Ernst Jan de Place Hansen
Einar Aassved Hansen
Manouchehr Hassanzadeh
Henrik Stang

Determination of the Fracture Energy of Concrete: A Comparison of the Three-Point Bend Test on Notched Beam and the Wedge-Splitting Test

NORDTEST Project No 1327-97

SP Swedish National Testing and Research Institute
Building Technology
SP REPORT 1998:09

Abstract

Determination of the fracture energy of concrete: A comparison of the three-point bend test on notched beam and the wedge-splitting test

The fracture energy of concrete, mortar and similar materials can be determined by means of various methods. The three-point bend test (TPBT) on notched beam and the wedge-splitting test (WST) are two methods for determination of the fracture energy.

This report, which is a compilation of the results of a Nordtest joint-project, compares the above methods with regard to the results of measurements, the reliability of measurements, and the simplicity of application. The project started in January 1997, with the Department of Structural Engineering and Materials at the Technical University of Denmark (DTU), the Department of Cement and Concrete at SINTEF Civil and Environmental Engineering - Norway, and SP Swedish National Testing and Research Institute, as participants.

The fracture energies obtained from the TPBT are, on average, somewhat higher than those obtained from the WST. The average difference is, however, insignificant, i.e. the methods yield similar results.

Since the true fracture energy is not known, the test methods cannot be compared on the basis of their validity. However, the methods show the same reliability.

It was found that the WST specimens are lighter and easier to handle than the TPBT specimens. Furthermore, the experimental set-up in the WST is faster than the TPBT. Sawing the groove, however, prolongs the preparation time for WST specimens.

The TPBT is recommended by RILEM, and is widely applied. Hence, the TPBT is recommended to be used as the primary choice.

Key words: Fracture mechanics, fracture energy, three-point bend test, wedge-splitting test, concrete, mortar.

SP Sveriges Provnings- och Forskningsinstitut
SP Rapport 1998:09
ISBN 91-7848-714-5
ISSN 0284-5172
Borås 1998

SP Swedish National Testing and Research Institute
SP Report 1998:09

Postal address:
Box 857, SE-501 15 BORÅS,
Sweden
Telephone + 46 33 16 50 00
Telefax + 46 33 13 55 02
E-mail: info@sp.se
Internet: www.sp.se

Contents

Preface

This NORDTEST project was started in January 1997. The objective of the project was to compare two test methods which are used to determine the fracture energy of concrete, mortar and similar materials. The methods were:

- three-point bend test on notched beam, and
- wedge-splitting test.

The objectives of the project were to compare the test methods with regard to the results of measurements, the reliability of measurements, and the simplicity of application.

The project was carried out by the following persons:

Ernst Jan de Place Hansen, DTU
Einar Aassved Hansen, SINTEF
Manouchehr Hassanzadeh, SP, project co-ordinator
Henrik Stang, DTU

This report was compiled by Manouchehr Hassanzadeh, and has been approved by the participants of the project.

Summary

The fracture energy of concrete, mortar and similar materials can be determined by means of various methods. The three-point bend test (TPBT) on notched beam and the wedge-splitting test (WST) are two of these.

A NORDTEST project was started in January 1997. The objectives of the project were to compare the test methods with regard to the results of measurements, the reliability of measurements, and the simplicity of application.

The TPBT method is recommended by the RILEM Technical Committee 50-FMC. The WST method, previously proposed by Linsbauer and Tschegg, has been used by Brühwiler et al. as an alternative method for determination of fracture energy.

Three research laboratories participated in the project: the laboratories at the Department of Building Technology at SP Swedish National Testing and Research Institute, the Department of Cement and Concrete at the Norwegian Research Institute SINTEF Civil and Environmental Engineering, and the Department of Structural Engineering and Materials at the Technical University of Denmark (DTU).

The comparative tests were performed with three different concrete qualities. Each laboratory performed the comparative tests with one concrete quality.

The fracture energies of three different concrete mixes with different water-cement ratios were determined by means of both methods. Each laboratory tested one concrete mix. Three water-cement ratios were planned to be used, namely 0,40 (SP), 0,45 (DTU) and 0,50 (SINTEF).

Each laboratory manufactured its own specimens by means of locally available materials in accordance with following requirements:

- the cement shall be ordinary Portland cement
- the aggregate shall be composed of 50% particles < 8 mm and 50% particles > 8 mm
- the maximum aggregate size shall be 16 mm, and
- the water content should be such that the slump of the mixes falls between 50 and 100 mm.

It was planned that the fracture energy of each concrete mix should be determined by means of four bending tests and four wedge-splitting tests at the age of 28 days.

The required beam size, i.e. length, depth and width were 0,84 *m*, 0,1 *m* and 0,1 *m*. The notch dept was equal to half the beam depth.

The shape of the WST specimen was cubic with 0,1 *m* edge size. The ligament depth and width were 0,05 *m* and 0,1 *m* respectively, i.e. the same as the beam.

The loading rate was chosen so that the maximum load was reached within about 30 - 60 seconds.

The fracture energies obtained from the TPBT are, on average, somewhat higher than those obtained from the WST. The average difference is, however, insignificant, i.e. the methods yield similar results.

Since the true fracture energy is not known, the test methods can not be compared on the basis of their validity. However, the methods show the same reliability.

It was found that the WST specimens are lighter and easier to handle than the TPBT specimens. Furthermore, the experimental set-up in the WST is faster than the TPBT. Sawing the groove, however, prolongs the preparation time for WST specimens.

The TPBT is recommended by RILEM, and is widely applied. Hence, the TPBT is recommended to be used as the primary choice.

1 Introduction

Applications of fracture mechanical models to both plain and reinforced concrete structures are increasing. Several successful applications of the fracture mechanical models have been reported in recent years. The areas of the applications have been concrete pipes, tension failure of reinforced concrete members, shear failure of reinforced beams, pullout failure of reinforcing bars and anchor bolts, and concrete dams, Shah et. al. (1995).

Some of the fracture mechanical models require material properties which can be obtained from the three-point bend tests performed with notched beams. Hillerborg's "Fictitious Crack Model", Bazant's "Crack Band Model" and "Size Effect Law", and Shah's "Two Parameter Model" are among the models which utilise three-point bend tests for determining the model parameters. Although the models may not extract the same material properties from the three-point bend tests, the tests are essential for the applications of the models.

The essential issues in fracture mechanical tests are to get a crack to initiate at a predetermined location, and to propagate in a controllable and stable manner. These conditions can normally be achieved only in a notched specimen. In the case of fracture mechanics applied to concrete the specimen can be a notched beam, a notched bar, a notched prism or a notched body with an arbitrary shape.

As far as the measurement of the real material properties is concerned, the geometry and the size of the specimen are not decisive factors. The most decisive factor in this context is the stability of the tests, which is determined by the geometry and the size of the specimen. Since the majority of the fracture mechanical properties are not pure material properties, some influences from the geometry and the size of the specimen can be expected. This is the reason why this project was initiated to compare the outcome of the two different test methods. The methods differ not only in specimen geometry, but also in the internal state of stress.

The objectives of the project were to compare the test methods with regard to the results of measurements, the reliability of measurements and the simplicity of application.

2 Test methods

2.1 Three-point bend test

The RILEM Technical Committee 50-FMC proposed a draft recommendation to measure the fracture energy G_F (*N/m*) of the concrete, mortar and similar materials, Appendix A. In this method the fracture energy is measured by means of a displacement controlled stable three-point bend test on a centrally notched beam (TPBT), Figure 1. The fracture energy is calculated by means of equation 1 using the load-displacement curve obtained from the test, Figure 2.

$$G_F = (W_0 + (m_1 + 2m_2) g \delta_0) / (b(d - a)) \qquad N/m \qquad (1)$$

W_0 is the area according to Figure 2 (*N/m*). δ_0 is displacement at the final failure of the beam (*m*). m_1 is weight of the beam between the supports (*kg*), calculated as the beam weight multiplied by l/L. l is the distance between the supports (span), and L is the beam length. m_2 is weight of the part of the loading arrangement which is not attached to the testing machine, but follows the beam until failure (*kg*). g is acceleration due to gravity, 9,81 m/s^2. b, d and a are defined in Figure 1.

The notch depth (a) is equal to half the beam depth. The size of the beam - i.e. l, d, and b - depends on the maximum size of aggregate (D_{max}). l, d and b are 0,8, 0,1 and 0,1 *m* when D_{max} = 16 *mm*; and 1,13, 0,2 and 0,1 *m* when 16 *mm* < D_{max} = 32 *mm*, see also Table 1 in appendix A.

The weight of the beam increases with increased D_{max}. In the above mentioned cases, the weights of the beams are approximately 20 and 54 *kg*, i.e. the difficulty to handle the beams increases with increased D_{max}.

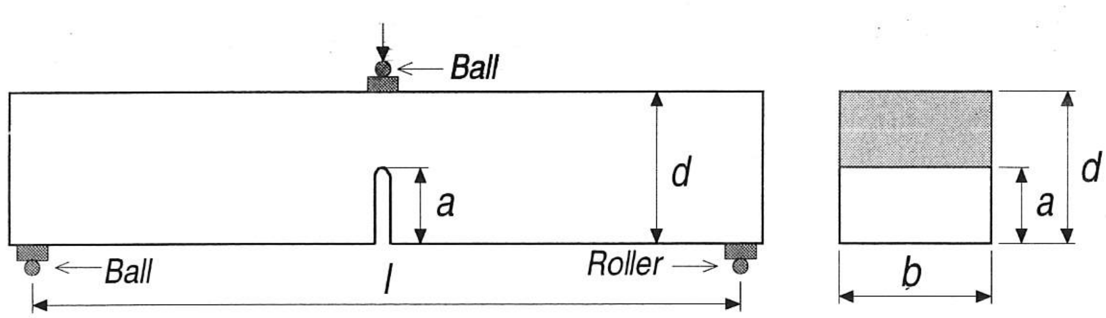


Figure 1 Determination of fracture energy by means of three-point bend test.

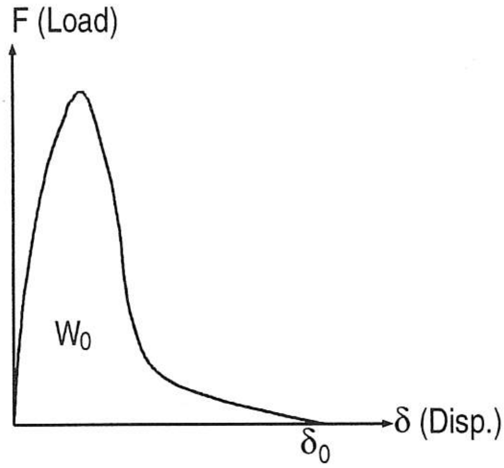


Figure 2 *Load-displacement curve.*

2.2 Wedge-splitting test

The wedge-splitting test is an alternative method for determination of the fracture energy. The principle of the wedge-splitting test is schematically presented in Figure 3. A groove and a notch are introduced in to the specimen. The specimen is placed on a line support fixed to the lower plate of the testing machine, Figure 3a. Two massive steel loading devices equipped with roller or needle bearings on each side are placed on the top of the specimen, Figure 3b. A steel profile with two identical wedges is fixed at the upper plate of the testing machine. The actuator of the testing machine is moved so that the wedge enters between the bearings, which results in a horizontal splitting force component, Figure 3c.

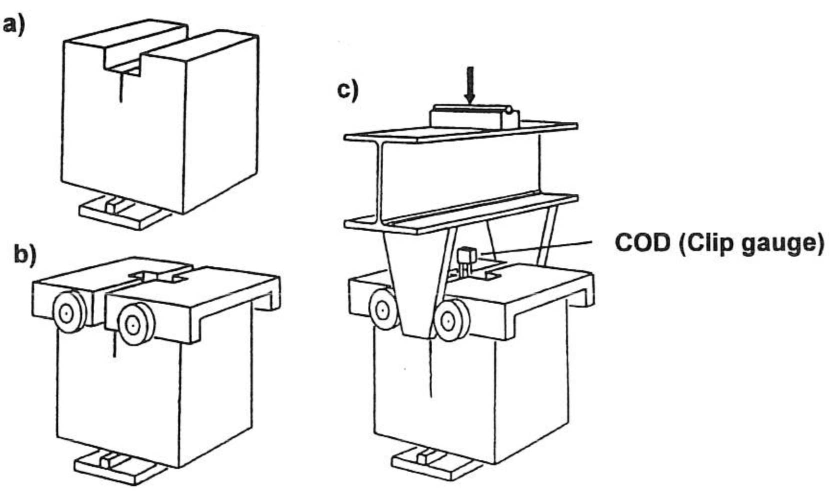


Figure 3 *Principle of the wedge-splitting test: (a) test specimen on a linear support; (b) placing of two loading devices with rollers; (c) the wedges are pressed between rollers in order to split the specimen into two halves. Brühwiler and Wittmann (1990).*

The test is performed in displacement-controlled mode. The load in the vertical direction, F_v, and the crack opening displacement, COD, are monitored during testing. The COD is measured by means of a transducer or a clip gauge at the level where the splitting force acts on the specimen, i.e. at the axles of the rollers. The horizontal component of the force acting on the bearings, F_s, can be calculated by means of formula 2.

$$F_s = \frac{F_v}{2\tan\alpha} \cdot \frac{1 - f\tan\alpha}{1 + f\cot\alpha} \approx \frac{F_v}{2(1 + f\cot\alpha)\tan\alpha} \tag{2a}$$

$$F_s \approx \frac{F_v}{2\tan\alpha} \qquad f\tan\alpha << 1 \tag{2b}$$

The area under the F_s - COD curve is the fracture energy of the specimen (W_0, Nm), Figure 4. The fracture energy of the material (G_F) is obtained by dividing the W_0 by the area of the ligament.

It should be noted that the weight of the test devices which are not attached to the testing machine, but follow the specimen during the fracture process, is not included in formula 2. However it can be accounted for by replacing F_v with F_{vt}.

$$F_{vt} = F_v + 2m_2\, g\, \delta_0 \tag{2c}$$

Furthermore, it should be noted that the results of fracture energy measurements by means of WST will not be consistent with the results of tests based on the RILEM method if the additional weight of the devices is not accounted for in the same way as above. Equation 2c can be derived with the same assumptions as those used for the RILEM method. The assumptions can be found in Petersson (1981).

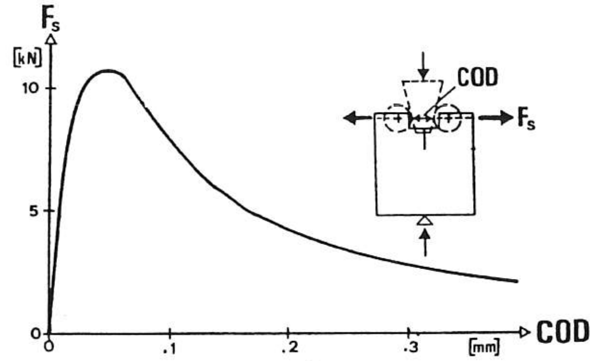


Figure 4 The F_s - COD curve. Brühwiler and Wittmann (1990).

3 Materials and specimens

3.1 Concrete mix

Fracture energies of three different concrete mixes with different water-cement ratios have been determined by means of both the TPBT and WST methods. Each laboratory tested one concrete mix. Three water-cement ratios were planned to be used: 0,40 (SP), 0,45 (DTU) and 0,50 (SINTEF).

Each laboratory manufactured its own specimens by means of locally available materials, and in accordance with the following requirements:

- the cement shall be ordinary Portland cement
- the aggregate shall be composed of 50% particles < 8 mm and 50% particles > 8 mm
- the maximum aggregate size shall be 16 mm, and
- the water content should be such that the slump of the mixes falls between 50 and 100 mm.

3.2 Specimens

It was planned that the fracture energy of each concrete mix should be determined by means of four bending tests and four wedge-splitting tests at the age of 28 days.

The required beam size, i.e. length, depth and width were 0,84 *m*, 0,1 *m* and 0,1 *m*. The required notch depth was equal to half the beam depth.

The required shape of the WST specimen was cubic with 0,1 *m* edge size. The required ligament depth and width were 0,05 *m* and 0,1 *m*, i.e. the same as the beam.

The loading rate was to be chosen so that the maximum load was reached within about 30 - 60 seconds.

4 Tests performed at the laboratories

4.1 Manufacturing and preparation of specimens

This section provides a brief description of the manufacturing and preparation of the specimens. For a detailed presentation, see appendices B, C and D.

Table 1 describes the compositions and the properties of the fresh concrete.

Table 1 The compositions and the properties of the fresh concrete.

	DTU		SINTEF	SP
Cement, kg/m^3	298[1)]		367[2)]	401[3)]
Silica fume, kg/m^3	15,5			
Fly ash, kg/m^3	65			
Water, kg/m^3	141		183	161,0
Water-cement ratio	0,47		0,50	0,40
Effective W-C ratio	0,45[4)]			
Sand, mostly < 8 *mm*	694		901	797
Crushed aggregates 4-8 *mm*	350			
Crushed aggregates 8-11 *mm*			450	
Crushed aggregates 8-16 *mm*	700		450	1031
Air entraining agent, kg/m^3	0,48			
Superplasticizer, kg/m^3	0,824			10,0[5)]
Plasticizer, kg/m^3	0,412			
Batch size, m^3	?[6)]	?[7)]	0,05	0,1
Slump, *mm*	100	100	145	85
Air content, %	6,0		1,7	1,4
Density, kg/m^3	2310	2300	2377	2423

1) Aalborg Portland Lavalkalicement. 2) Norcem Anlegg. 3) Slite Standard. 4) Calculated with the efficiency factors 1and 0 for silica fume and fly ash respectively. 5) Melcrete 35% dry substance. 6) Batch 1. 7) Batch 2.

Table 1 shows that the composition and the properties of the fresh concrete deviate from the required values in some cases. However, the deviations are not considered to be crucial for the objectives of the project. Furthermore, as shown in the table, DTU has used two batches of concrete for manufacturing the specimens. This procedure may have some influence on the test results, which is discussed later.

The details regarding the manufacturing and the preparation of specimens are presented in Table 2.

Table 2 The details regarding the manufacturing and the preparation of specimens.

Specimen		DTU	SINTEF	SP
Beams for TPBT	Number of specimens	7	4	4
	Number of tests	5	4	4
	Number of successful tests	5	3	4
	Specimen size, *mm*	840 x 100 x 100	840 x 100 x 100	1000 x 100 x 100
	Age at demoulding, *days*	1	1	1
	Conditioning after demoulding	in water	in water	in water
	Age at testing, *days*	45	34	29
	Way of manufacturing the notch	sawing	sawing	sawing
	Direction of the notch	D[1)]	C[1)]	C[1)]
Cubes for WST	Number of specimens	8	4	4
	Number of tests	5	4	4
	Number of successful tests	5	4	4
	Specimen size, *mm*	100 x 100 x 100	100 x 100 x 100	100 x 100 x 100
	Way of manufacturing	cut from a beam	casting	casting
	Age at demoulding, *days*	1	1	1
	Conditioning after demoulding	in water	in water	in water
	Age at testing, *days*	45	34	28
	Way of manufacturing the notch	sawing	sawing	sawing
	Direction of the notch	A[1)]	A[1)]	B[1*)]
	Way of manufacturing the groove	sawing	sawing	sawing
Cubes for compressive tests	Number of specimens		2	3
	Number of tests		2	3
	Specimen size, *mm*		100 x 100 x 100	100 x 100 x 100
	Age at demoulding, *days*		1	1
	Conditioning after demoulding		in water	in water
	Age at testing, *days*		34	28
Cylinders for compressive tests	Number of specimens	5		
	Number of tests	5		
	Specimen size, *mm*	ϕ100 x 200		
	Age at demoulding, *days*	1		
	Conditioning after demoulding	in water		
	Age at testing, *days*	45		

1) Refers to the Figure 5. 1) Also refers to the Figure 5, but a cube is used instead of a beam.*

The laboratories differ somewhat in procedures regarding the manufacturing and preparation of specimens, see Table 2. The major differences are as follows:

1. the length of the beam (mould size),
2. the age at testing,
3. the manufacturing of WST specimen, and
4. the direction of the notches and the grooves.

