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Traceable Measurement of the Magnetic Susceptibility of Weakly-Magnetic Mass Standards

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Abstract

Magnetic forces can adversely affect precise weighings. The performance of a magnetic susceptometer, intended amongst others for the determination of the magnetic properties of mass standards, is investigated systematically in this work. Measurements are made with the susceptometer of a variety of weakly-magnetised objects of different materials over a range of relative permeabilities between 1.0013 and 1.12, including novel permeability standards.

Key words:

Magnetic, susceptibility, mass, weighing, standards

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1 Magnetism and weighing

The magnetic properties of weakly-magnetic items need to be determined accurately for a wide range of applications covering electricity generation, equipment in industry and the home, as well as the most advanced electronic and information technology systems [1]. One way of achieving the desired accuracy is through traceability to recognised standards.

Magnetic forces can, in particular, adversely affect precise weighings. Without systematic investigation, these spurious forces cannot be distinguished from the gravitational forces to be compared in the determination of mass [2]. Recent international recommendations on the accuracy classification of mass standards stipulate limits on magnetic properties [3].

One method recommended [3] of determining these magnetic properties refers to a magnetic susceptometer [2]. The present paper reports a systematic investigation aimed at verifying the performance of such a magnetic susceptometer and its attendant theory by measurement of a variety of weakly-magnetic objects of different materials ("nonmagnetic" stainless steel, Alacrite X.H.S., etc.) over a range of relative permeability up to 1.12.

2 Magnetic susceptometer

In order to determine the magnetic properties of weakly-magnetic bodies such as stainless-steel mass standards used in precision weighing, Davis [2] recently developed a novel susceptometer using a commercial mass comparator for measurement of the magnetic force (typically several micronewtons) exerted on the object of interest when placed centrally above and in the strong field gradient of an axially magnetised, vertically positioned permanent magnet. In figure 1 is shown a schematic of the susceptometer: full details may be found in reference [2].

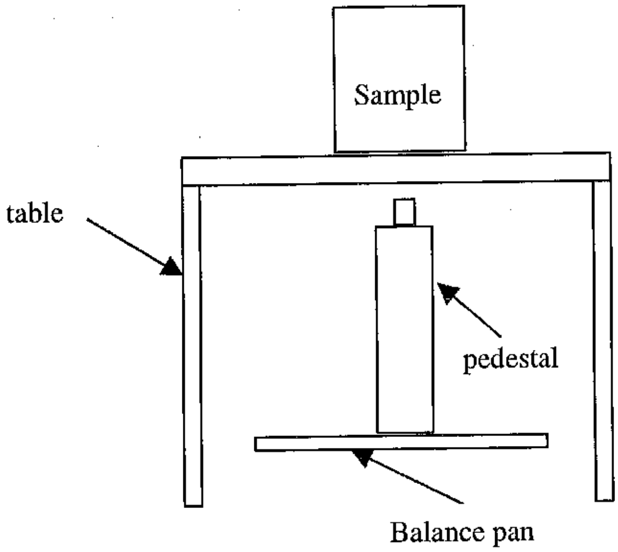

Figure 1. Schematic view of the susceptometer.

Despite the high remanence of the permanent magnet (about 1 T), the object itself is two or three centimetres from the magnet and only experiences flux densities of at most a few millitesla. This can be a distinct advantage over better-known susceptometers based on other measurement principles, such as the magnetic balance and fluxgate-type [4], since low fields are far less likely to magnetise permanently the samples under test. In addition, the shape of the test piece can be taken into account so that, for instance, finished products (such as mass standards) can be tested.

A basic expression for the force FM, exerted in the z direction by a magnetic field acting on a body with magnetisation M is given by equation (1):

$$F_M = -\mu_0 \int M \bullet \frac{\partial H}{\partial z} \mathrm{d}V \qquad (1)$$

In the case of the Davis susceptometer, H is the magnetic field strength produced by the susceptometer magnet and the integral is taken over the volume, V, of the sample to be measured. Here μ0 = 4π x 10-7 N·A-2. We assume that the quantity M is the vector sum of the induced magnetisation χH due to the magnet (where χ is the volume magnetic susceptibility of the sample, assumed to be homogeneous and nearly zero), the induced magnetization χHE due to local ambient fields (e.g. that of the earth) and any permanent magnetisation M' inherent in the sample. To make the calculations manageable, the forces between a cylindrical magnet and a cylindrical mass standard are calculated under several simplifying assumptions. Full details are given in reference [2]. Values for the susceptibility and permanent magnetisation of the sample are determined by measuring the change in magnetic forces when the magnet is reversed. This is because the terms involving χHE and M' change sign upon reversal of the magnet while the term involving χH does not.

