even though no uncertainty budget was presented some investigations of dynamic repeatability and linearity were reported. Repeatability was calculated by several applications of (approximately) the same negative step-force and the obtained eigenfrequencies of the fitted models were compared. A spread of about one percent was obtained. The question of how much of this that should be attributed to the transducer and how much should be attributed to the method to obtain the eigenfrequencies was left unanswered. Dynamic linearity is estimated by using negative step-forces of different magnitudes and comparing obtained model eigenfrequencies. Differences of the order of five percent were obtained.

![Figure 28. Schematic picture of the negative step-force generator](image)
3.3.4 Secondary methods
In this section methods of dynamic calibration are reviewed that use reference transducers or methods which lack direct traceability to SI-units. If a reference transducer is used as a transfer standard the secondary method may provide traceability if the reference transducer is calibrated by a primary method and if the results of this calibration can be transferred (using some model for the error propagation) to the secondary calibration method.

Material testing machines are calibrated statically by use of reference transducers (transfer standards). The idea to dynamically calibrate the machines using reference transducers is therefore quite natural. An instrumented reference specimen was developed to fulfil this task. The specimen (see Figure 29) was designed to measure force simultaneously in two different ways: by use of strain gages and by use of capacitance gages. The two methods were chosen to be as dissimilar as possible.

![Figure 29. Transfer standard proposed for dynamic calibration of material testing machines](image)

The reference specimen was mounted in a material testing machine as shown in Figure 30. When using the machine to realize dynamic loads (up to 15 kN and 100 Hz), the difference between output from the strain gage bridge and the capacitive bridge was less than 0.5 %. True traceability is lacking but if one believes that the difference between the methods is sufficient to eliminate systematic errors the reference specimen could be used to calibrate material testing machines with an uncertainty composed of 0.5 % error due to the transfer standard. The reference specimen was later dynamically calibrated by PTB using their primary harmonic method. Also a force transducer calibrated at PTB was installed in series with the reference specimen (Figure 29) and calibrated using the set-up of Figure 30. Differences of the order of 0.5 % were obtained that are within the uncertainty of the methods.

Methods for dynamic calibration and verification of material testing machines using instrumented reference specimens were also discussed in Reference 16. Available standards to use are ISO 4965:1979 (E), ASTM E467-98a and MIL-STD-1312B.
The method to compare the output of the transducer being calibrated to the output from a transducer previously calibrated with a harmonic primary method can be illustrated by Figure 31. The transfer standard and the force transducer being calibrated are mounted in series in a load frame and dynamic loads are generated by use of a shaker. The load frame is mounted on soft springs. The set-up is modelled by the discrete model shown to the right in Figure 31. In this model mass $m_0$ represents the mass of mounting parts and the base mass of force transducer 1, $m_1$ represents mounting parts as well as end masses of both transducers and $m_2$ represents mounting parts and base mass of force transducer 2. The force being measured by the force transducers are assumed to be linearly dependent on the spring forces

$$ F_1 = k_1(x_1 - x_0), \quad F_2 = k_2(x_2 - x_1) $$

and the most important equation of motion can be written (if damping is neglected)

$$ F_2 = F_1 + m_1\ddot{x}_1 $$

If the electrical output from the transducers are written as (in which $S$ are dynamic sensitivities)

$$ U_1 = S_1 F_1, \quad U_2 = S_2 F_2 $$

the equation relating the two transducers can be rewritten as

$$ U_2 = S_2 \left( \frac{U_1}{S_1} + m_1\ddot{x}_1 \right) $$

If force transducer 1 is the reference transducer this equation can be used to calculate the sensitivity of force transducer 2. The cited reference does not show any results of this approach although several problems with the approach are discussed.
A secondary approach to calibrate the force transducers used in pendulum impact machines (see Figure 32) was described. The reference transducer was fixed at the position in which the test specimen normally is placed. Unfortunately the reference transducer was only statically calibrated. To reduce the high-frequency content of the transducer signals a compliant nylon bumper was fixed to the reference transducer. The signals obtained during and after impact from the two transducers were compared using different pendulum angles and nylon bumpers. Some improvements to the proposed method that have to be made before it can be used for calibrations are:

- A method for calibration of the reference transducer must be found
- If a compliant bumper is to be used its dynamic characteristics must be completely determined before the calibration
- The difference in signals from the two transducers must be divided into the difference due to the dynamic behaviour of elements apart from the transducer and that due to the transducer being calibrated itself
- A way to report the results must be devised
- An uncertainty budget must be developed

Another approach to obtain and subsequently use a calibrated impact measuring system for material testing has been proposed. The method recognises that the force measured by the instrumented striker is different from the true contact force between striker and test sample. A linear relation between the two forces (appropriate for elastic deformations) is assumed and modelled by an unknown frequency-dependent transfer function. If this transfer function was known the true impact force could be calculated (with some uncertainty) using the inverse Fourier transform.