The first three factors are not considered to be crucial to the objectives of the project. However, the fourth factor may influence the test results, specially when the directions of the notches in WST deviate from those in TPBT.

Figure 5 shows the various directions of the notches and grooves with respect to the horizontal surface at casting. The direction which is shown in part *C* is the direction recommended by RILEM for TPBT. Hence, in order to make WST consistent with the TPBT according to the RILEM recommendation, the groove and the notch must be made as shown in part B. The notches in DTU's and SINTEF's WST specimens have the same direction as shown in part *A*. Furthermore, the notches in DTU's TPBT specimens have the same direction as shown in part *D*.

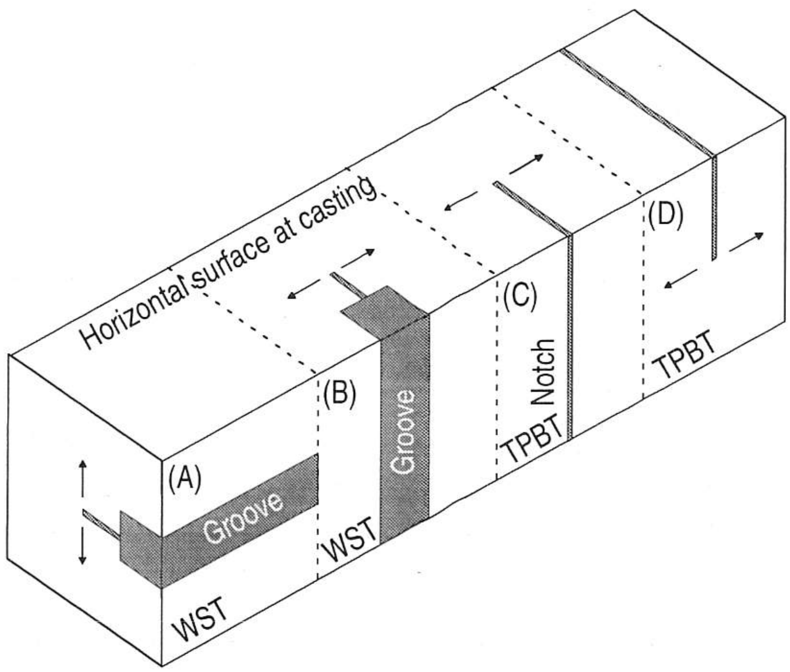


Figure 5 The direction of the notches and the grooves.

4.2 Measurements

All tests have been performed by means of servo-controlled closed-loop testing machines. The stability of the tests was satisfactory in all tests.

All tests were performed in displacement-controlled mode. The displacement rates were such that the maximum loads were reached within 30 - 75 seconds. The displacements were measured by means of various types of transducers, which are summarised in Table 3.

Table 3 *Measurements of displacements.*

		DTU	SINTEF	SP
TPBT	Disp. for control of the tests	COD[1)]	deflection	deflection
	Trans. for control of the tests	clip gauge	LVDT[2)]	LVDT
	Disp. for load - disp. curve	type *A*[3)]	type *B*[3)]	type *A*
	Trans. for displacement measurements	LVDT	LVDT	LVDT
WST	Disp. for control of the tests	COD	COD	COD
	Trans. for control of the tests	clip gauge	clip gauge	LVDT
	Disp. for load - disp. curve	type *C*[4)]	type *C*	type *D*[4)]
	Trans. for displacement measurements	clip gauge	clip gauge	LVDT

1) Crack Opening Displacement. 2) Linear Variable Differential Transducer. 3) See Figure 6. 4) See Figure 7.

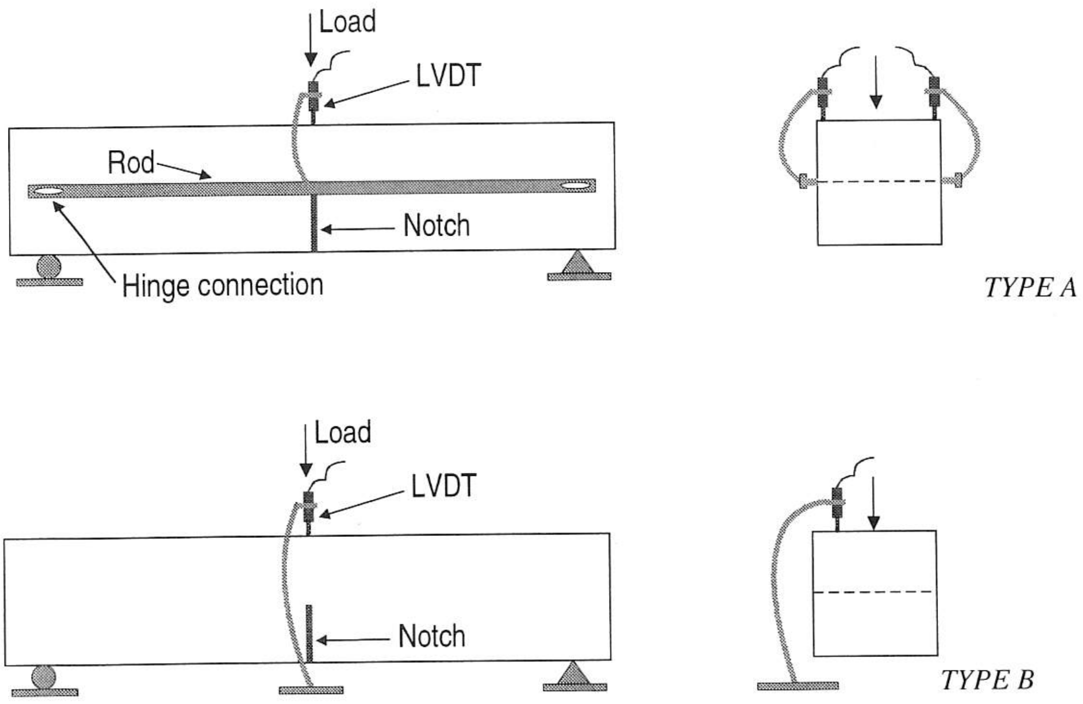


Figure 6 *Principle of type A and B displacement measurements. In type A the test is controlled by means of either a LVDT (SP) or a clip gauge mounted in the notch (COD according to DTU). In type B the test is controlled by means of LVDT.*

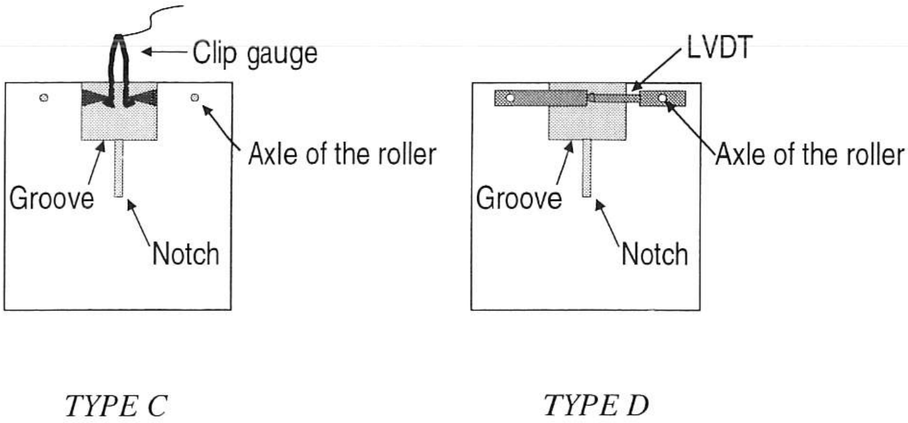


Figure 7 *Principles of type C and D displacement measurements.*

4.3 Quality control, compressive strength of concrete

The results of compressive tests are presented in Table 4. It should be noted that the results of DTU refer to the cylinder strength. The cylinder strength may be converted to the cube strength through multiplication by factor 1,35, Ewertson and Pettersson (1990).

Table 4 *Compressive strength.*

	DTU		SINTEF	SP
	Batch 1	Batch 2		
Compressive strength (cube), MPa	55,2	58,9	57,6	71,6
Compressive strength (cylinder), MPa	40,9	43,6		

4.4 Results of fracture energy determined by means of TPBT

Results of the fracture energy measurements by means of the TPBT are shown in Table 5. The table separately presents the work of fracture per unit surface of ligament conducted by the external load (G_{FF}), the weight of the beam (G_{Fm1}) and the weight of the devices which are not attached to the testing machine (G_{Fm2}). The other test data can be found in the appendices B, C and D.

Table 5 Results of fracture energy determined by means of TPBT.

Specimen no. as reported		DTU		SINTEF	SP
		Batch 1	Batch 2		
1	G_{FF}, N/m		67,9	70,1	85,5
	G_{Fm1}, N/m		40,8	57,0	47,3
	G_{Fm2}, N/m		≈ 0	≈ 0	1,2
	G_F, N/m		108,7	127,1	134,0
2	G_{FF}, N/m	66,9		68.4	101,6
	G_{Fm1}, N/m	43,1		36,2	56,6
	G_{Fm2}, N/m	≈ 0		≈ 0	1,6
	G_F, N/m	110,0		104,6	159,8
3	G_{FF}, N/m		81,4	44,8	102,0
	G_{Fm1}, N/m		35,3	26,9	58,2
	G_{Fm2}, N/m		≈ 0	≈ 0	1,6
	G_F, N/m		116,7	71,7	161,8
4	G_{FF}, N/m	87,3			109,7
	G_{Fm1}, N/m	50,5			48,9
	G_{Fm2}, N/m	≈ 0			1,4
	G_F, N/m	137,8			160,0
5	G_{FF}, N/m	89,8			
	G_{Fm1}, N/m	41,7			
	G_{Fm2}, N/m	≈ 0			
	G_F, N/m	131,5			
Mean G_F, N/m		126,4	112,7	101,1	153,9
STD G_F, N/m		14,6	5,7	27,9	13,3
Mean G_F, N/m		120,9			
STD G_F, N/m		13,1			

4.5 Results of fracture energy determined by means of WST

Results of fracture energy measurements by means of WST are shown in Table 6. The table presents separately the work of fracture per unit surface of ligament conducted by the external load (G_{FF}), and the weight of the devices which are not attached to the testing machine (G_{Fm2}). The other test data can be found in the appendices B, C and D.

Table 6 Results of fracture energy determined by means of WST.

Specimen no. as reported		DTU		SINTEF	SP
		Batch 1	Batch 2		
1	G_{FF}, *N/m*		49,0	80,4	114,8
	G_{Fm2}, *N/m*		9,7	21,0	28,1
	G_F, *N/m*		58,7	101,4	142,9
2	G_{FF}, *N/m*		60,0	63,5	132,8
	G_{Fm2}, *N/m*		12,8	17,7	28,7
	G_F, *N/m*		72,8	81,2	161,5
3	G_{FF}, *N/m*		65,8	72,4	113,4
	G_{Fm2}, *N/m*		12,6	20,2	31,1
	G_F, *N/m*		78,4	92,6	144,5
4	G_{FF}, *N/m*		65,5	69,1	125,3
	G_{Fm2}, *N/m*		17,5	22,4	26,0
	G_F, *N/m*		83,0	91,5	151,3
5	G_{FF}, *N/m*		67,3		
	G_{Fm2}, *N/m*		12,8		
	G_F, *N/m*		80,1		
Mean G_F, *N/m*			74,6	91,7	150,1
STD G_F, *N/m*			9,6	8,3	8,5

The results of the wedge-splitting tests of both DTU and SINTEF are not consistent with the TPBT recommended by RILEM. The reason for this is that DTU and SINTEF used the following formula instead of 2c:

$$F_{vt} = F_v + m_2 g \delta_0 \qquad (3)$$

4.6 Concluding remarks of the laboratories

This section presents the concluding remarks of each laboratory.

4.6.1 DTU

The experimental setup in the WST method is significantly faster to work with although the sawing procedure is more time consuming. Only when using laboratory prepared specimens can time be saved by designing the formwork in such a way that no groove has to be sawn.

The standard deviation of the results in the two methods is about the same.

Significantly larger fracture energy is measured using the TPBT method.

It is expected that the TPBT method gives larger fracture energy. More energy can be dissipated in the beam because of the size (compared with the WST specimen).

We question equation (1). It seems more correct to only use half of the weight of the beam including the loading arrangement represented by m_2. At failure, the displacement (in the vertical direction) of the centre of gravity of the two halves of the beam is half the displacement of the centre of the beam. By using

$$G_f = (W_0 + 0.5mg\,\delta_0) / A_{lig} \qquad (4)$$

the following values for G_F are given:
88.3 N/m, 88.4 N/m, 99.0 N/m, 113 N/m and 111 N/m
(mean value 100 N/m, standard deviation 11.7 N/m ≈ 12 %)

4.6.2 SINTEF

No concluding remarks.

4.6.3 SP

The G_F of the present concrete is 153,9 N/m (std = 13,3 N/m) and 150,1 N/m (std = 8,5 N/m) according to the TPBT and WST respectively. According to these tests, both methods show similar results.

Our experiences during this project are as follows:

- The requirements regarding the stiffness and performance of the testing machine are more severe in the WST than the TPBT.
- WST specimens are smaller and lighter, which is advantageous with regard to storage and handling.
- The grooves in the WST specimens were, in these tests, saw cut which required approximately 15 minutes additional preparing time for each specimen.

5 Comparison of the test methods

The test results obtained by the three laboratories are summarised in the previous section. This section presents the compilation of the results and the comparison of the test methods.

As mentioned previously, the results of the wedge-splitting tests are, in some cases, not consistent with the RILEM method. Hence, in the following sections the test methods are compared on the basis of two different sets of WST values. The first set is based on the results reported by the laboratories and the second set is based on the WST values adjusted with regard to the RILEM method.

5.1 The results reported by the laboratories

Figure 8 shows the results of the fracture energy measurements and the corresponding standard deviations as reported by the laboratories.

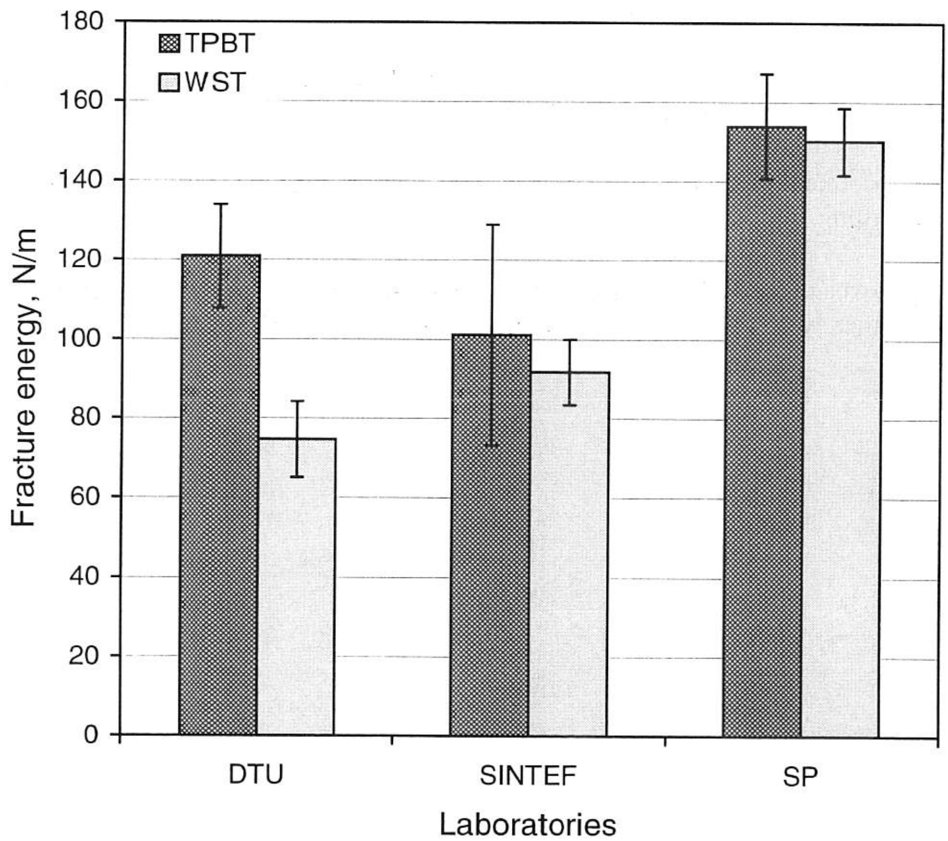


Figure 8 Results of the fracture energy tests.

If the test methods are not different, then the following condition must be satisfied:

$$\Delta G_F = G_{F,TPBT} - G_{F,WST} = 0 \tag{5}$$

It is obvious from the figure that condition 5 will not be satisfied due to the scatter of the test results. Therefore, some statistical tools must be used in order to verify whether or not the test methods measure the fracture energy similarly. With the assumptions that the fracture energy is a random variable and that the test results are not correlated the mean value, the 95% confidence interval, the variance, and the standard error of the differences in fracture energy measurements (ΔG_{F},) can be estimated. The analysis is described in appendix E. The estimated values are shown in Table 7.

Table 7 *The mean value, the 95% confidence interval, the variance, and the standard errors of the difference in fracture energy measurements.*

Mean difference, ΔG_F *N/m*	Variance $(N/m)^2$	Standard error *N/m*
21,9 ± 25	145,3	12,1

The confidence interval has been calculated using the t-distribution. The probability of ΔG_F to be lower than a certain value can also be calculated by means of the t-distribution. The results are shown in Table 8.

Table 8 *The probability of ΔG_F being lower than a certain value.*

$\Delta G_F < 0$ *N/m*	$\Delta G_F < 5$ *N/m*	$\Delta G_F < 10$ *N/m*	$\Delta G_F < 15$ *N/m*
4%	9%	17%	28%

The results of the analysis show that the results of the test methods are not equal. Furthermore, the results show that the fracture energy determined by means of the TPBT is greater than that determined by means of the WST.

5.2 The adjusted results

The WST results of the DTU and SINTEF have been recalculated in order to be consistent with the RILEM TPBT. An additional measure has also been taken, which leaves the results of DTU's batch 1 out of the consideration. Although this measure decreases the number of TPBT and the accuracy of the analysis, it was necessary in order to make the results of DTU's WST and TPBT comparable with each other. The results of the analysis are shown in Figure 9.

Regarding DTU's specimens, the direction of the notches of the WST and the TPBT specimens are not the same. What influence this may have on the results is not known.