3 Traceable magnetic measurements

The Davis susceptometer has been adopted and studied by a number of laboratories for the determination of the susceptibility and permanent magnetisation of weakly magnetic objects, particularly mass standards. Systematic investigation of the susceptometer includes work at the Bureau National de Métrologie-Laboratoire National d'Essais (BNM-LNE) [5], which confirmed geometric terms derived from Eq. (1), and the Korea Research Institute of Standards and Science (KRISS) [6], which confirmed that the earth's magnetic field has an observable effect that is accounted for in the theory of operation of the susceptometer. However, neither of these studies is directly concerned with the issue of overall accuracy. Strategies for achieving accurate results with traceability to SI units were outlined in [2] but, until this work, have never been tested adequately.

3.1 Performance of Davis susceptometer

Independent investigations have been made in the present work at the Bureau International des Poids et Mesures (BIPM) and the SP Swedish National Testing and Research Institute (SP) of two Davis susceptometers with samples over a range of relative permeabilities between 1.0013 and 1.12. Two alternative modes of operation of the Davis susceptometer have been compared and contrasted, which establish traceability in different ways. A comparison is then made with other, independent traceability routes, in particular using novel permeability standards made of iron particles in an acrylic matrix, supplied by the National Physical Laboratory (NPL) [7]

At SP, traceability in the susceptometer measurements was attempted through an "absolute" or "ab initio" calibration mode, requiring the determination of the dipole moment, dm, of the susceptometer magnet and determination of the distance, z0, between magnet and object. The dipole moment of the magnet used was determined by successive, pairwise measurements of the magnetic forces between the magnet and other similar magnets using the susceptometer; a procedure described in section §4.2.2 of reference [2]. The distance was measured mechanically with calibrated Vernier calipers. The method yields estimates of the sample susceptibility without the need of an independent permeability standard, and therefore depends heavily on the theoretical model (Eq. (1) and its approximations), as already pointed out in reference [2].

In contrast, at the BIPM the susceptibility χX of any unknown sample X is determined by a measurement of the ratio of the magnetic force exerted by the susceptometer magnet on the sample to the corresponding force on an object of known susceptibility, □S. Traceability can be obtained using a independent permeability standard when available. Alternatively, a local standard S (in the present case, a disk of Alacrite X.H.S.) may be used, the susceptibility χS of which may be determined ab initio by fitting measured force values to a theoretical curve versus distance (bootstrap calibration described in section §4.2 of reference [2]). This latter step still depends heavily on the theoretical model, but subsequent ratio measurements are far less model dependent than the "ab initio" method employed in this study at SP. Note that fitting of the magnetic force as a function of distance in this method assumes that the susceptibility of the sample is independent of applied magnetic field. Similarly, the susceptibility of an externally calibrated standard must have negligible field dependence over its range of use.

3.2 Uncertainty budget

At SP, where susceptibility measurements are made using the “ab initio” method, relatively large standard uncertainties (typically about 10%, see below) in the deduced magnetic properties were found. The method has a reproducibility worse than the repeatability. In the current investigation we have evaluated the reproducibility by making the susceptibility measurements using different mass comparators (fig. 1). Typical relative standard uncertainty components (ur) are quoted in the second column Table 1 for one of the NPL standards, known as the NPL005 block (where “005” refers to the nominal susceptibility)

	$u_r(\chi)$	$u_r(\chi)$
Uncertainty component	SP (*ab initio*)	BIPM (substitution)
susceptibility of standard (*type B*)	-	3.2 %
repeated measurement of magnetic force of unknown X (*type A*)	0.4 %	0.4 %
repeated measurement of magnetic force of standard S (*type A*)	-	0.4 %
dipole moment ($u(dm) = 0.001$ A·m^2) (*type B*)	2.9 %	0.2 %
distance ($u(z_0) = 0.3$ mm) (*type B*)	10 %	-

Table 1. Uncertainty budgets.