The unknown transfer function was determined by striking a pendulum hammer instrumented with semiconductor strain gages, see Figure 33, at a horizontally supported rod that was also instrumented with strain gages attached at 100 mm from the impact end. The transfer function of the rod was determined previously by striking the rod with another rod of the same diameter and using the theory of one-dimensional longitudinal wave motion. Having the transfer function from impact force to strain measurements the impact force could be calculated using the inverse Fourier transform. Numerical problems with this inverse approach were evident in the case reported. No attempt to estimate measurement uncertainty was presented.

Figure 31. Harmonic calibration of force transducers using the comparison method
Another two-step method based on elastic wave motion was presented\textsuperscript{55} for dynamic forces between 100 N and 10 kN with the aim to determine force transducer frequency response up to about 10 kHz. In the first step the frequency characteristics of the used semiconductor strain gages and the amplifiers are determined by an impact experiment. In the second step these characteristics are used in a split-Hopkinson impact test with the force transducer to be calibrated mounted between to bars. Uncertainty in force was estimated to be 0.7%.

A system for measuring cutting forces was developed\textsuperscript{10} using an optical fibre sensor integrated in the tool shank (see Figure 34). Fairly linear (1%) output was demonstrated. A dynamic calibration was claimed based on a procedure using a standard force-sensing hammer. The hammer was used to excite the tool shank at the position where the cutting
force acts in normal operation. Output from the hammer as well as the optical sensor could be used but the only information presented was the frequency content of the optical fibre output. The procedure was very far from what could be called a true dynamic calibration.

Figure 34. Measurement of cutting force $F_z$ by use of an optical fibre sensor

A somewhat more complete (compared to Ref. 10) approach to calibrate a cutting force dynamometer was later reported. In the dynamometer table being used piezoelectric sensors were installed enabling measurements of three orthogonal forces. The idea used was to determine the frequency response from input forces, $F_i$, at the dynamometer table to output signals, $f$, from the sensors incorporated in the table. Using matrix notation the relation between input and output forces can be written

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = [T] \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$  \hspace{1cm} (22)$$

in which $[T]$ is a complex frequency-dependent matrix of transfer functions. If this matrix of transfer functions was known, the wanted input forces could be determined by premultiplying the input signals by the matrix inverse. This approach accounts for the dynamics between input and output as well as corrects for cross-talk components. To determine the frequency response elements, an instrumented impact hammer was used and considered to work in its quasi-static frequency range (supposedly it had a flat response curve in the considered frequency range, <2500 Hz). This assumption made it possible to use the static sensitivity of the hammer. Presented results made it evident that the approach extended the useable frequency range of the measurement system but since no uncertainty analysis was performed it was not possible to state any quantitative measures of the performance of the system.

Similar experiments, shown in Figure 35, were reported in another article. Frequency-dependent transfer functions from exciting force to sensor outputs were obtained by exciting the structural system by pseudo-random noise from a shaker. The exciting force was measured by a piezoelectric reference transducer. In the frequency region considered (less than 800 Hz) the reference transducer was considered to work quasistatically. Actually also the transducer to be calibrated was assumed to operate in a quasistatical region which in effect means that only the dynamic effects due to surrounding (bar, base bar and foundation) elements are investigated. This is important to accomplish when performing dynamic calibrations in order to investigate the influence of the actual installation but by itself it is not something that could be called a method of dynamic calibration of sensors. Especially not since no uncertainty estimates were presented. The
article could be used as a demonstrator of an interesting special case - namely the one in which the sensor output differs between static and dynamic measurements and the reason for this is dynamic properties not of the sensor but of the surrounding structural elements.

Figure 35. Calibration of a multi-component piezoelectric transducer

A method which may be classified somewhere between primary and secondary was outlined\textsuperscript{57}. In this paper a multi-component piezoelectric transducer measuring three forces and three moments was investigated. The frequency-dependent transfer matrix from exciting loads to output signals was obtained by impacting a body with known inertial properties connected to the force transducer. By measuring accelerations on the impacted body and using the known inertial properties, the true loads could be calculated. As usual in the structural dynamics community not a single word was mentioned concerning measurement uncertainty.
4 Other methods to increase accuracy and to reduce uncertainty

The reason for performing a calibration is to increase measurement accuracy by using the calibration data, if deemed necessary, to correct the measurement values. Remaining in the uncertainty budget is then the uncertainty of the corrections and the uncertainty due to differences between calibration situation and actual measurement situation.