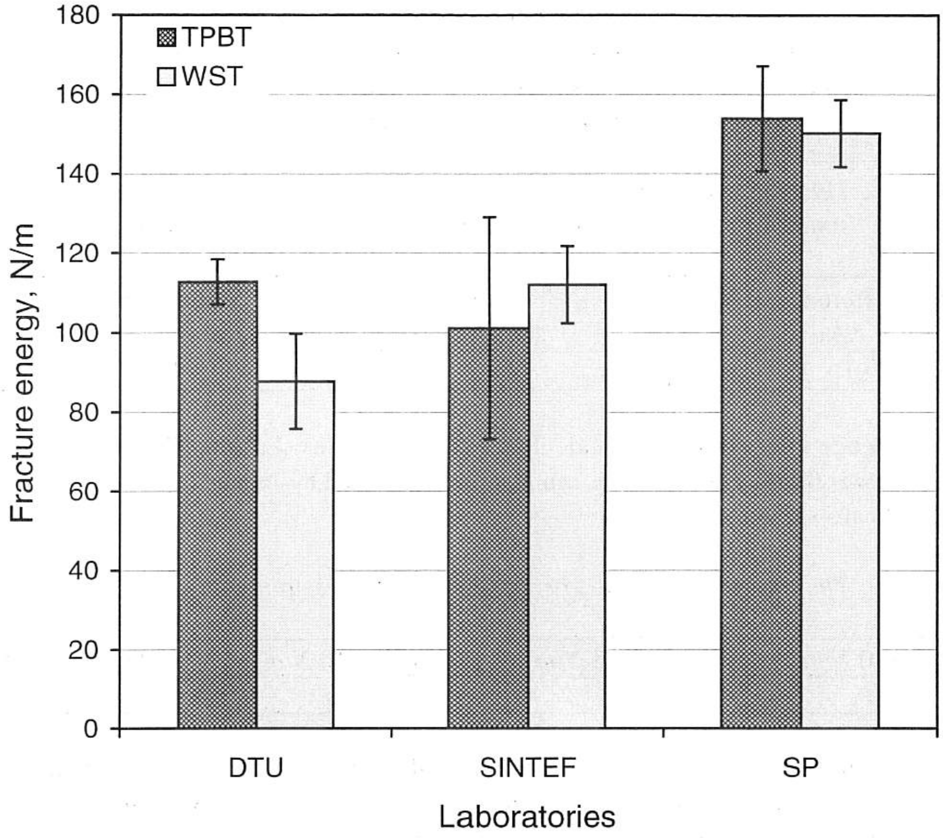


Figure 9 *Results of the fracture energy tests.*

The estimated ΔG_F and the corresponding variance and standard error are shown in Table 9.

Table 9 *The mean value, the 95% confidence interval, the variance, and the standard errors of the difference in fracture energy measurements.*

Mean difference, ΔG_F *N/m*	Variance $(N/m)^2$	Standard error *N/m*
7,5 ± 22	116,1	10,8

The probability of ΔG_F being lower than certain values is shown in Table 10.

Table 10 *The probability of $\Delta G_{F,TPBT}$ being lower than a certain value.*

$\Delta G_F < 0$ *N/m*	$\Delta G_F < 2$ *N/m*	$\Delta G_F < 3$ *N/m*	$\Delta G_F < 4$ *N/m*
25%	31%	34%	37%

Table 11 shows that the adjustment of the test results has decreased the difference between the results of the TPBT and the WST. The results show that when the results from the three laboratories are analysed together the average difference is 7,5 *N/m*, with ±22 *N/m* confidence intervals. This difference is insignificant.

Since the true fracture energy is not known, the test methods can not be compared on the basis of their validity.

The reliability of the test methods can be compared on the basis of their standard error and the coefficient of variation. These values are shown in Table 11. The results show that the methods have the same reliability.

Table 11 *Comparison of the methods on the basis of the standard error and the coefficient of variation.*

	TPBT	WST
Standard error, *N/m*	26,1	25,0
Coefficient of variation, *%*	21	22

6 Concluding remarks

The fracture energies obtained from the TPBT are, on average, somewhat higher than those obtained from the WST. The average difference is, however, insignificant, i.e. the methods yield similar results.

Since the true fracture energy is not known, the test methods can not be compared on the basis of their validity. However, the methods show the same reliability.

It was found that the WST specimens are lighter and easier to handle than the TPBT specimens. Furthermore, the experimental set-up in the WST is faster than the TPBT. Sawing the groove, however, prolongs the preparation time for WST specimens.

The TPBT is recommended by RILEM, and is widely applied. Hence, the TPBT is recommended to be used as the primary choice.

References

Box G. E. P., Hunter, W. G. and Hunter, J. S. (1978) *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, John Wiley & Sons, inc.

Brühwiler, E. and Whittmann, F. H. (1990) "The Wedge-Splitting Test, a New Method of Performing Stable Fracture Mechanics Tests", in *Engineering Fracture Mechanics* 35, 117-125.

Ewertson, C. and Pettersson, R (1990) "Comparison of Nordic Methods for Determination of the Compressive Strength of Concrete", SP REPORT 1990:28E, Swedish National Testing and Research Institute, Building Technology, Borås, Sweden.

Petersson, P. E. (1981) "Crack Growth and Development of Fracture Zones in plain Concrete and Similar Materials". Report TVBM-1006, Division of Building Materials, Lund Institute of Technology, Lund, Sweden.

RILEM Draft recommendation, 50-FMC, (1985) "Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams", in *Materials and Structures*. 18, 287-290.

Shah, S. P., Swartz, S. E. and Ouyang, C. (1995) *Fracture Mechanics of Concrete: Applications of Fracture Mechanics to Concrete, Rock, and Other Quasi-Brittle Materials*, John Wiley & Sons, inc.

Appendix A, RILEM draft recommendation

FMC1 Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams

1985 TC50-FMC

Draft: *Materials and Structures*, Vol. 18, No. 106, July–August 1985. **Final:** Endorsed May 1993, no modification. **French version:** *Materials and Structures*, Vol. 18, No. 106, July–August 1985.

CONTENTS

1. SCOPE

This Recommendation specifies a method for the determination of the fracture energy (G_F) of mortar and concrete by means of stable three-point bend tests on notched beams. The fracture energy is defined as the amount of energy necessary to create one unit area of a crack. The area of a crack is defined as the projected area on a plane parallel to the main crack direction.

This test method is not recommended for fibre-reinforced concrete.

2. SPECIMENS

The specimens shall be beams with a central notch according to figure 1. The depth of the beam as tested shall be horizontal during casting. The size of the beam shall depend on the maximum size of the aggregate, D_{max}, according to table I.

If the maximum aggregate size exceeds 64 mm the depth and width shall be increased proportional to this size and the length and span increased proportional to the square root of this size.

The notch shall always have a depth which is equal to half the beam depth ± 5 mm. It is recommended that it is sawn under wet conditions at least one day before the test. It may also be cast but this is only recommended if a suitable saw is not available. If the notch is cast it is recommended to make the insert in the mould wedge-shaped with an angle of 10-15° in order to make it possible to loosen the mould without damaging the beam. If the notch is cast the beam must be handled very carefully during demoulding.

TABLE I

SIZES OF SPECIMENS.

D_{max} (mm)	Depth d (mm)	Width b (mm)	Length L (mm)	Span l (mm)
1-16	100 ± 5	100 ± 5	840 ± 10	800 ± 5
16.1-32	200 ± 5	100 ± 5	1,190 ± 10	1,130 ± 5
32.1-48	300 ± 5	150 ± 5	1,450 ± 10	1,385 ± 5
48.1-64	400 ± 5	200 ± 5	1,640 ± 10	1,600 ± 5

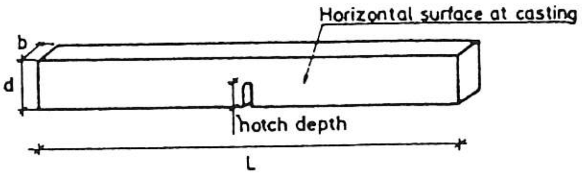


Fig. 1

The notch width at the tip should be less than 10 mm.

If the fracture energy of concrete of an existing structure has to be determined, it is suggested that suitable specimens are cut from drilled cores. A notch has to be sawn and metal or concrete arms have to be glued to the specimen, so that the total specimen fulfils the geometrical conditions given in table I. The diameter of the core has to be adjusted to the maximum aggregate size.

3. CONDITIONS OF STORAGE

During the period of curing the ambient temperature shall be 20 ± 2°C.

The specimen shall remain in the mould, protected from shock, vibrations and drying, for at least 16 hours.

After removal from the mould, the specimen shall be stored in lime-saturated water until less than 30 minutes before testing.

4. APPARATUS

4.1. Testing machine

The testing machine has to be stiff enough or furnished with a closed-loop servo control in order to make it possible to perform stable tests. A test can be regarded as stable in the load and the deformation change slowly during the whole test, i. e. without any sudden jump. Unless the complete load-time curve is recorded the check of stability must be made visually during the test, as the load-deformation curve does not always reveal an instability.

If the testing machine has no closed-loop servo control the required stiffness is about 10 kN/mm for the smallest standard beam. The corresponding stiffnesses of the larger standard beams are about 25, 70, and 150 kN/mm respectively. These figures only indicate the order of magnitude of the required stiffnesses, and large variations may occur depending on the properties of the tested concrete. Whenever there are problems in running the test in a stable way, crack mouth opening displacement shall be used as the control value for deformation rate.

4.2. Supports and loading arrangement

The supports and loading arrangements shall be such that the forces acting on the beam are statically determinate, e. g. according to figure 2.

4.3. Deformation measurement

The deformation of the center of the beam shall be determined with regard to a line between two points on the beam above the supports.

Alternatively the deformation of the load-point with respect to the support may be measured if the load-application and supports are arranged in such a way that non-elastic deformations at these points are less than 0.01 mm.

The deformation shall be measured with an accuracy of at least 0.01 mm.

5. PROCEDURE

The test is performed with an approximately constant rate of deformation, which is chosen so that the maximum load is reached within about 30-60 seconds after the start of the test. The deformation of the center of the beam and the corresponding load are registered until the beam is completely separated into two halves.

In case the deformation is not measured directly on the specimen it is recommended that before measuring the load-deformation curve the load is cycled 3 times between 5% and 25% of the expected maximum load.

The load shall be measured with an accuracy of at least 2% of the maximum value in the test.

In connection with the test, the weight of the beam and the weight of the part of the loading arrangement which is not attached to the testing machine shall be determined.

The area of the ligament, A_{lig}, shall be measured. It is defined as the projection of the fracture zone on a plane perpendicular to the beam axis.

The length L of the beam as well as the span l during the test shall be measured with an accuracy of at least 1 mm.

6. TEST RESULTS

The load-deformation curve is corrected for eventual non-linearities at low loads, see dashed line in figure 3. The energy W_o, represented by the area under the curve, is measured as well as the deformation δ_o at final fracture.

The fracture energy is calculated from the equation:

$$G_F = (W_o + mg\delta_o)/A_{lig} \quad [\text{N/m (J/m}^2)],$$

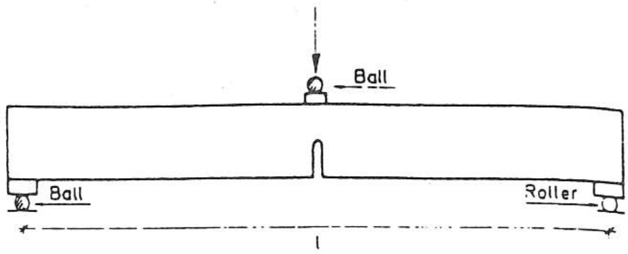


Fig. 2

where:

W_o = area according to figure 3 (N/m);

$m = m_1 + 2m_2$ (kg);

m_1 = weight of the beam between the supports, calculated as the beam weight multiplied by l/L;

m_2 = weight of the part of the loading arrangement which is not attached to the machine, but follows the beam until failure;

g = acceleration due to gravity, 9.81 m/s^2;

δ = deformation at the final failure of the beam (m);

A_{lig} = area of the ligament as defined above (m^2).

7. TEST REPORT

7.1. Basic information, always to be mentioned in the report

- identification number of specimen;
- dimensions of the specimen: b, d, L, l, A_{lig};
- method of making the notch;
- weight of the specimen;
- weight m_1;
- weight m_2;
- W_o;
- δ_o;
- F_{max} = maximum applied load, excluding m_2;
- accuracy of deformation measurements;
- how the stability has been checked;

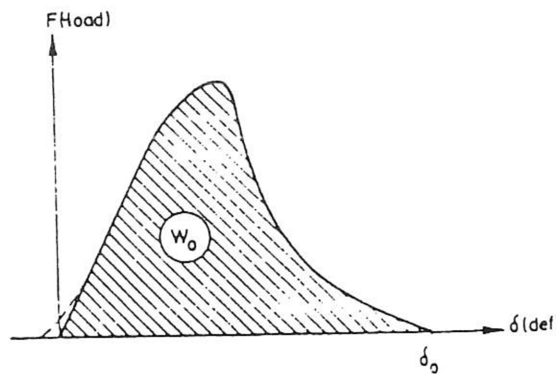


Fig. 3

- observed indications of instability;
- G_F.

7.2. Additional information

- type of concrete;
- type of cement;
- type of aggregate;
- cement content and mix proportions of concrete;
- age at testing;
- compressive strength, including method of determination;
- tensile strength, including method of determination;
- elastic modulus, including method of determination;
- density;
- any unusual features or interesting observations.

Appendix B, results of DTU

DETERMINATION OF THE FRACTURE ENERGY OF CONCRETE -

COMPARISON OF THE TPBT AND THE WST METHOD

Ernst Jan de Place Hansen
Henrik Stang

BKM, DTU, September 1997

1. Introduction

The following tests are carried out as part of a Nordtest project entitled "Determination of fracture energy of concrete - comparison of the Three-Point Bend Test (TPBT) and the Wedge Splitting Test (WST)" at the Technical University of Denmark (DTU).

No standardized method exists for the determination of fracture energy of concrete. Today two different commonly accepted methods are used. The first is the three-point bend test recommended by RILEM and based on the work of the RILEM TC50-FMC Technical Committee [1]. The second is known as the "wedge splitting test" originally suggested by Brühwiler and Wittmann [2]. The test specimens in the first test are notched beams while the specimens in the second test are much more compact - typically cubes or cylinders.

The purpose of the project is to suggest a Nordtest-method. The two methods will be compared in relation to handling and precision (repeatability, reproducability).

2. Test specimens

A total of seven 100x100x840 mm beams are casted in wooden forms.

Five beams are prepared for the three-point bend test (TPBT) by sawing a notch as shown in Figure 1 in [1]. The test description [1] specifies that the notch should be sawn from a side perpendicular to the top surface at casting. For practical reasons the notch is sawn from the top surface during casting at DTU.

One beam is sawn into eight 100x100x100 mm specimens for the Wedge Splitting Test (WST), each with a groove and a notch as shown in Figure 3A in [2]. The groove and the notch are sawn parallel to the top surface during casting at DTU.

The last beam is kept as a spare beam.

Table 1. Specimen no., size [mm] and weight, TPBT.

Specimen no.	I	II	III	IV	V
Width	102	101	101	101	102
Depth	101	100	100	100	101
Length	842	841	841	841	841
Length between supports	800	800	800	800	800
Area of the ligament [mm^2]	51x102	49x101	50x101	49x101	51x102
Weight [kg]	20.26	19.86	19.74	19.88	19.98

Table 2. Specimen no. and size [mm], WST.

Specimen no.	1	2	3	5	6
Width	100	100	101	100	101
Height	101	101	101	101	101
Depth	100	100	100	100	100
Ligament length	51	51	50	51	50

Specimen no.4 failed before the test or at instrumentation (see load-COD curve in appendix 2).

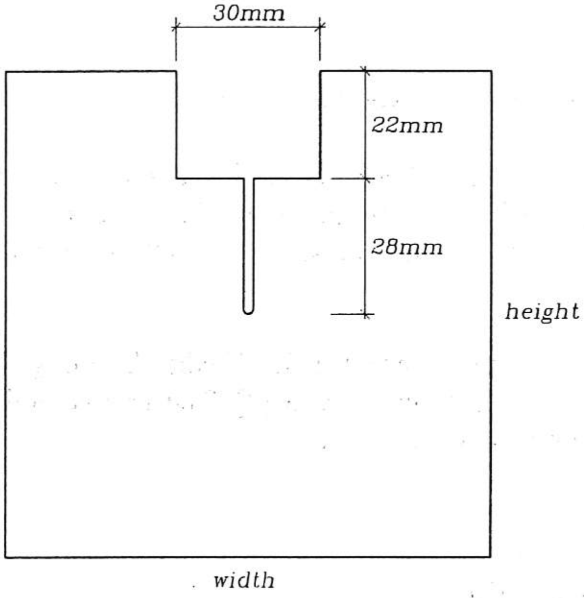


Figure 1. Geometry of specimens (WST) with typical values for the size of the groove and the notch [mm].

3. Materials

Table 3. Recipe, amounts in [kg/m^3].

Materials [kg/m^3]	
Cement (ASTM Type V)	298
Fly Ash	65
Micro Silica - slurry	15.5
Water	140.5
Air entraining agent	0.480
Superplasticizer	0.824
Plasticizer	0.412
Fine aggregate	694
Coarse aggregate 4-8 mm	350
Coarse aggregate 8-16 mm	700

Coarse and fine aggregates, cement and fly ash is poured into the mixer in the given order and mixed for two minutes. Water, slurry and the additives are mixed together and added to the mixer. The mixing continues for another three minutes.

After casting the beams are kept in the forms until the following day where they are transferred to water storage until the tests are performed. One beam is sawn into 100x100x100 mm specimens for the WST-test during the water storage.

Two batches are mixed. The specimens II, IV and V for the TPBT-test are from batch 1, the specimens I and III for the TPBT-test and all the specimens for the WST-test are from batch 2. Compressive strength was measured on five 100x200 mm cylinders, three from batch 1 and two from batch 2.

Table 4. Slump, air content and density.

Batch	1	2
Slump [mm]	100	100
Air content [%] *	6	no measurement
Density [kg/m^3]	2310	2300

*: The air meter (pressure type) failed during the measurement.

4. Test setup - TPBT

The test is performed in a electro mechanical universal testing machine INSTRON 6025 equipped for closed loop testing. In the test a 100 kN static / 50 kN dynamic load cell was used.

Figure 2 shows the test setup for the three-point bend test. The beam is supported by two supports allowing for rotation both parallel to and in the axial direction of the beam. Furthermore small axial movements were allowed according to [1].