The corresponding uncertainties for the BIPM measurements of the NPL005 block, using the “ratio” method, where the susceptibility χ_X is determined relative to χ_S , are given in the third column of Table 1. With this method, if the shape of the permeability standard is well chosen, then uncertainty in the dipole moment of the susceptometer magnet is in fact unimportant as is the assumption that its field is that of a pure dipole [2]. Note also that the assumed susceptibility of S is common to all measurements made with S and therefore the uncertainty of the BIPM measurements can be reduced as soon as a better value of χ_S becomes available.

3.3 Study of NPL iron:acrylic permeability reference blocks

The novel NPL permeability reference blocks have the attractive possibility of being able to be made with a permeability value of choice over a range of values. The NPL also provides calibration of these blocks with low uncertainty, over a range of magnetic fields, corresponding to flux densities of a few millitesla to several hundred millitesla.

3.3.1 Spatial variations of susceptibility

The present investigation of the Davis susceptometer has revealed that the susceptibility of a typical NPL reference block appears to be different on the upper and lower end faces of the cylinder. The difference is small but, nevertheless, lies outside the quoted uncertainties, particularly for the samples having the smallest susceptibility (0.005) where the difference reaches 2% of the nominal susceptibility. We have compared the calibration assigned by the NPL to our results averaged over the top and bottom of each sample. We can find no plausible explanation for the observed difference.

3.3.2 Field dependent susceptibilities

One goal of the NPL programme was to create standards whose susceptibility was less strongly dependent on applied magnetic field than conventional standards [7].
In the present work, measurements were made of the susceptibility of the reference blocks at different magnetic flux densities in the range 0.4 mT to 6 mT, by varying the distance between the sample and the permanent magnet of the susceptometer (z0). For the reference blocks with the smallest permeability, a distinct flux density dependence of the susceptibility was observed, as shown in Figure 2. The uncertainty limits indicated in the figure are based on type A repeatability values.

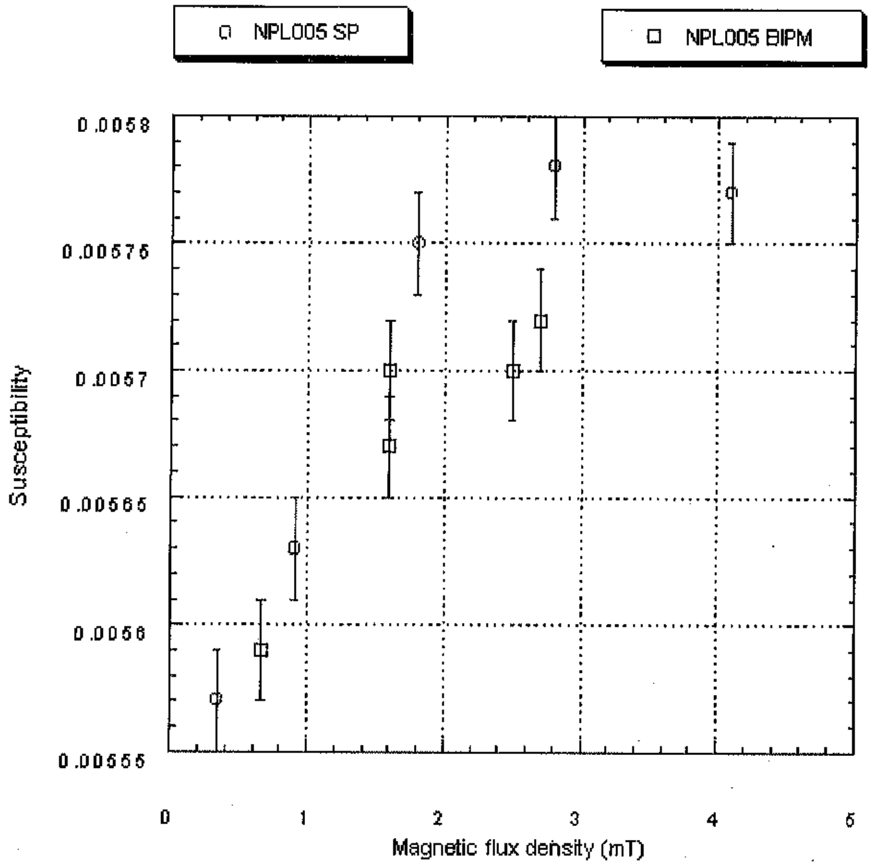

Figure 2. Dependence of susceptibility on magnetic flux density for permeability reference block NPL005.