It must be emphasized that a calibration performed under conditions matching as closely as possible the ones during actual use is a necessary prerequisite for reducing the complexity of the work with estimating measurement uncertainty. In theory most things possible to be measured could be calculated but this is only so if an infinite amount of information was readily at hand to put into the models and an eternity were available to develop the models. In the present author’s opinion the most efficient way of obtaining dynamic force measurement data with high accuracy and low uncertainty is through a combination of conditioning of data, well-suited calibrations and computational models to bring the calibration data into the uncertainty budget of an actual experiment.

In this section some ideas concerning conditioning (increasing accuracy) of measurement data and how computational models could be used to correct measurement data and therefore to reduce measurement uncertainty are reviewed.

4.1 Compensation methods

The idea behind compensation is that the raw measurement signals for some reason do not reflect the true dynamic forces being sought for. If there is a known systematic difference between measured signal and true force and if this difference could be quantified by a measured parameter the original signal could automatically be compensated by the known measurable systematic influence. This could be done internally to the measurement system and could in some cases be switched on or off. If this is the case it is important that calibration is performed with the compensation algorithm working in the same way as when performing subsequent actual measurements.

To start the discussion about compensation of force transducer output the idealised model of Figure 7 is reused. The force we want to measure is $F_1$ but the force transducer provides an output that is proportional only to the elastic force. From the equation (derived from the equation of motion) relating $F_1$ and the elastic force $F_{el}$ we see that the difference is due to inertial and damping force contributions. The example modelled in Figure 7 is simpler than what is encountered in many situations since quite often there is a more general dynamic system situated in the force path between force introduction and force transducer. In Figure 7 this path is modelled only by an inertia which can thought of as representing the inertia of the transducer end mass and the inertia of the force path between force introduction and transducer.

The simplest possible way to account for the difference between true excitation force and transducer output (proportional to elastic force) is to measure the acceleration of mass $m_1$, that is $\ddot{x}_1$, and if the mass is known multiply this with the acceleration signal and add this signal to the transducer output to get the compensated output

$$F_{comp} = F_{el} + m_1 \ddot{x}_1$$  (23)
Even if the mass is not exactly known and even if the acceleration measurement is not perfect the compensated force should be closer to \( F_1 \) than \( F_{el} \). When the mass is unknown a convenient method\(^5\) to determine the scale factor of the acceleration signal is to first calibrate the transducer statically and then to subject it to an impulsive load. The scale factor for acceleration is then adjusted so that the compensated signal is as close to zero as possible in the free oscillation period realized after impact. This approach was later\(^5\) successfully demonstrated and the useable frequency range of a transducer was extended from about 80 Hz to about 700 Hz. Some additional considerations regarding accuracy of the accelerometer signal and application details for measurement of cutting forces were presented\(^6\).

The more general problem when there is a complicated dynamic system in the force path could in theory be handled by a compensation system realizing the inverse (implying pole-zero cancellations) of the force path transfer function. Experimental evidence was provided\(^5\) for an additional extension of the useable frequency range of a force measurement system.

Inertia compensation using the same approach as described above was later discussed\(^13,14\) for the application in cyclic material testing.

The idea to realize a perfect (flat frequency response up to a very high frequency) transducer using system identification and digital filters has been discussed\(^41,62\). An approach that used a direct optimisation (without system identification as a separate step) to produce a compensating digital filter was elaborated upon\(^62\). The technique of high gain negative feedback to tailor the dynamic response of a measurement system has been discussed in several textbooks, of which Ref. 63 is one.

Methods of system identification both in time and frequency domain as applied to transducer calibration have been discussed\(^64\). This is an important area because it concerns how the result of the calibration should be presented to the user. In this respect a dynamic system model could be regarded in the same way as the interpolation curve given in some static force calibration methods.

The measurement of dynamic (as well a static) force can be cast as a so-called inverse problem. From a measurement of for instance elastic strain the task is to estimate the dynamic force acting on a part of a structure. In the dynamic case the transfer function from force application to sensing element must be known to solve the inverse problem. Work on this topic has been presented\(^65\) for the case of finding the optimal location of strain sensors on an elastic beam governed by Euler-Bernoulli beam theory. The inversion algorithm can be incorporated in the sensor electronics.