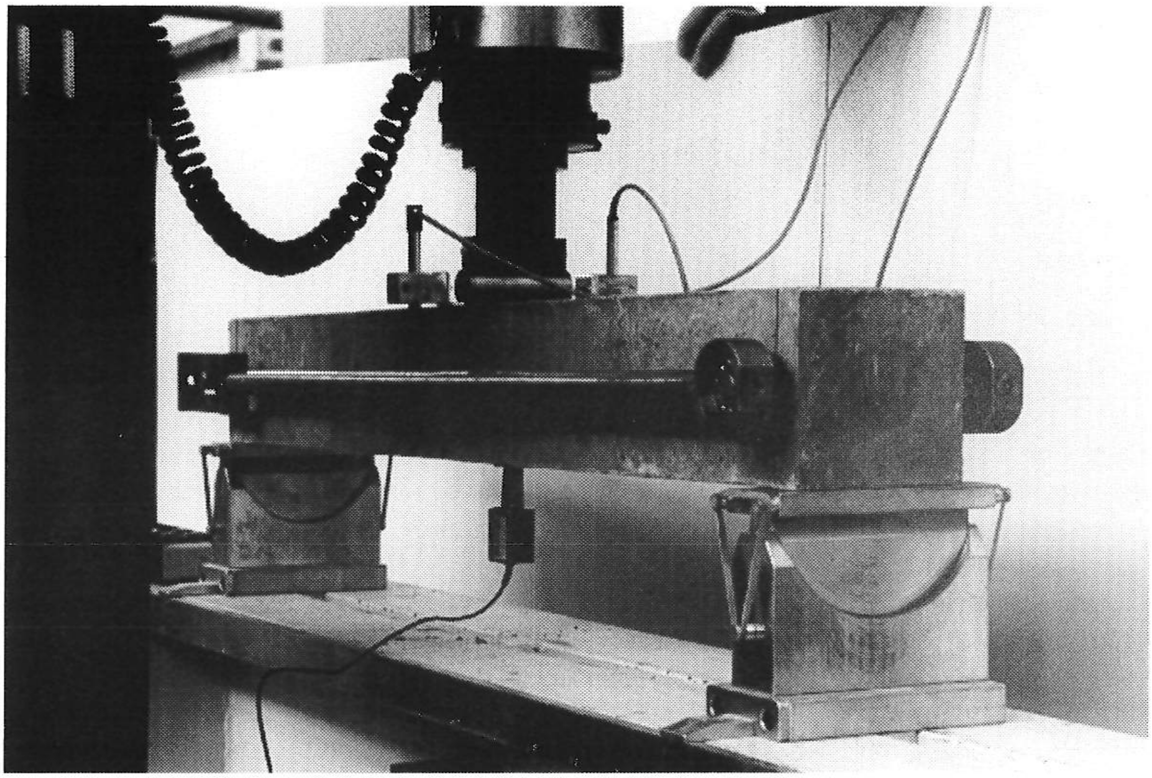


Figure 2. Test setup, TPBT.

The deformation of the center of the beam is measured directly off the specimen using two rigid yokes fixed with hinges glued to the beam directly over the supports, one hinge on each side allowing for deformation in the axial direction. The deformation is measured using two LVDT's fixed to the top surface of the beam - one on each side - measuring the displacement of the yoke, cf. figure 2.

The crack opening displacement (COD) is measured with a clip gauge (INSTRON Type 2670-116) fixed between aluminium pieces glued on each side of the notch.

When the aluminium pieces necessary for the deformation and COD measurements are mounted, the beam is placed on the supports, the yokes from which the deformation is measured, the clip gauge and the LVDT's are fixed and the test is started. Two pieces of wood are placed under the beam to protect the clip gauge when the specimen fails (not shown in Figure 2).

The preparation of each beam including gluing and instrumentation (excluding sawing the notch) takes 1-1½ hour.

The test is controlled by the signal from the clip gauge. A constant test speed was used (beam no.1: 0.1 mm/min, beam no.2-5: 0.05 mm/min) resulting in a loading history with 30 (beam no.1) to 70 seconds to peak load.

5. Test setup - WST

The test is performed in a electro mechanical universal testing machine INSTRON 6022 equipped for closed loop testing. In the test a 10 kN static load cell is used.

Figure 3 shows the test setup for the WST-test according to Figure 1c in [2].

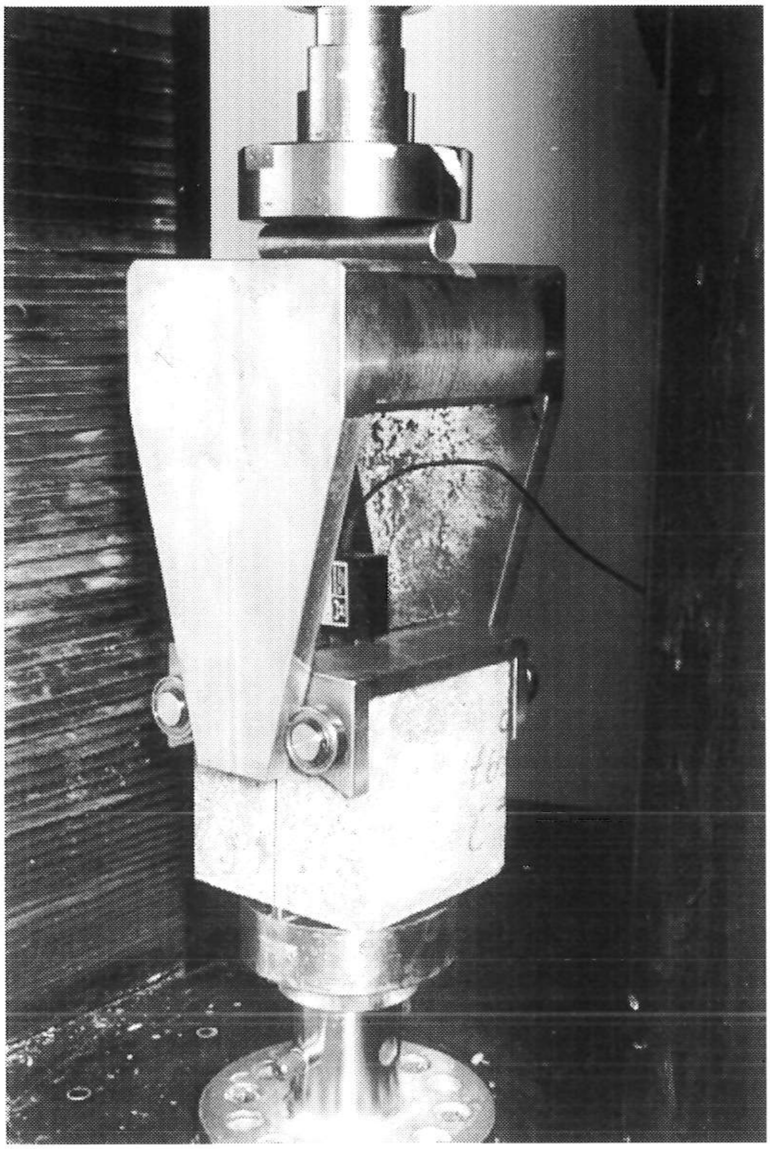


Figure 3. Test setup, WST.

The COD is measured with a clip gauge (INSTRON Type 2670-116) fixed between aluminium pieces glued on each side of the notch.

The specimen and the wedge are loaded between fixed loading platens in the test machine. The specimen mounted with the clip gauge and the loading devices is placed on linear support on the lower loading platen. The wedge with a linear support on top is placed on the loading devices and the upper loading platen is lowered. A steel piece is placed between the wedge and the lower loading platen to protect the clip gauge from being destroyed when the specimen fails (not shown in Figure 3).

The preparation of the specimens including gluing and instrumentation (excluding sawing) takes 15-20 minutes.

The test is controlled using the signal from the clip gauge. A rate of 0.025 mm/min is used resulting in a loading history with 60-75 seconds to peak load.

6. Results

Compressive strength

Table 5. Compressive strength [MPa], 100x200 mm cylinders.

Age at test	Batch 1		Batch 2
28 days	34.5	35.5	37.1
45 days *	40.9		43.6

*: The WST- and TPBT-tests were performed when the concrete was 42-49 days old (cf. table 6 and 7).

TPBT-test

The fracture energy (G_F) is calculated from the equation

$$G_F = (W_0 + mg\delta_0)/A_{lig} \tag{1}$$

where W_0 is the area under the load-displacement curve, δ_0 is the deformation at the final failure of the beam, A_{lig} is the area of the ligament, g is the acceleration due to gravity (9.81 m/s^2) and $m = m_1 + 2m_2$, where m_1 is the weight of the beam between the supports and m_2 is the weight of the part of the loading arrangement which is not attached to the machine, but follows the beam until failure.

Since not all tests were stopped under the same conditions it was decided to define the end of test at a residual load of 30 N. The displacement at this point is chosen as δ_0. The duration of the TPBT-test was 7-15 minutes.

Load-deformation curves: see appendix 1.

Table 6. Results from the TPBT-tests.

Specimen no.	I	II	III	IV	V
Age at test [days]	42	43	43	45	45
m_1 [kg]	19.25	18.89	18.78	18.91	19.01
m_2 [kg]	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0
W_0 [Nm]	0.353	0.331	0.411	0.432	0.467
δ_0 [mm]	1.124	1.151	0.967	1.348	1.164
F_{max} [N]	968	826	1056	1039	1088
G_F [N/m]	114	115	121	144	137
mean value [N/m]			126		
std.deviation [N/m]			13.6 (11 %)		

F_{max} is the maximum applied load, excluding m_1

WST-test
The fracture energy (G_F) is calculated from the equation

$$G_F = W/A_{lig} \tag{2}$$

where W is the area under the F_S-COD curve and A_{lig} is the area of the ligament. The splitting force F_S is the horizontal component of the force F_V acting on the rollers, i.e. including the load (m_3) which comes from the weight of wedge and the linear support on top of it.

$$F_S = \frac{F_V}{2\ tg\alpha} \tag{3}$$

where α is the wedge angle. In the test setup at DTU the wedge angle was 15°.

Since not all tests were stopped under the same conditions it was decided to define the end of test at a residual load of 30 N. The COD at this point is chosen as COD_0. The effect of this approximation on the magnitude of the fracture energy is shown to be limited. Duration of the WST-tests was 19-46 minutes (specimen no.5 endured 46 minutes).

Load-COD curves: see appendix 2.

Table 7. Results of the WST-tests.

Specimen no.	1	2	3	5	6
Age at test [days]	43	43	43	45	49
m_3 [kg]	6.25	6.25	6.25	6.25	6.25
W_0 [Nm]	0.134	0.164	0.178	0.179	0.182
COD_0 [mm]	0.431	0.571	0.557	0.781	0.565
F_{max} [N]	1124	1132	1112	1003	1061
G_F [N/m]	58.4	72.9	79.3	83.1	81.0
mean value [N/m]			75		
std. deviation [N/m]			9.8 (13%)		

Specimen no.4 failed before the test or at instrumentation (see load-COD curve in appendix 2). W_0 is based on F_v excluding m_3. F_{max} is the maximum applied load (F_v), excluding m_3.

Stability
Both test setups had a satisfying stability. Examples are shown in Figure 4 and 5.

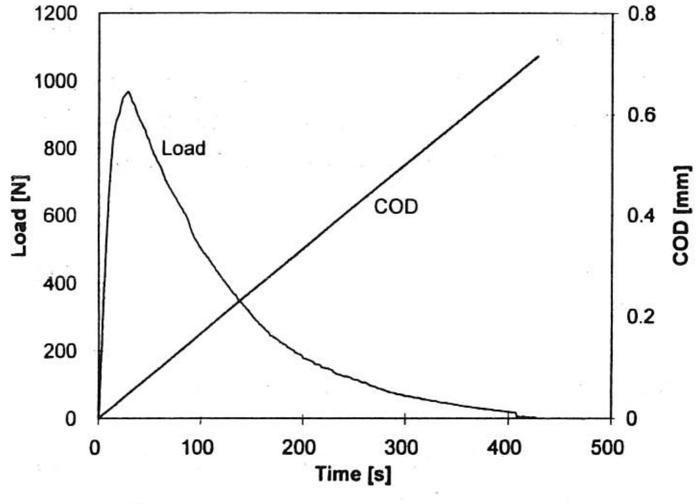


Figure 4. Load-time and load-COD history, TPBT (specimen no.I).

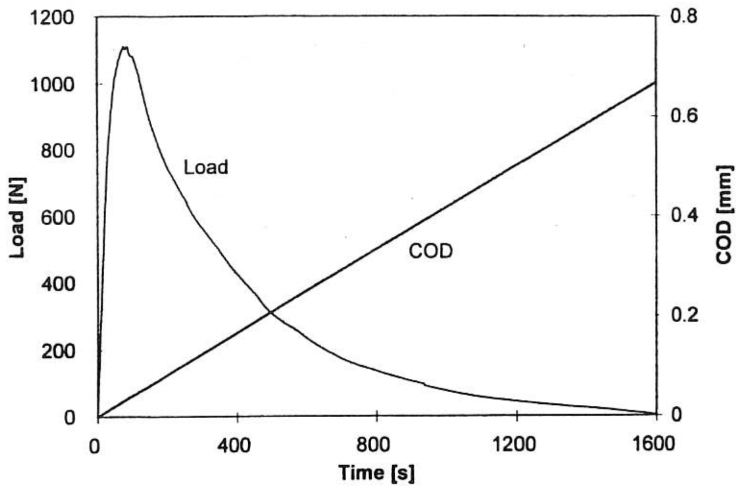


Figure 5. Load-time and load-COD history, WST (specimen no.3).

7. Concluding remarks

The experimental setup in the WST method is significantly faster to work with although the sawing procedure is more time consuming. Only when using laboratory prepared specimens time can be saved by designing the formwork in such a way that no groove have to sawn.

The standard deviation of the results in the two methods is about the same.

Significantly larger fracture energy is measured using the TPBT method.

It is expected that the TPBT method gives larger fracture energy. More energy can be dissipated in the beam because of the size (compared with the WST specimen).

We are questioning equation (1). It seems more correct only to use half of the weight of the beam including the loading arrangement represented by m_2. At failure the displacement (in vertical direction) of the centre of gravity of the two halves of the beam is half the displacement of the centre of the beam. By using

$$G_f = (W_0 + 0.5 m g \delta_0) / A_{lig} \qquad (4)$$

the following values for G_F are given:
88.3 N/m, 88.4 N/m, 99.0 N/m, 113 N/m and 111 N/m
(mean value 100 N/m, standard deviation 11.7 N/m $\approx$ 12 %)

8. Literature

[1] Determination of the fracture energy of mortar and concrete by means of the three-point bend tests on notched beams. *Materials and Structures*, vol.18, no.106, pp.99-101, july-aug 1985.

[2] Brühwiler E. & Wittmann, F.H.: The wedge splitting test, a new method of performing stable fracture mechanics tests. *Engineering Fracture Mechanics*, vol.35, no. 1/2/3, pp.117-125, 1990.