These observations may be compared those made by the NPL [4] of the permeability of typical reference blocks at different values of magnetic flux density. In going from high (typically 120 mT) towards lower flux densities, most blocks display a permeability which initially increases, approaching an asymptote at low flux densities (say 5 mT). For the NPL005 block for example, a permeability of 0.00540(2) is quoted in the NPL calibration certificate at a field strength of 5 kA/m (6.3 mT flux density). The present work suggests that this apparent asymptote is in fact a maximum, and that the permeability indeed appears to decrease again towards even lower field strengths. According to the NPL, one should expect a shallow peak in the susceptibility with respect to increasing flux density and is a feature due to rapid movement of the walls of magnetic domains [8].

3.4 Other Blocks

In order to test the susceptometer over a larger range of susceptibility, studies were also made of two additional NPL permeability blocks similar in shape and fabrication to NPL005, of relatively large susceptibilities (up to 0.12) as well as two Alacrite samples with nominal susceptibility as small as 0.001.

The susceptibility of the NPL12 block, nominally 0.12, is, perhaps, too large to serve as a calibration standard for the susceptometer. The determination of large susceptibilities ($\chi > 0.1$) is problematic with the Davis susceptometer because the calculation of the right-hand-side of Eq. (1) in the present case has assumed that the total magnetic flux density within the sample is simply $(1+\chi)\mu0H$ [2]. For susceptibilities that are still small compared to 1, this simplification leads to a value of χ that differs from the exact solution by a term of order $\chi2$. The exact solution is known for a sphere [9] where one sees that the simplification leads to an underestimate of χ. The first-order corrections to the underestimate range from $\chi2/3$ to $\chi2/2$ depending on the radius of the sphere relative to the distance of the dipole from its surface. Both laboratories found no significant difference in the susceptibility of the NPL12 block measured at different values of maximum flux density up to 4 mT. Neither was the susceptibility found to be different from top to bottom of the block. The BIPM result obtained with the ratio method with respect to the Alacrite reference of 0.1218(13) (with no account taken of the uncertainty either of the reference or the bias due to the high susceptibility of the "unknown") can be compared with the ab initio value determined at SP to be 0.1238(104). The susceptibility quoted by the NPL is 0.1220(4).

The BIPM found with the ratio method for the NPL02 sample, a top/bottom difference in the susceptibility of about 1.4 %. At the same time, results showed no significant dependence of □ on maximum flux density over the investigated range (0.67 mT to 6.6 mT). An initial estimate of susceptibility, with respect to the assumed value of the BIPM Alacrite reference, is 0.02694(10) where the standard uncertainty shown is the reproducibility of the measurements. Although smaller in susceptibility than the NPL12 block, the magnitude of the susceptibility of the NPL02 block is still large enough (see previous paragraph) to force us to include an additional component of about 3 %, which we assume is centred around a bias estimated as +1.5 %. Hence the final BIPM value (not including the uncertainty of the Alacrite reference) is 0.02734(83). The susceptibility quoted by the NPL is 0.02659(9). On the basis of this result, we would infer that the susceptibility that had been assigned previously to the BIPM Alacrite reference [2] is systematically large by 2.8 % with a relative standard uncertainty of 3 %.

Finally, measurements were also made of the low susceptibilities of two pieces BNM 13 and BNM 14 of the superalloy Alacrite XHS. The susceptibilities measured at BIPM were 0.001275(41) and 0.001325(42) which may be compared with corresponding values at SP of 0.001267(113) and 0.001329(118).

4 Conclusion

Two strategies for calibrating the BIPM susceptometer have been critically evaluated and compared with results obtained from calibrated standards. The susceptometer has good internal consistency, which may be exploited when checking the magnetic properties of weakly-magnetic objects, such as mass standards (as recommended internationally [3]). To take advantage of the lower uncertainties obtained with the susceptometer in its relative mode of operation compared with the ab initio mode, the availability of traceable permeability standards is crucial. Standards investigated here include those at low susceptibility values made of Alacrite (typically $\chi \approx 0.0013$), which would be particularly suitable if they had a calibration directly traceable to SI units. On the other hand, the novel NPL iron-acrylic blocks with $\chi \approx 0.005$ gave small systematic differences in the measured susceptibility across the block and as a function of the applied magnetic field. These effects were, however, negligible for the blocks of higher susceptibility and the NPL blocks can in general be recommended as standards for calibration of the Davis suceptometer. We have also presented some information showing to what extent demagnetisation effects are seen in calibrated reference samples where the susceptibility is $\chi \approx 0.12$.

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