4.2 Computational methods

As mentioned above successful dynamic force measurements should rely on a combination of theoretical computational considerations and well-chosen dynamic calibrations. The first question to answer for the measurement engineer is when a static calibration is sufficient for his purposes. The answer to this question depends on a number of factors such as the tolerable measurement error, the type of forces that should be measured (amplitudes and frequency content), the dynamic properties of the measurement system by itself and the dynamic properties of the structures in which the transducer is installed. At least qualitative computational models could be developed to help answering this basic question and also in gaining insight into what kind of measurement signals one may encounter.
As an example consider the simple transducer model in Figure 7 and assume that the lower mass is stationary (assuming a perfectly rigid foundation). If the force to be measured is a stationary harmonic force of angular frequency $\omega$ the relation between elastic force (as measured by the transducer) and the true force $F_i$ can be written as

$$\frac{F_{el}}{F_i} = \frac{1}{1 - \left(\frac{\omega}{\omega_c}\right)^2 + i \frac{2\xi}{\omega_c} \omega}$$  \hspace{1cm} (24)

If $\alpha$ is introduced as the relative amplitude error defined by

$$\left|\frac{F_{el}}{F_i}\right|^2 = (1 + \alpha)^2$$  \hspace{1cm} (25)

the relative amplitude error can be expressed as

$$\alpha = \sqrt{\frac{1}{1 - \left(\frac{\omega}{\omega_c}\right)^2 + i \frac{2\xi}{\omega_c} \omega} - 1}$$  \hspace{1cm} (26)

This function is plotted in Figure 36 below. The graph can be used for rough estimations. For instance, if one wants to measure harmonic forces with a relative error less than 5%, one should not try to measure forces (using the static sensitivity) having frequencies much above 20% of the lowest transducer eigenfrequency. Having obtained a good estimation (with uncertainty) of the transducer eigenfrequency (through a modal analysis or from product data sheets) this could be used to correct the measurement signals and thereby obtain a better estimation of the true dynamic force. If this not leads to sufficiently low measurement uncertainties, a true dynamic calibration must be performed. At least once a true dynamic calibration should be performed also to verify the computational models used to correct the measurement signals.

The example above was of course very simplified and the characteristic frequency used was the lowest eigenfrequency of the transducer with fixed lower base. In reality the transducer is installed in a structural environment with dynamic properties. The frequency of interest in this case is the lowest eigenfrequency of the complete structure with transducer that contains modal energy of the transducer. This frequency may be much lower than the lowest eigenfrequency of the transducer by itself and it is this frequency that should be used in Figure 36 to check whether a static calibration is sufficient. The frequency could be found by a modal analysis of the complete structure or by components and then using a computational substructure analysis. One could also think of cases when it is more convenient, at least in an early product development phase, to determine the modal data of the surrounding structures by fully computational means such as FE-methods.
A few papers have been published regarding the interaction between the transducer and the structure in which the transducer is mounted. These are concerned with harmonic loads, see Refs. 66 and 67.

Another complication is that not all forces are stationary harmonic ones. Typical transient forces are impulsive loads. To gain some qualitative insight into what may happen when one tries to measure impulsive loads with a transducer possible to represent by the simple model in Figure 7, the governing ODE could be numerically solved. One example of such a solution is seen in Figure 37. In this case the transducer model is hit by a sudden force pulse of half-sine form. A comparison between the calculated transducer output and the truly applied force is shown. It can be seen that although the impulsive loading is not extremely fast as compared to transducer dynamics (ten times slower than the characteristic time of the transducer) the calculated measurement error is above 5% at several times. Also the peak force is in error by about 5%. This serves as an illustration of the fact that there is a difference between harmonic and general loads that has to be accounted for.

Another area which seems to be very far from developed is to use detailed FE-models to predict the dynamic output from different transducers. This should be possible to use in some cases a complement to dynamic calibrations. At least for companies that develop transducers.
Figure 37. Calculated force indicated by the force transducer compared to the applied half-sine force
5 The need for further work

Having come this far in the report it should be evident that most of the work remains to be done in the area of dynamic calibrations of force transducers. The point of view that should be taken is to put oneself into the position of the measurement engineer. A lot of measurement data is easy to collect but then the relevance (the measurement uncertainty) of the data should be ascertained. A lot of questions arise:

- Is a static calibration sufficient?
- If a static calibration is sufficient how does one correct the measurement data to account for dynamic effects so that measurement uncertainty is minimized?
- In what form should results from a dynamic calibration be reported so that the measurement engineer can use them without needing a PhD-degree and a lot of spare time?
- How should the results from a primary dynamic calibration be transferred by use of transfer standards without increasing the measurement uncertainty to an extent that renders the procedure meaningless?
- How should one account for the difference in boundary conditions between dynamic calibration and dynamic use?