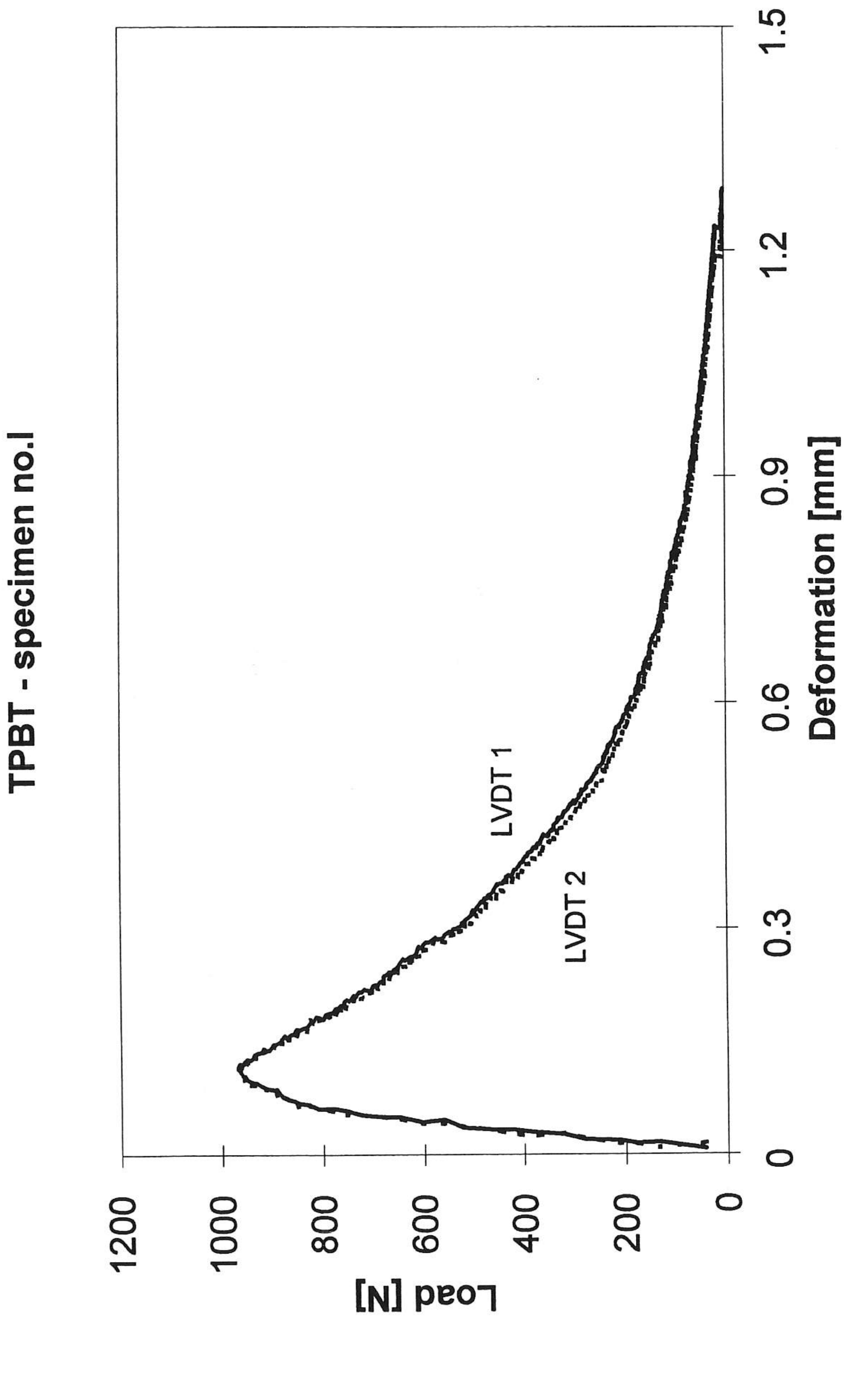
List of appendices

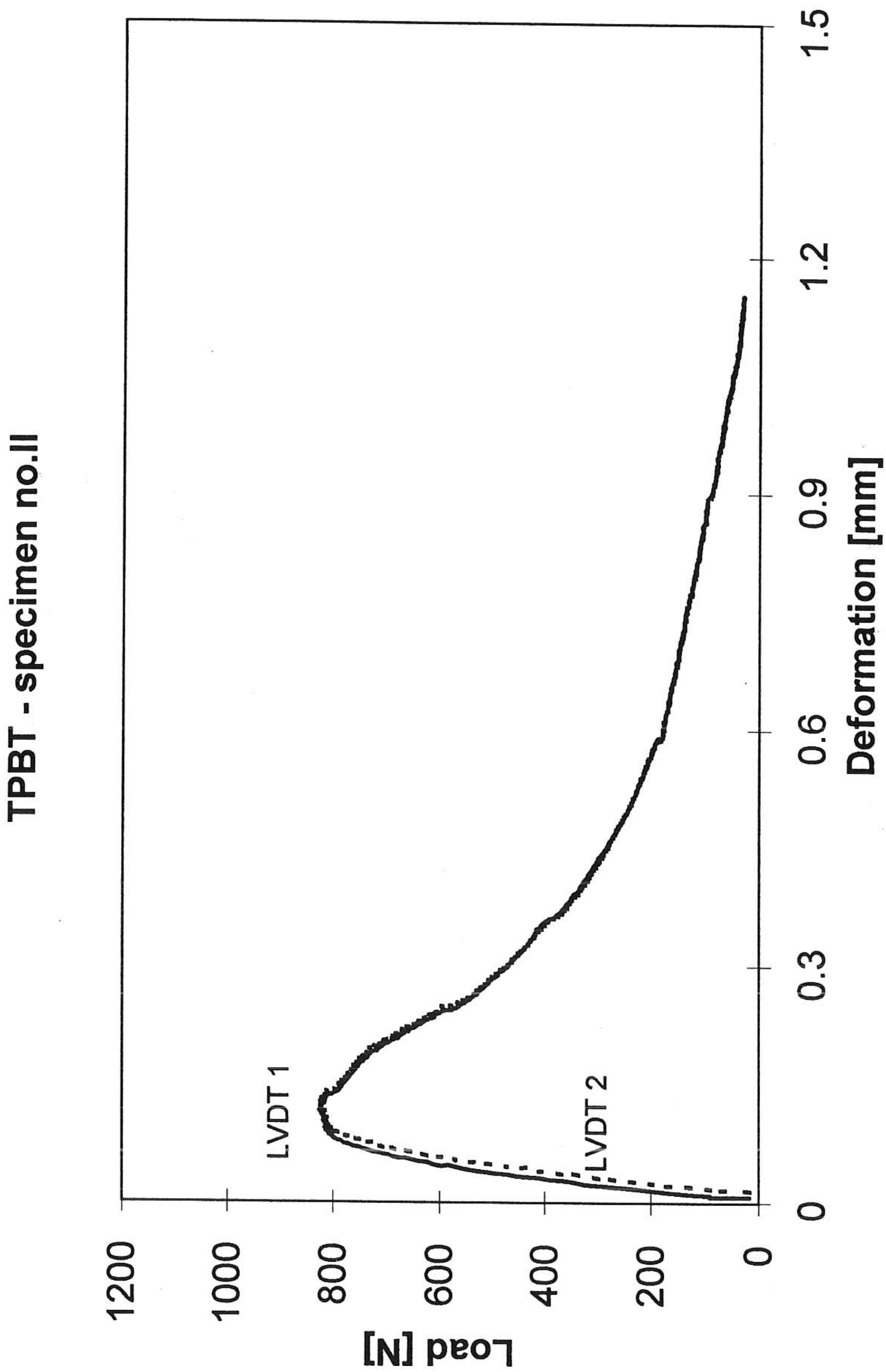
Appendix 1 - Raw data (curves) TPBT

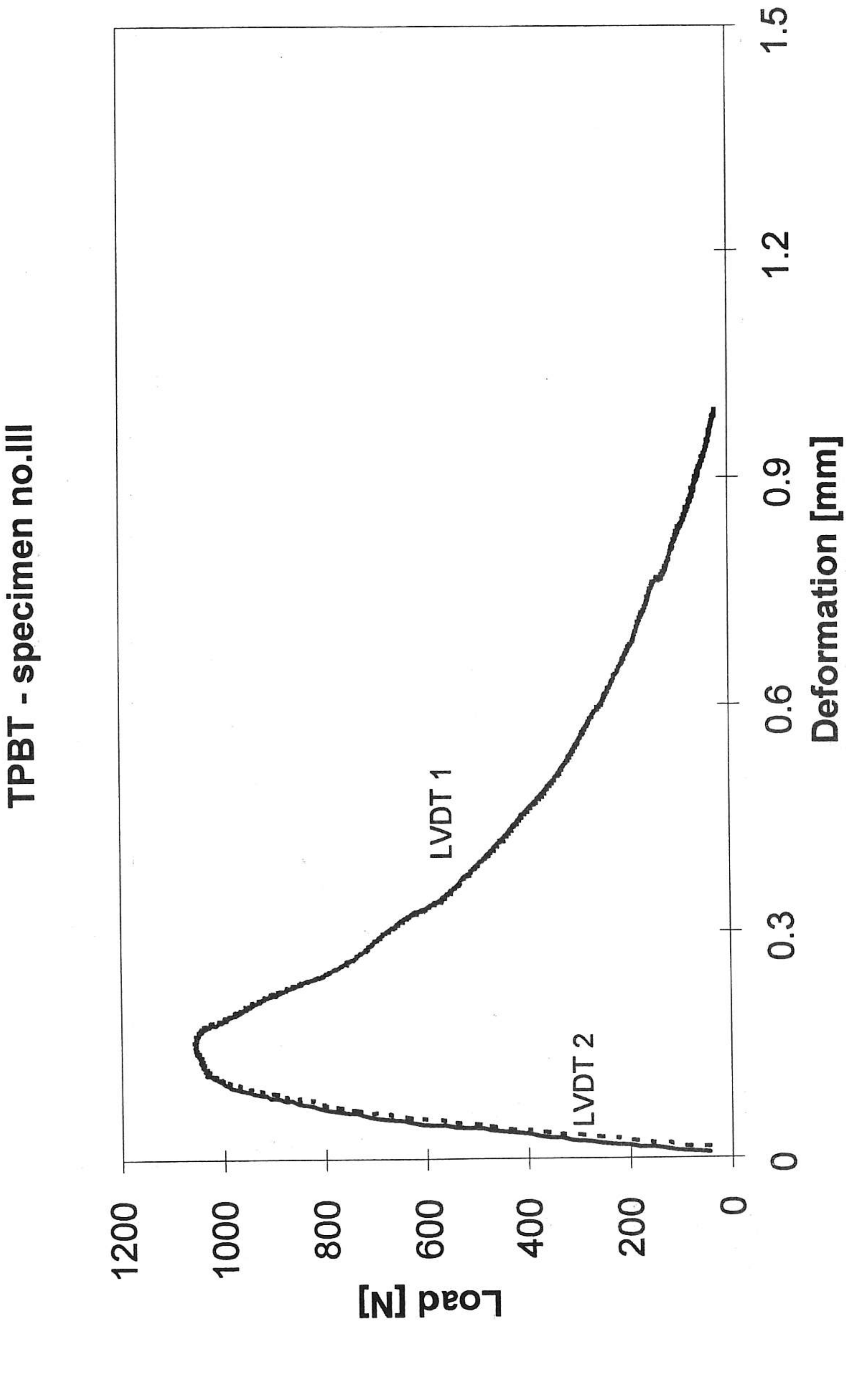
Appendix 2 - Raw data (curves) WST

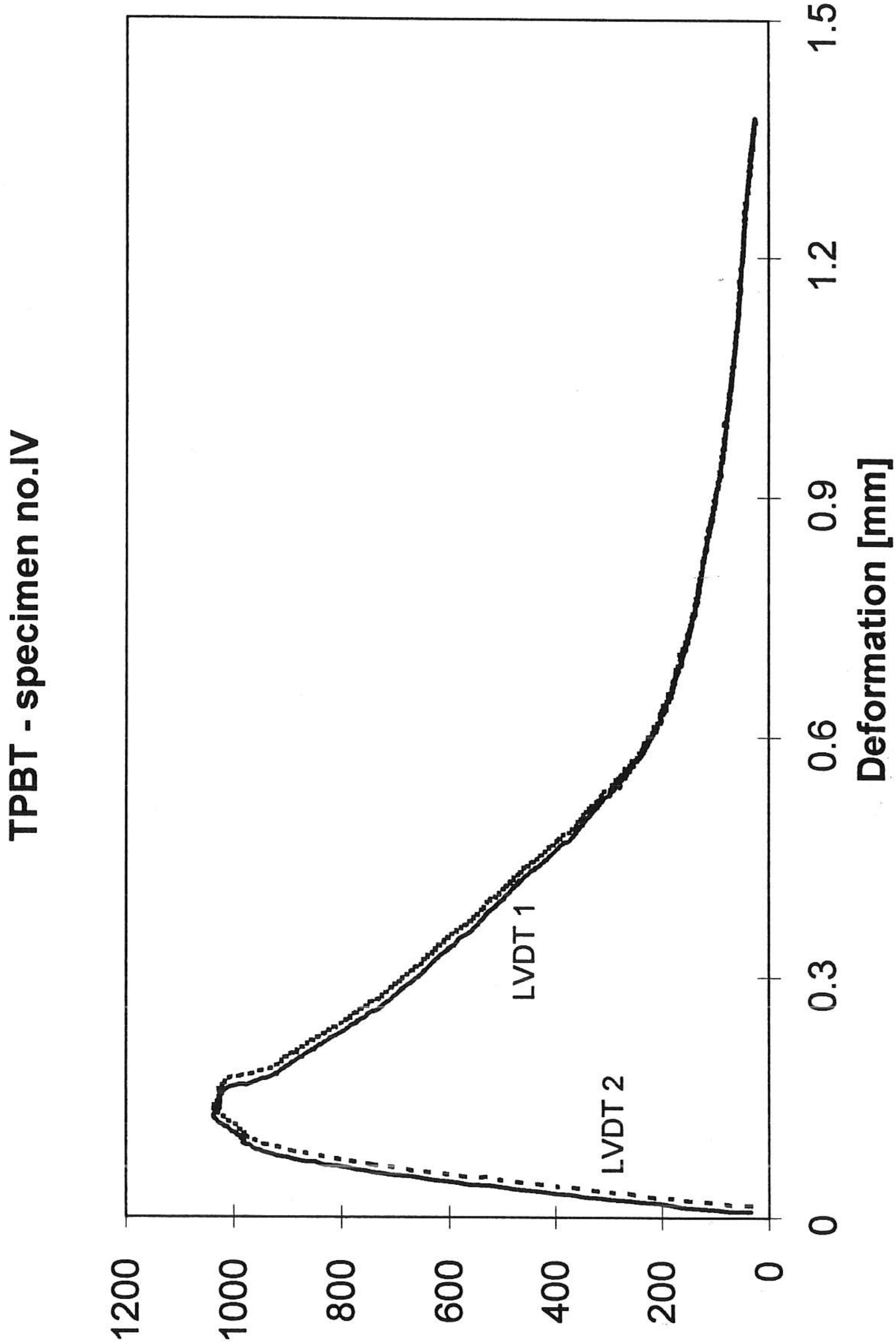
Appendix 1

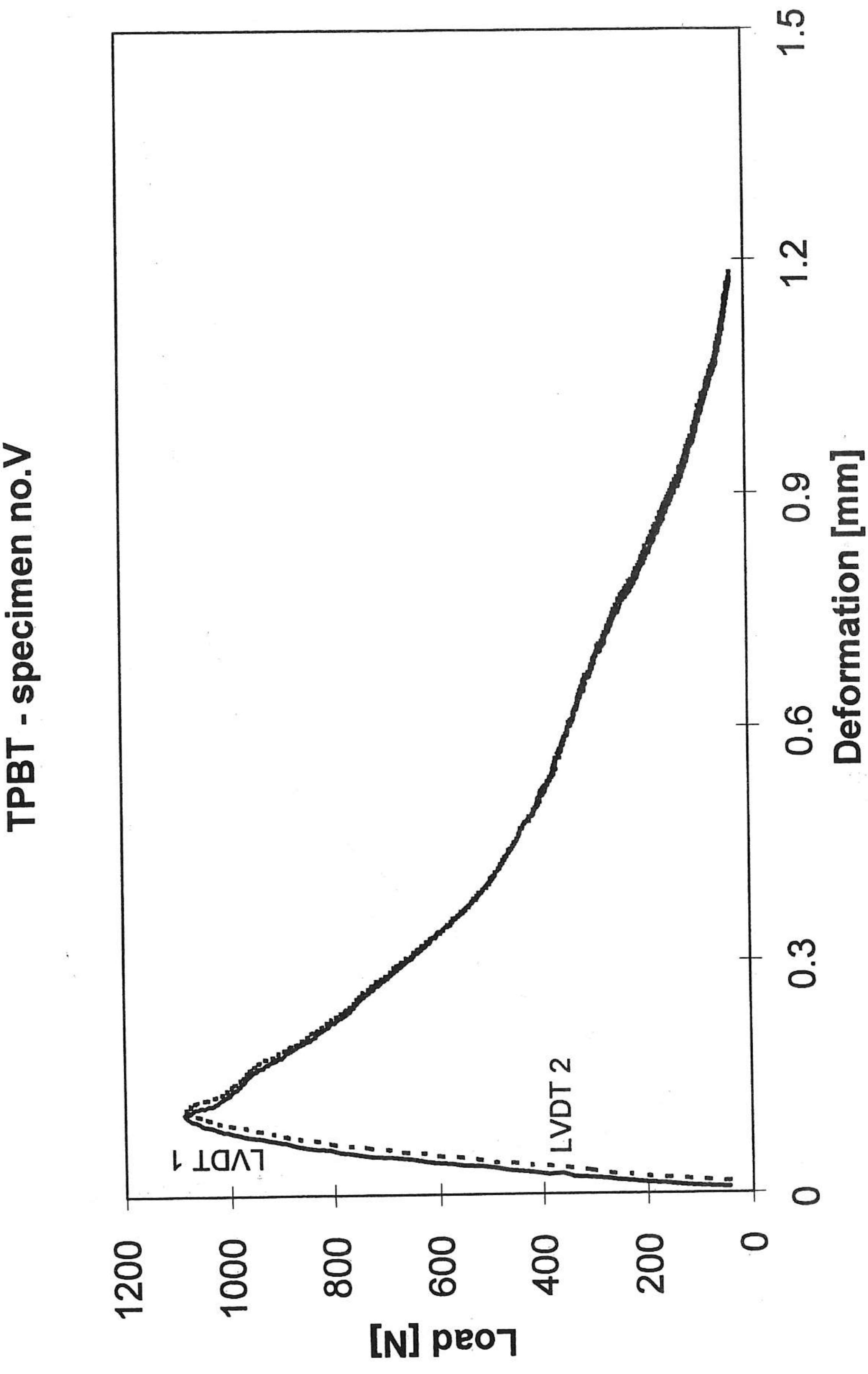
Raw data (curves) TPBT


TPBT - specimen no.l
Load [N]
0
200
400
600
800
1000
1200
Deformation [mm]
0
0.3
0.6
0.9
1.2
1.5
LVDT 1
LVDT 2


TPBT - specimen no.II
Load [N]
1200
1000
800
600
400
200
0
Deformation [mm]
0
0.3
0.6
0.9
1.2
1.5
LVDT 1
LVDT 2

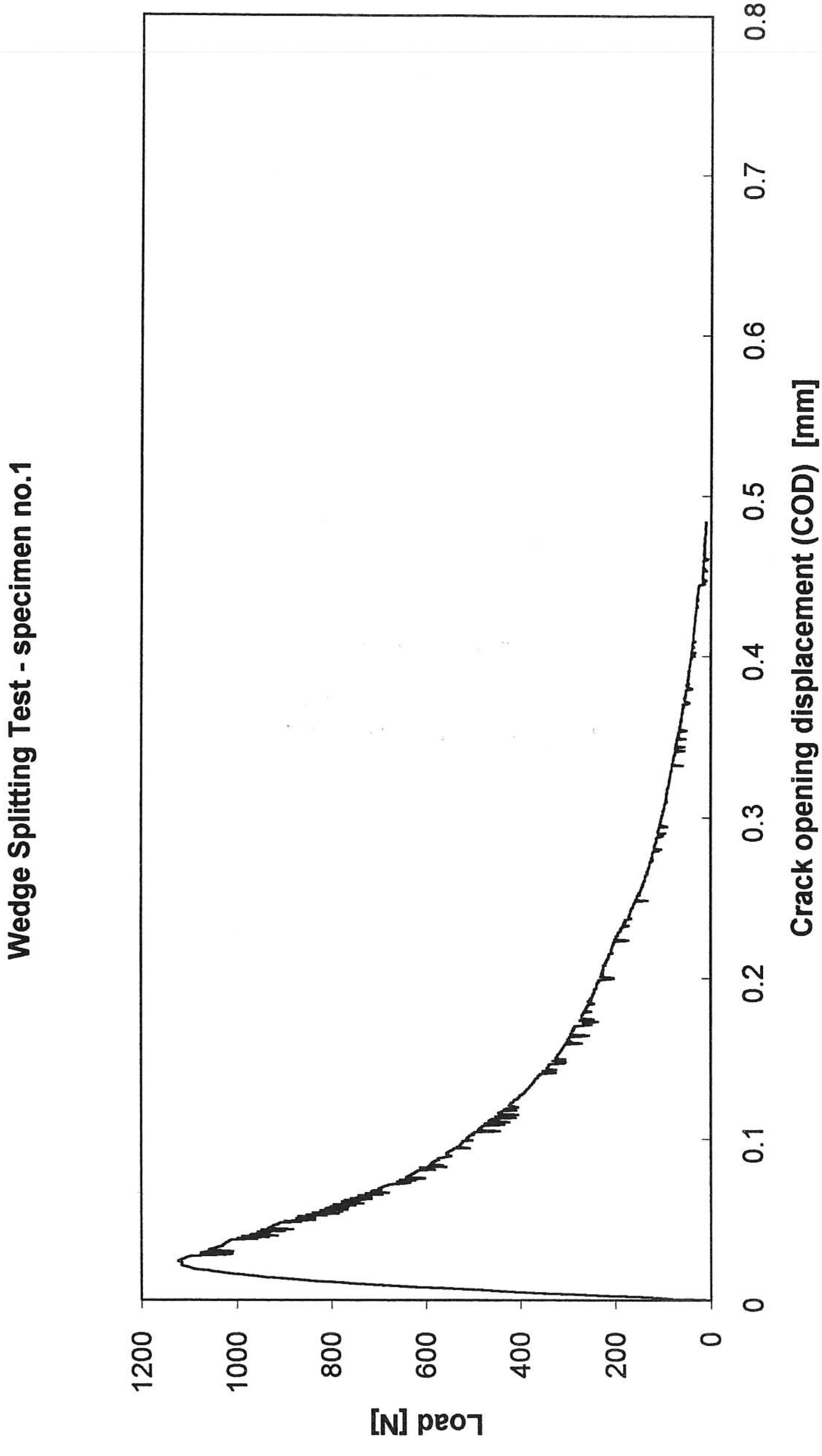

TPBT - specimen no.III
Load [N]
1200
1000
800
600
400
200
0
0
0.3
0.6
0.9
1.2
1.5
Deformation [mm]
LVDT 1
LVDT 2


TPBT - specimen no.IV
Load [N]
1200
1000
800
600
400
200
0
0
0.3
0.6
0.9
1.2
1.5
Deformation [mm]
LVDT 1
LVDT 2

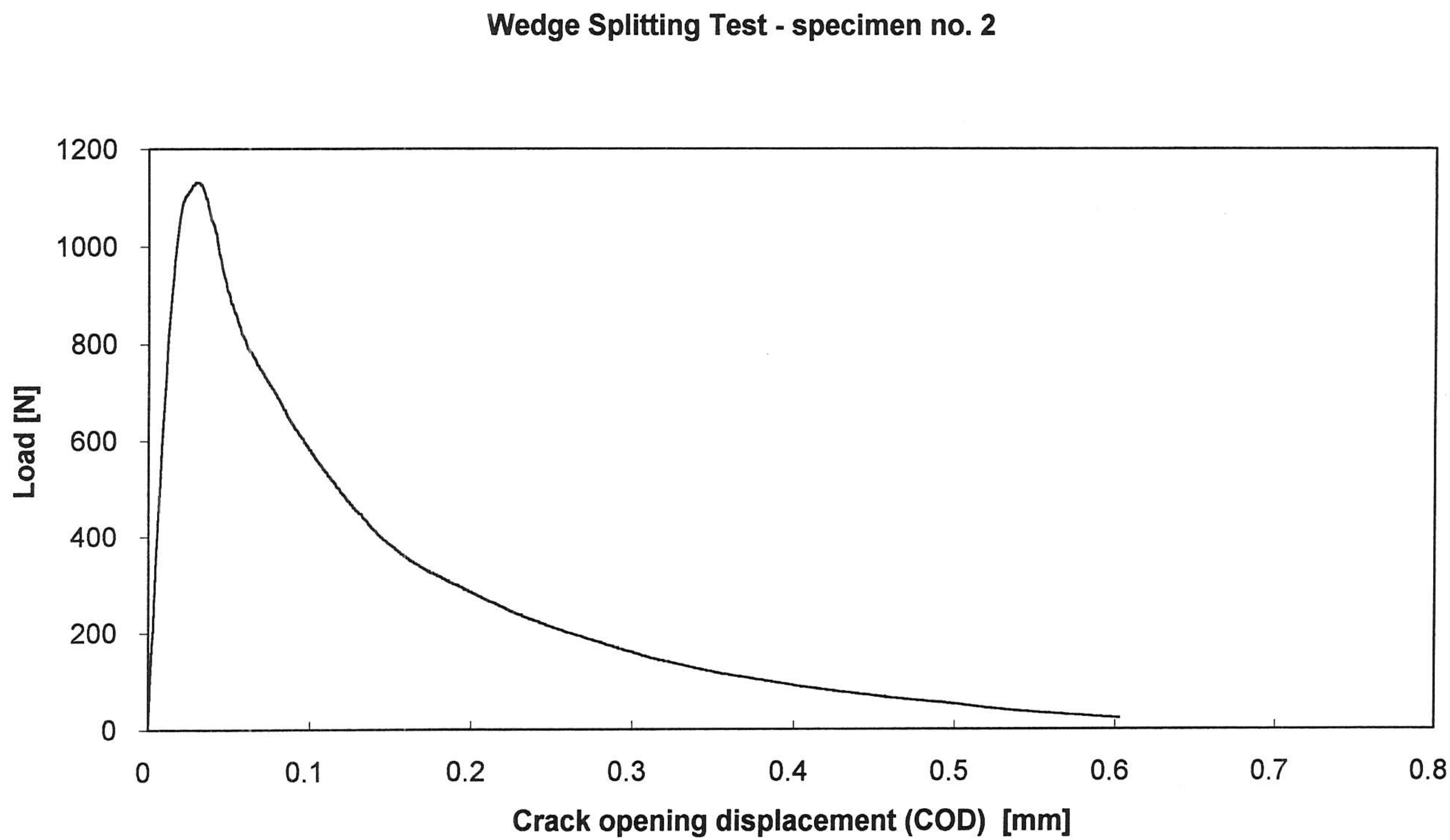

TPBT - specimen no.V
Load [N]
1200
1000
800
600
400
200
0
Deformation [mm]
0
0.3
0.6
0.9
1.2
1.5
LVDT 1
LVDT 2

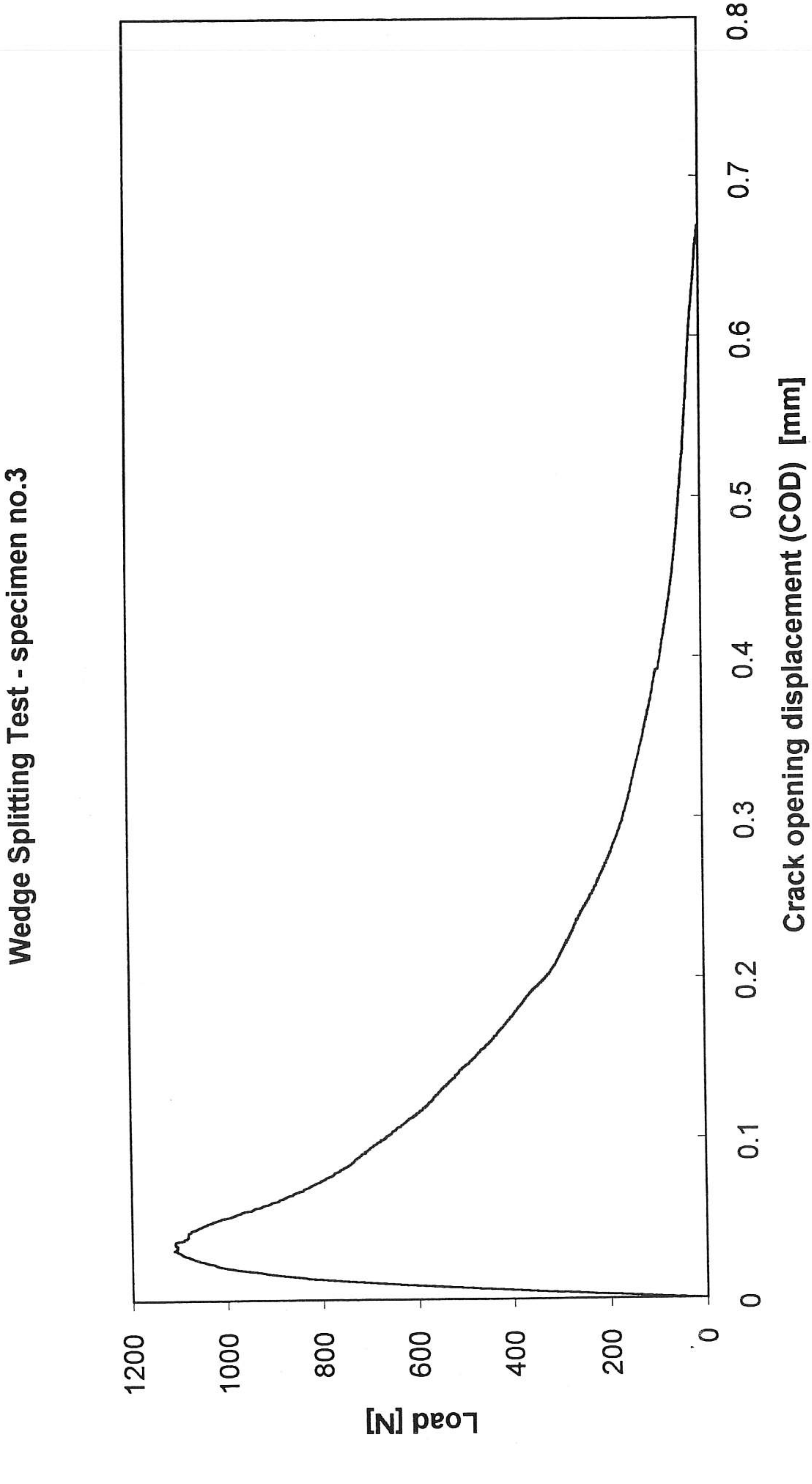
Appendix 2

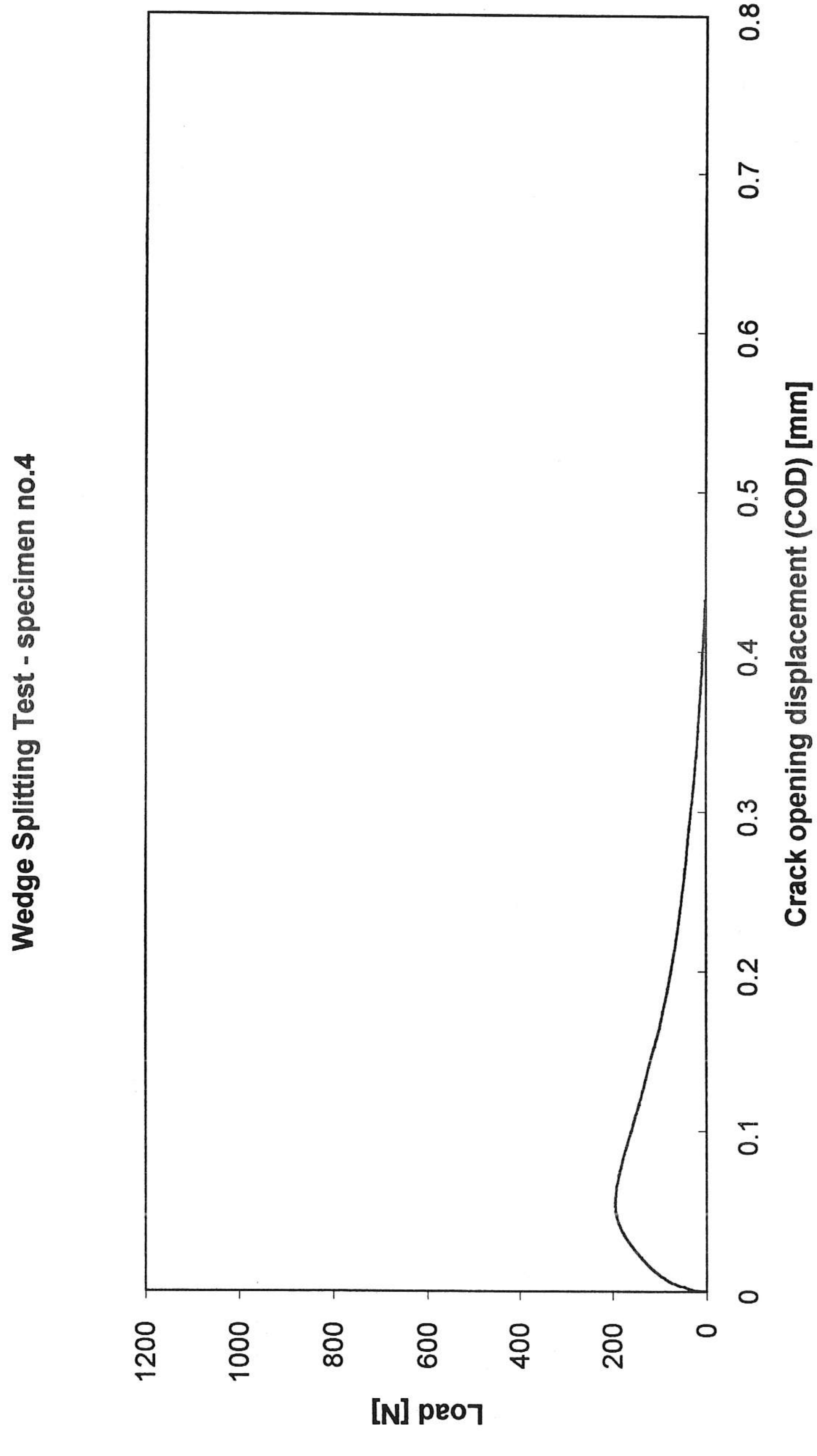
Raw data (curves) WST



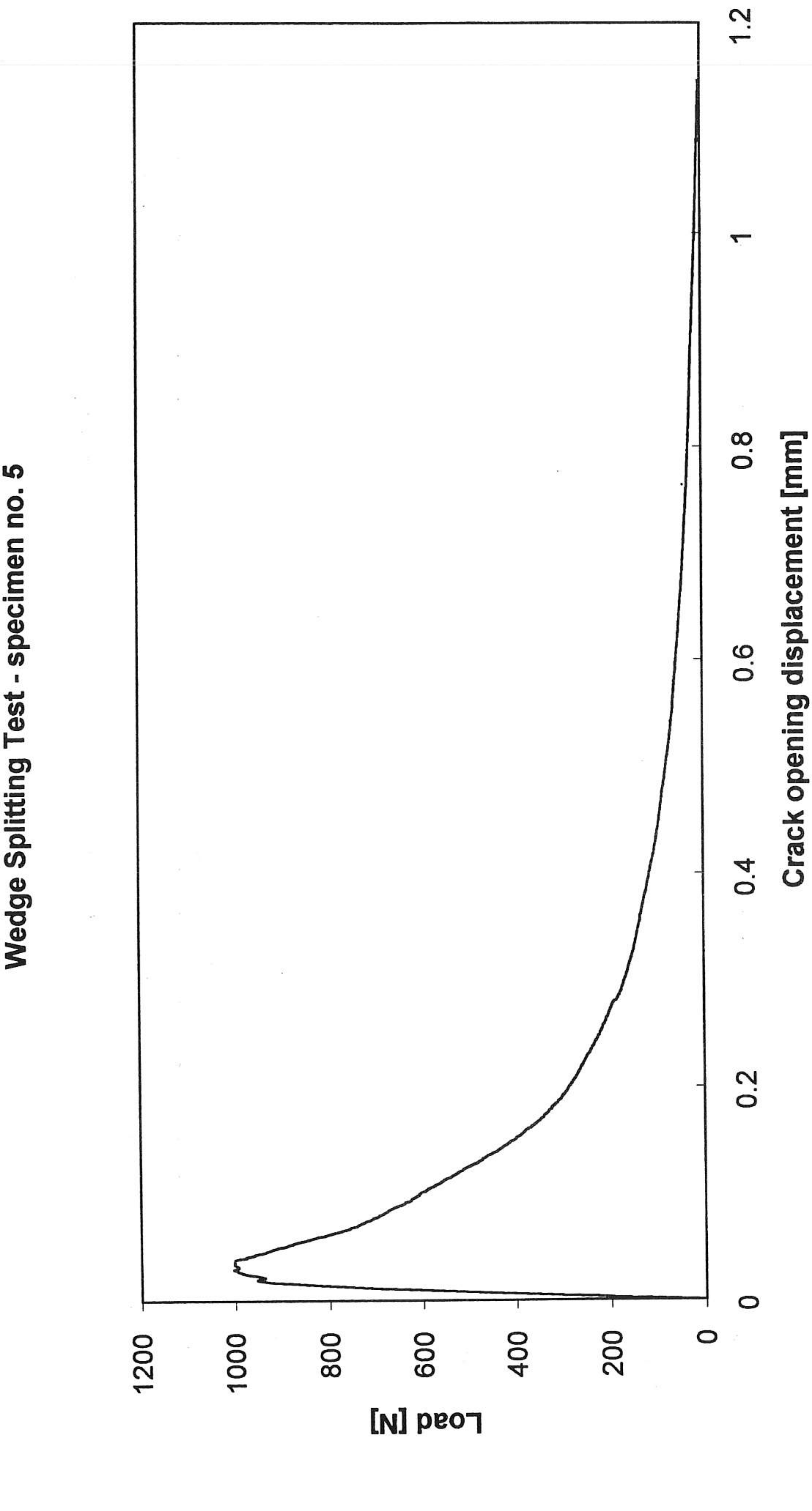
Wedge Splitting Test - specimen no.1
Load [N]
0
200
400
600
800
1000
1200
Crack opening displacement (COD) [mm]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

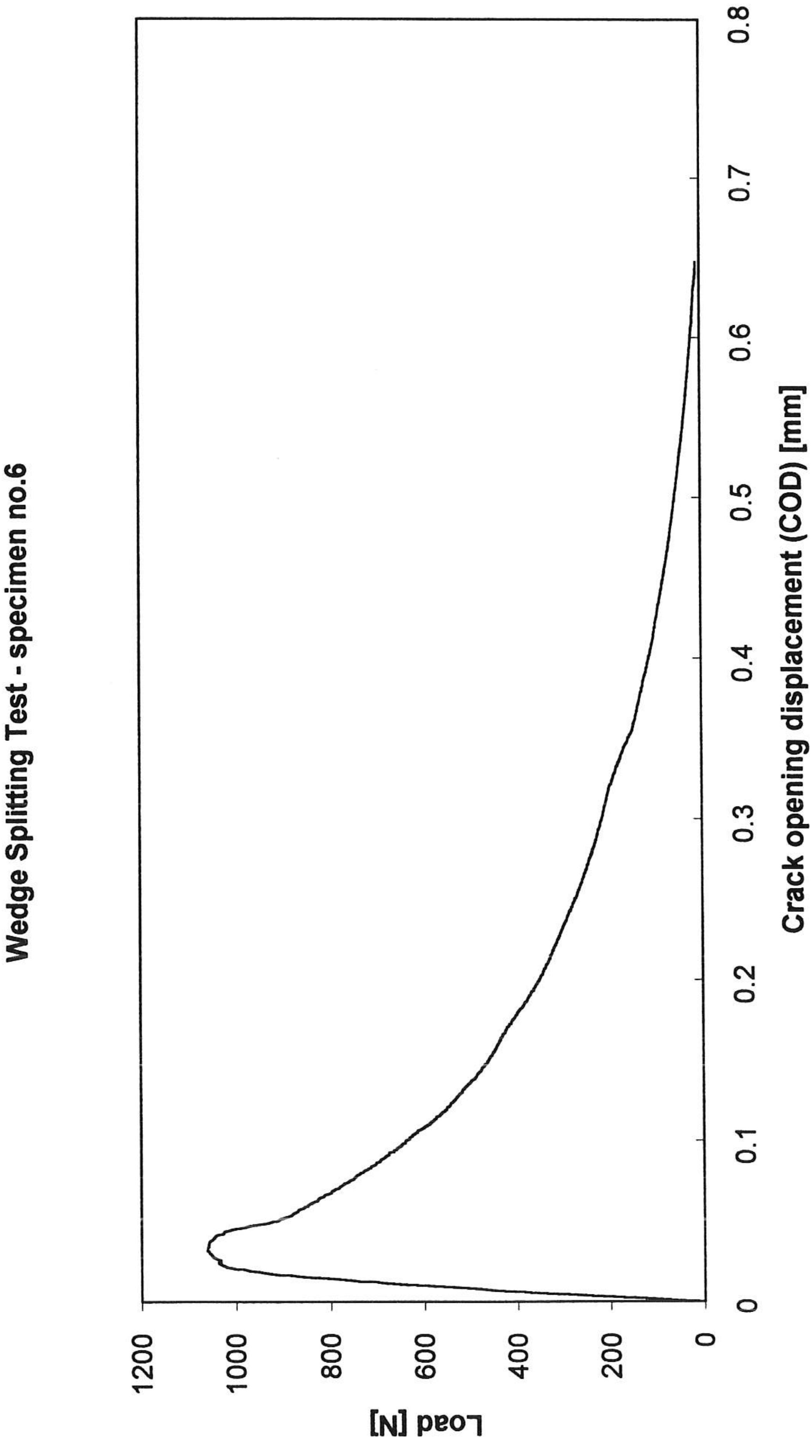

Wedge Splitting Test - specimen no. 2
1200
1000
800
600
400
200
0
Load [N]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Crack opening displacement (COD) [mm]


Wedge Splitting Test - specimen no.3
Load [N]
1200
1000
800
600
400
200
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Crack opening displacement (COD) [mm]



Wedge Splitting Test - specimen no.4
Load [N]
0
200
400
600
800
1000
1200
Crack opening displacement (COD) [mm]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


Wedge Splitting Test - specimen no. 5
Load [N]
0
200
400
600
800
1000
1200
Crack opening displacement [mm]
0
0.2
0.4
0.6
0.8
1
1.2


Wedge Splitting Test - specimen no.6
Load [N]
0
200
400
600
800
1000
1200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Crack opening displacement (COD) [mm]

Appendix C, results of SINTEF

NOTAT

SINTEF

SINTEF Bygg og miljøteknikk
Sement og betong

Postadresse: 7034 Trondheim
Besøksadresse: Rich. Birkelands vei 3
Telefon: 73 59 52 24
Telefaks: 73 59 71 36

Foretaksnr: NO 948 007 029 MVA

EN KOPI SENDES SENTRALARKIV

GJELDER
Bestemmelse av betongens bruddenergi

GÅR TIL	BEHANDLING	UTTALELSE	ORIENTERING	ETTER AVTALE
Manouchehr Hassanzadeh, SP	x			

07 10

ARKIVKODE	GRADERING
	Fortrolig

ELEKTRONISK ARKIVKODE
XEAHB001.DOC

PROSJEKTNR.	DATO	SAKSBEARBEIDER/FORFATTER	ANTALL SIDER
22m100.00	1997-07-15	Einar Aassved Hansen	3 + 8

1 Bakgrunn

Sveriges Provnings och Forskningsinstitut (SP) i Borås er prosjektleder for et Nordtest prosjekt på sammenligning av to metoder for måling av betongens bruddenergi. I tillegg til SP bidrar også SINTEF og Danmarks Tekniske Universitet (DTU) med eksperimentelle undersøkelser.