None of these questions have a satisfying solution today. Some (a few) of them have been discussed by academic researchers but the results of the discussions are not in a form that makes them useable for a measurement engineer.

5.1 Calibration methods

Some of the developments that have to be performed apart from the general questions above are discussed below.

5.1.1 Primary methods

Harmonic methods have to be simplified so that the measurement engineer understands how to use the calibration data. This has to be demonstrated by simple examples. The methods must be made available for higher forces but methods must also be found to extrapolate the data to higher forces. Measurement uncertainty must be lowered if it should be meaningful to even discuss secondary methods. At the same time simpler methods must be found if there should be any demand for services. The cost (and uncertainty) target should be the same as for accelerometer calibrations.

Methods for impulsive calibrations must be extended to higher forces and lower pulse times. The methods presented for material impact testing must be furnished with developed and validated uncertainty budgets.

The negative-step method coupled to system identification seems to be promising. An uncertainty budget should be developed and more experiments performed.

5.1.2 Secondary methods

Secondary calibrations with instrumented reference specimens in material testing seems possible to develop as a service in fairly short time since relevant standards have been developed. In this case no problem with difference in boundary conditions is apparent.
Good transfer standards and the means to calibrate them remains to be developed. This task seems perfect for a small research project (0.5 man-year).

The comparison method for harmonic calibration has to be developed so that it is apparent what the expected measurement uncertainty will be. It could well be that the uncertainty in the end when one has to account also for the difference in boundary conditions is too high. The method must also be extended to higher forces.

Methods working with identification of transfer function and using the inverse of it to get the true force are worth developing further but they must be furnished with some metrological ideas such as measurement uncertainty. The importance of the inherent linearity assumption when working with transfer functions must be put to test.

One thing in common for all secondary methods is that there is a need for better-characterized transfer standards.

### 5.1.3 Computational methods

By all means methods of system identification, inverse methods and active compensation should be embraced by the measurement community. Smart sensors performing this automatically and adapting to changing dynamics due to differences in boundary conditions are envisaged. More research is needed to understand the obtainable decrease in measurement uncertainty and the robustness of the methods.

The areas of pure metrology and modal testing must be united to find and verify practical methods to use and transfer the data from a dynamical calibration.

Simple guidelines should be developed to aid the measurement engineer in choosing between static and dynamic calibrations.

Dynamic transducer models should be developed since it will never possible to calibrate under all possible conditions. Mathematical models will be the link between calibration and actual measurement situations.
6 Conclusions

Some methods to measure dynamic forces have been presented and a lot of application areas in which dynamic forces are measured every day have been shown. It is very easy to obtain measurement data but is very hard for the measurement engineer to produce the necessary measurement traceability and uncertainty needed to report the result of the measurement.

In static force measurement the quality of the measurement result is guaranteed by a good calibration of the transducer combined with a sound uncertainty budget. In dynamic force measurements there is a missing link. Today a static calibration is used as a base for the measurement quality but there are no methods available to use to transfer the results from the static calibration to the dynamic use. Furthermore, there are no commercially available methods for dynamic calibration of force transducers. The academic methods presented have to be developed in several ways before being of use.

Work should be directed towards developing simple methods to determine whether a dynamic calibration is necessary in a specific case, computational methods of joining the areas of static calibration and modal testing, simplified dynamic calibrations of extended ranges and computational metrological structural dynamics methods to account for differences in boundary conditions.

When it comes to developing the means to evaluate and produce high-quality dynamic force measurement data, 90% of the work remains to be done.
References

2 www.kistler.ch
5 Kumme, R., "Influence of measuring amplifiers on dynamic force measurements", Proceedings of the 13th International Conference on Force and Mass Measurement, Helsinki (Finland), May 10th-14th, 1993, pp. 25-31
6 www.pcb.com
7 www.rdpe.com
8 www.synaptics.com
12 www.mts.com
17 www.instron.com
20 www.instron.com
23 www.volvo.com
24 motion.massload.com
30 www.kistler.ch
32 Stokic, D. and Vukobratovic, M, “Historical perspectives and state of the art in joint force sensory feedback control of manipulation robots”, Robotica, Vol. 11, 1993, pp. 149-157
33 www.robotic.dlr.de
36 www.nrc.ca
42 Kumme, R., “Error sources in dynamic force calibration”, Proceedings of the 13th IMEKO World Congress, Torino (Italy), September 5th-9th, 1994, pp. 259-264
67 Yeiping, X., Kongjie, S. And Xing, A., ”The effects of dynamometer and measured system on the accuracy of dynamic cutting force measurement”, ICPCG '94, Shanghai, China, September 26th-28th, 1994, pp. 218-223