Dette notatet redegjør for resultatene av forsøk med trepunkts bøyning av prisme med kjerv (TPBT) og splitting av terninger med kjerv (WST) utført ved SINTEF. Metodene er beskrevet i søknaden til Nordtest.

2 Materialer

SINTEF skulle gjør forsøk på en betong med v/c = 0.5, Dmax = 16 mm og synk mellom 5 og 10 cm.

Blanderesepten og oppveide mengder er gitt i vedlegg 1.

Den ferske betongen hadde følgende egenskaper:

synk:	14.5 cm
luftinnhold:	1.7%
densitet:	2 377 kg/m^3

Synkmålet ble større enn ønsket maksimal verdi på 10 cm uten bruk av tilsetningsstoff. Dette skyldes at sementen (Norcem Anlegg) har et relativt lavt vannbehov.

Prøvestykkene ble avformet etter et døgn og vannlagret frem til prøving. Prøvingen av samtlige prøvestykker ble utført ved en alder på 34 døgn (forsinkelsen skyldes at tildanning av belastningsutstyr for WST tok lengre tid enn planlagt).

Det ble støpt fire bjelker med dimensjon 100*100*840 mm og seks 100 mm terninger.

Kjerv i bjelker og utsparing og kjerv i terninger ble våtsaget en uke før prøving.

Trykkfastheten målt på to stk. 10 cm terning etter 34 døgn (alder for bruddenergimåling) var 57.6 MPa (henholdsvis 57.0 og 58.2 MPa).

3 Bruddenergi målt på tre punkts bjelker med kjerv

Forsøkene ble gjennomført på prismer med dimensjon 100*100*840 mm. Kjervdybden var 50 mm. Prøvestykkene ble opplagt på rullelager med senteravstand 800 mm.

Forsøkene ble gjennomført i en Instron 1126 prøvemaskin. Da opplagerne ikke deformeres ble nedbøyningen målt som forskyvning av bjelkens midtpunkt under lasten. Forskyvningene ble målt med en induktiv forkyvningsmåler av type Hottinger W50. Last og forskyvning ble registrert på PC.

Prøvestykkene ble belastet under deformasjonskontroll med en hastighet av lastpåføringshode på 0.1 mm/minutt.

Et av prøvestykkene ble ødelagt under montering i maskinen. Da det ikke var tildannet reserveprøver ble forsøkene gjennomført på tre prøvestykker.

Etter prøving ble prøvestykkene veiet og målt for bestemmelse av vekten m_1 og arealet A_{lig}. Det var ingen løse deler på prøvemaskinen, slik at $m_2 = 0$.

Last og forskyvningsmålingene ble behandlet i Excel regneark. Arealet W_o ble beregnet og maksimal nedbøyning δ_o registrert.

Resultatene er gitt i tabell 1.

Tabell 1 Resultater fra bruddenergimåling på bjelker

Bjelke nr	Vekt mellom opplegg m_1 (kg)	Bruddflateareal A_{lig} (mm*mm)	W_o (N/m)	δ_o (mm)	G_F (N/m)
1	19.57	49.6*101.8	70.1	1.50	127.1
2	19.68	49.7*102.0	68.4	0.95	104.8
3	19.47	49.6*101.3	44.8	0.71	71.8
Middel					101.2

4 Bruddenergi målt ved splitting av terninger

Forsøkene ble gjennomført på 100 mm terninger. I hver terning var det en uke før prøving saget en utsparing på 30*30*100 mm. Kjervdybden var 20 mm slik at gjenværende bruddareal var ca 50*100 mm.

Nødvendig prøveutstyr for WST ble tildannet etter tegninger mottatt fra SP med noen modifikasjoner. De to symmetriske delene som ligger an mot selve prøven ble tildannet av hele stålstykker og ikke skrudd sammen av flere deler.

Forsøkene ble gjennomført i samme prøvemaskin som bjelkene. Til tross for at SP anbefalt deformasjonsstyring via COD, ble det valgt å belaste med konstant deformasjon av lasthodet. Dette av hensyn til den aktuelle prøvemaskin Deformasjonshastighet var 0.1 mm/min. Det ga maksimal last etter ca 70 sekunder.

COD ble målt 8 mm under betongflaten på samme høyde som senter av boltene som overfører kraften fra kilen til stålstykkene som ligger an mot prøven. En Hottinger Denhungsaufnemher type DD1 med et måleområde på ±2.5 mm ble brukt. Last- og forskyvningssignalene ble registret direkte på PC. I tillegg ble last og vertikalforskyvning registrert på x-y skriver.

Vekt av prøveutstyret som ikke var festet til lasthodet var 9.0 kg.

Last og COD målingene ble overført til Excel regneark for behandling. Spaltekraften, F_s, ble beregnet som:

$$F_s = (F_v + 90\ N) / 2*tg\ (15.25) = 1.83 * (F_v + 90\ N)$$

der F_v er registrert vertikallast og 90 N vekt av løst prøveutstyr.

Energibidraget fra vekten av løst prøveutstyr er tatt med opp til målt COD ved brudd. Det er ikke gjort noen tillegg for bidrag etter maksimalt registrert COD.

Tabell 2 Resultater fra bruddenergimålinger på terninger

Prøve nr	Bruddflateareal A_{lig} (mm*mm)	Areal under Fs - COD kurve (Nm)	Bruddenergi G_F (N/m)
1	49.8*100.5	0.508	101.4
2	50.0*100.5	0.408	81.2
3	49.8*100.8	0.465	92.6
4	49.6*100.7	0.457	91.5
Middel			91.7

VEDLEGG

- Blandeskjema
- Registrerte last-nedbøyningskurver for bjelker
- Spaltekraft-COD kurver for terninger

Blandeskjema

Prosj./ Id.: Mal

Blandevolum:	50 liter
Dato:	5/6-97
Tidspunkt for vanntilsetning	10 02
Ansvarlig:	
Utført av:	

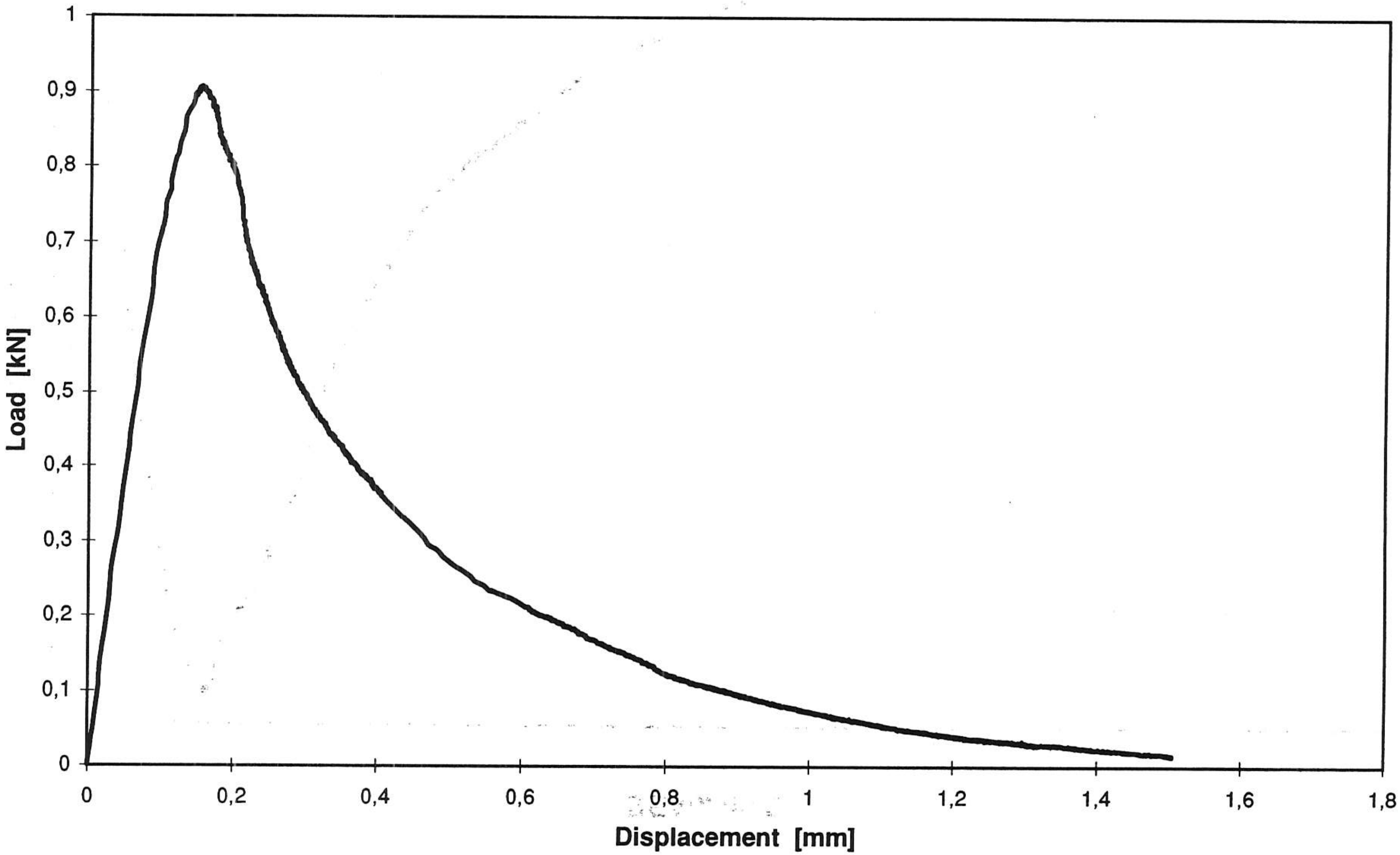
Materialer	**Resept** kg/m³	**Sats** kg	**Fukt** %	**Korr.** kg	**Oppveid** kg	
Sement Norcem Anlegg	366,6	18,329			18,329 ✓	
Silikastøv	0,0	0,000			0,000	
Flyveaske	0,0	0,000			0,000	
Fritt vann	183,3	9,164		-2,026	7,138	7,724 ✓
Absorbert vann	11,7	0,586			0,586	
Årdal 0-8	901,0	45,050	4,4	1,982	47,032 ✓	
Årdal 8-11	450,5	22,525	0,0	0,000	22,525 ✓	
Årdal 8-16	450,5	22,525	0,0	0,000	22,525 ✓	
Årdal 0-2 mm	0,0	0,000	0,0	0,000	0,000	
	0,0	0,000	0,0	0,000	0,000	
	0,0	0,000	0,0	0,000	0,000	
Scancem P	1,5	0,073	60	0,044	0,0[illegible]	
Tss2	0,0	0,000	60	0,000	0,000	
Tss3	0,0	0,000	60	0,000	0,000	
Tss4	0,0	0,000	0	0,000	0,000	

Fersk betong				
Tidspunkt etter vanntilsetning				
Synkmål	14,5			
Utbredelsesmål				
Luft	1,7 %			
Densitet 13.715	2377			

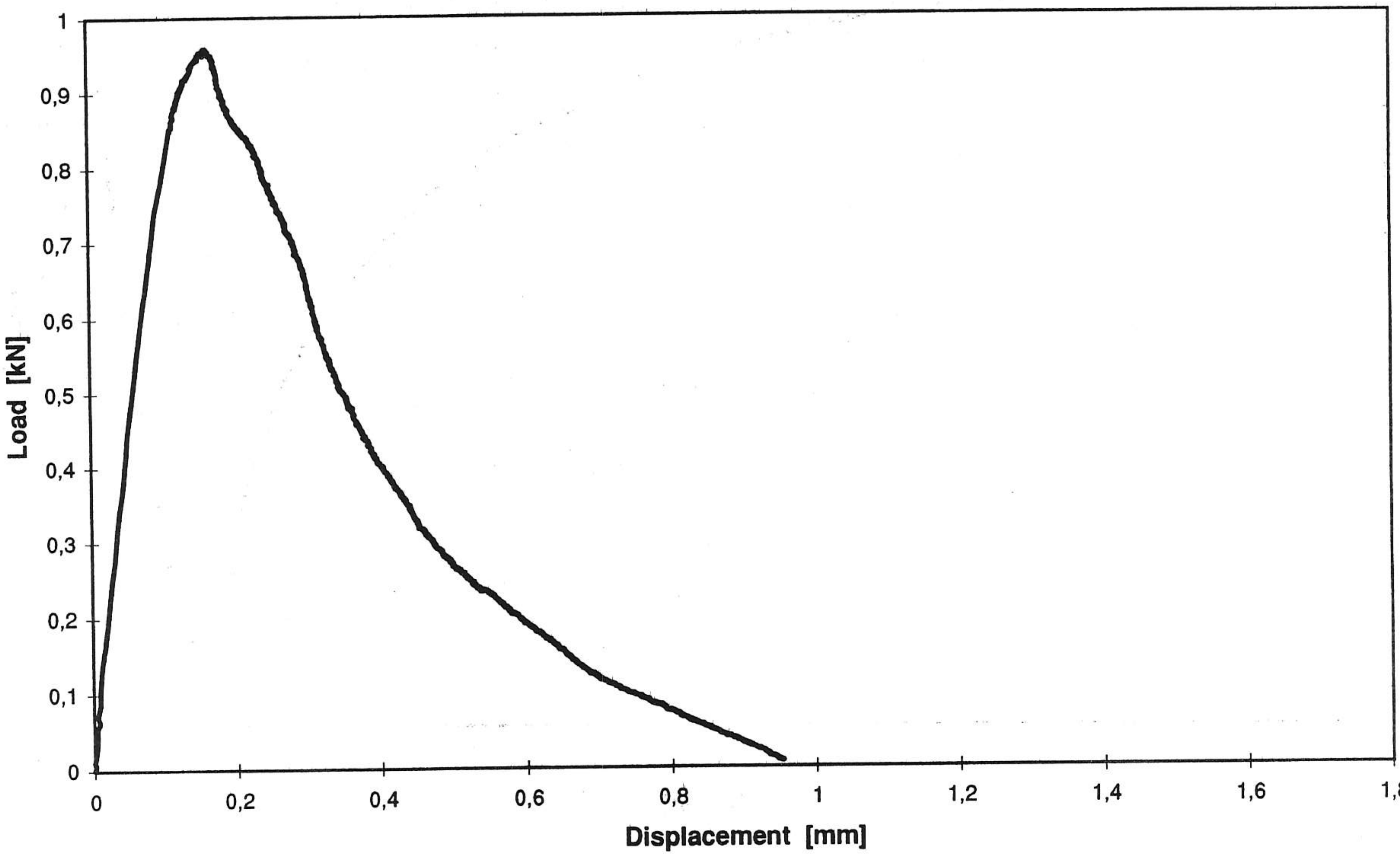
Prøvestykker (antall)				
Utstøpningstidspunkt	10 15			
Terninger	6			
150x300 sylindre				
100x200 sylindre				
100x100x840 prismer	4			

Tilsatt P til max 10 cm. min 5 cm.

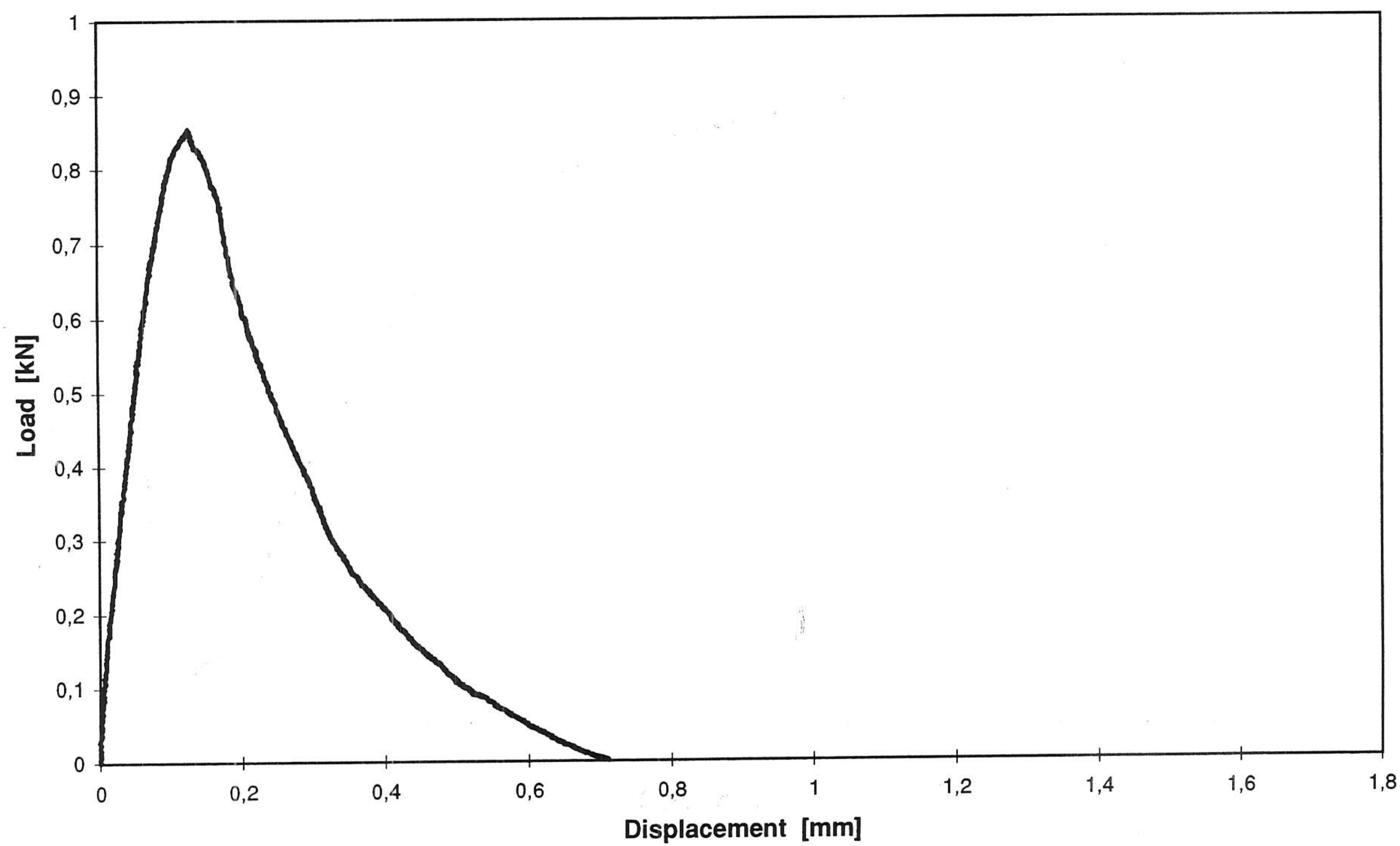
BEAM no 1


1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Load [kN]
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
Displacement [mm]

BEAM no 2

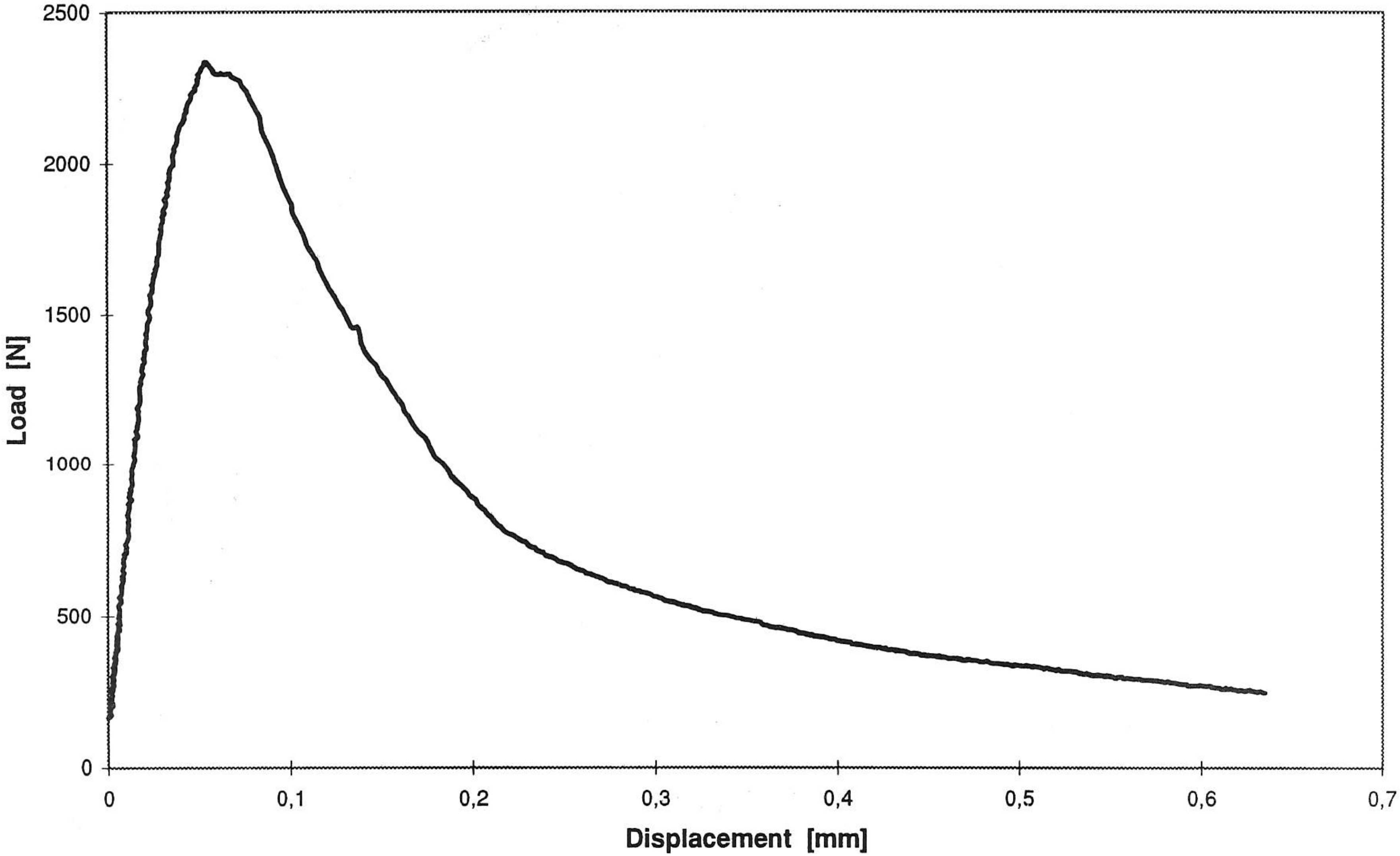

1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Load [kN]
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
Displacement [mm]

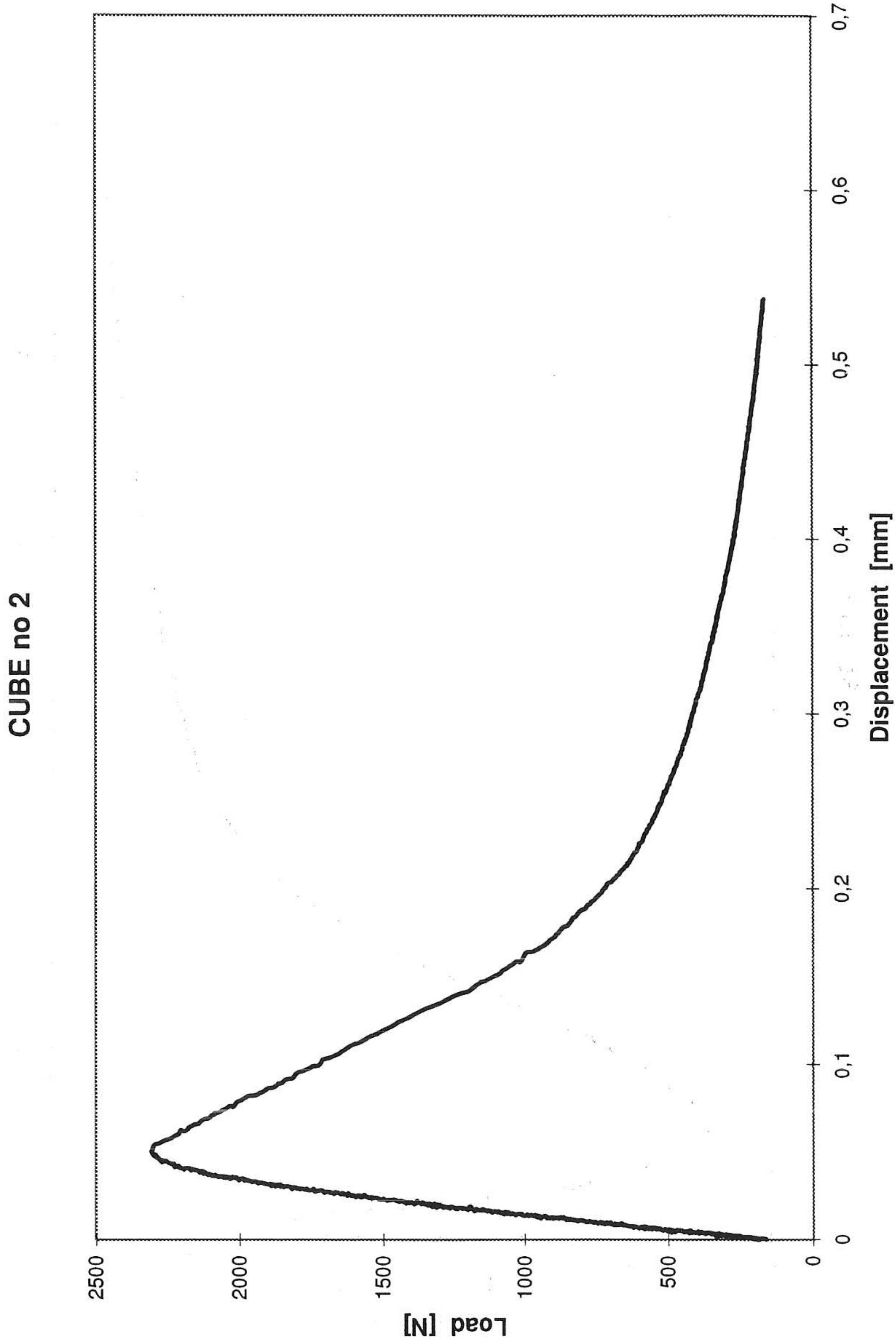
BEAM no 3



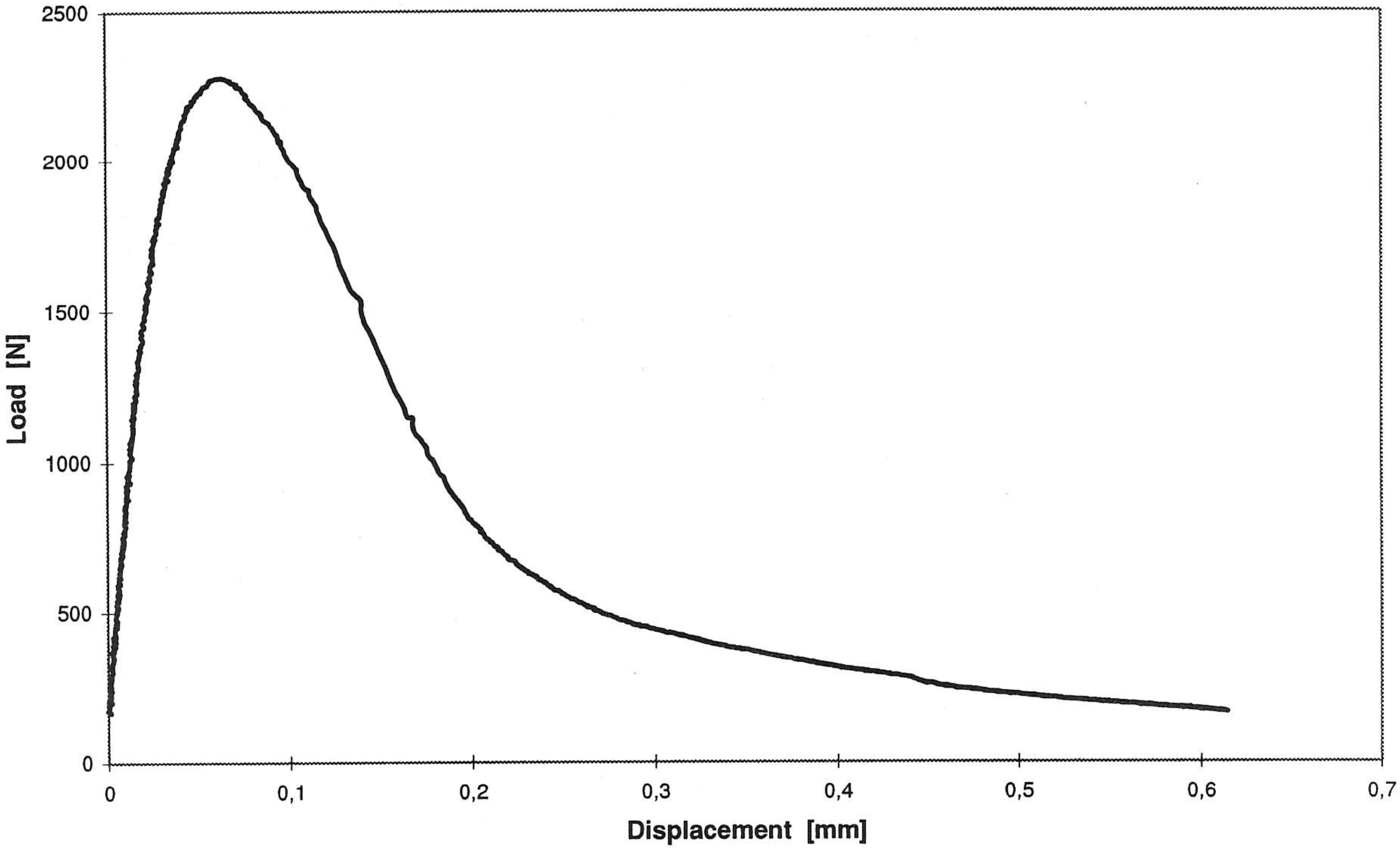
Load [kN]
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
Displacement [mm]

CUBE no 1


2500
2000
1500
1000
500
0
Load [N]
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
Displacement [mm]

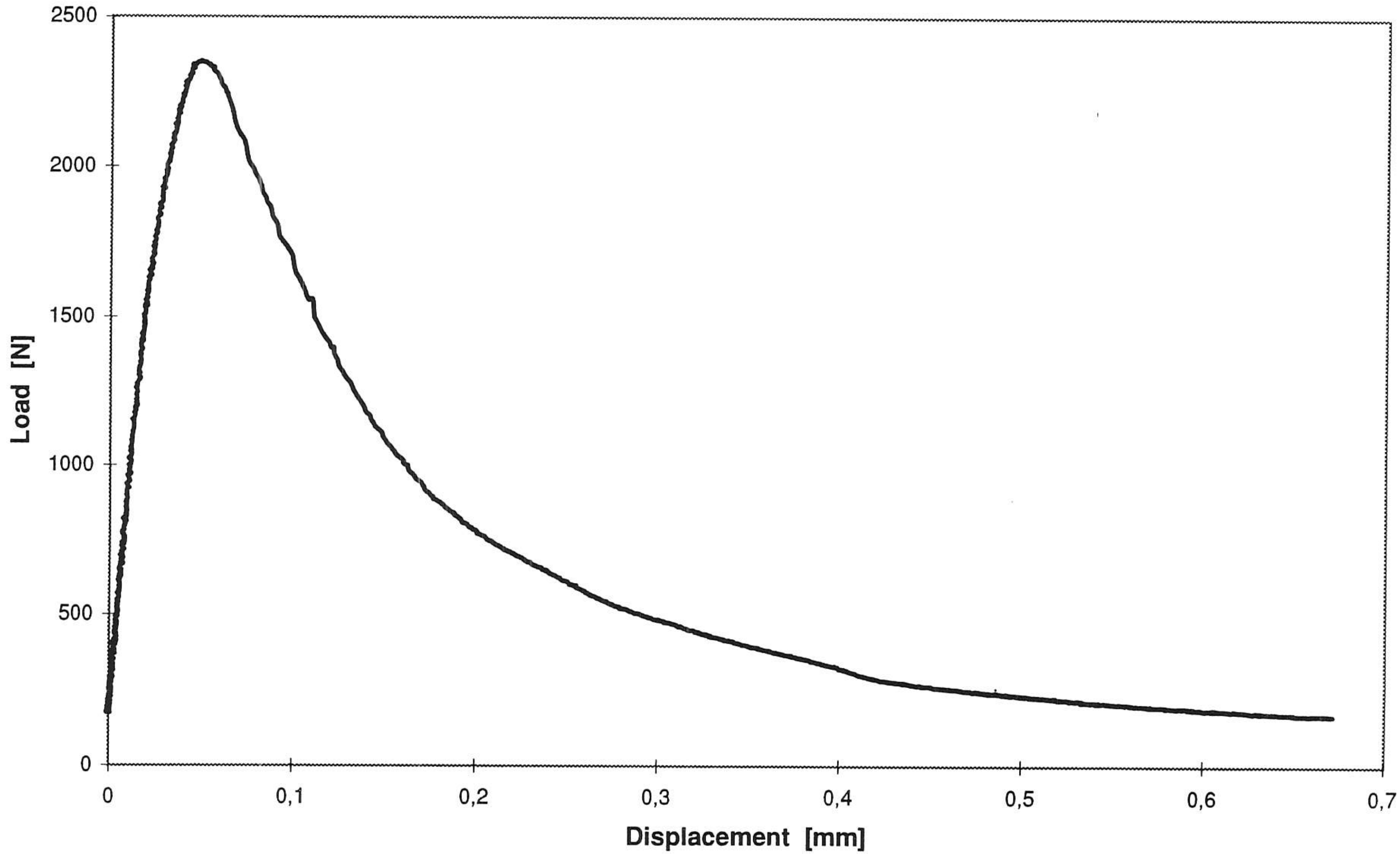

CUBE no 2
Load [N]
2500
2000
1500
1000
500
0
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
Displacement [mm]

CUBE no 3



Load [N]
2500
2000
1500
1000
500
0
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
Displacement [mm]

CUBE no 4



Load [N]
2500
2000
1500
1000
500
0
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
Displacement [mm]

Appendix D, results of SP

Manouchehr Hassanzadeh

Determination of the fracture energy of concrete: A comparison of the three-point bend test on notched beams and the wedge-splitting test

SP-AR
Byggnadsteknik
Borås 1997

Contents

Preface

This "NORDTEST" project was started in January 1997. The objective of the project was to compare two test methods which are used to determine the fracture energy of concrete. The test methods were:

- three-point bend test on notched beam, and
- wedge-splitting test

The first method is recommended by the RILEM Technical Committee 50-FMC. The second method, previously proposed by Linsbauer and Tschegg, has been used by Brühwiler et al. as an alternative method for determination of fracture energy.

Three departments participated in this project: the Department of Building Technology at the Swedish National Testing and Research Institute (SP), the Department of Cement and Concrete at SINTEF Civil and Environmental Engineering - Norway, and the Department of Structural Engineering and Materials at the Technical University of Denmark (DTU).

This report presents the results of the tests performed at the Swedish National Testing and Research Institute.

1 Introduction

The results presented in this report reflect the tests performed at the Swedish National Testing and Research Institute. The tests were performed by means of two methods: the three-point bend test on notched beams (TPBT) and wedge-splitting test (WST). The specimens made and prepared in accordance with the requirements stated in the project application. This section briefly presents the test methods and the requirements.

1.1 Test methods

1.1.1 Three-point bend test, RILEM method

The RILEM Technical Committee 50-FMC proposed a draft recommendation to measure the fracture energy, G_F *N/m*, of concrete, mortar and similar materials. In this method the fracture energy is measured by means of a displacement-controlled stable three-point bend test on a central notched beam, Figure 1. The fracture energy is calculated by means of equation 1 using the load-displacement curve obtained from the test, Figure 2.

$$G_F = (W_0 + (m_1 + 2m_2) g \delta_0) / (b(d - a)) \qquad N/m \qquad (1)$$

W_0 is area according to Figure 2 (*N/m*). δ_0 is displacement at the final failure of the beam (*m*). m_1 is weight of the beam between the supports (*kg*), calculated as the beam weight multiplied by *l/L*. *l* is the distance between the supports (span) and *L* is the beam length. m_2 is weight of the part of the loading arrangement which is not attached to the testing machine, but follows the beam until failure (*kg*). *g* is acceleration due to gravity, 9,81 m/s^2. *b*, *d* and *a* are defined in Figure 1.

The notch depth (*a*) is equal to half the beam depth. The size of beam (*l*, *d*, and *b*) depends on the maximum size of the aggregate (D_{max}). *l*, *d* and *b* are 0,8, 0,1 and 0,1 *m* when $D_{max} \leq 16$ *mm*; and 1,13, 0,2 and 0,1 *m* when 16 *mm* $< D_{max} \leq 32$ *mm*.

The weight of the beam increases with increased D_{max}. In the abovementioned cases, the weights of the beams are approximately 20 and 54 *kg*. As can be observed the difficulty of handling the beams increases with increased D_{max}.

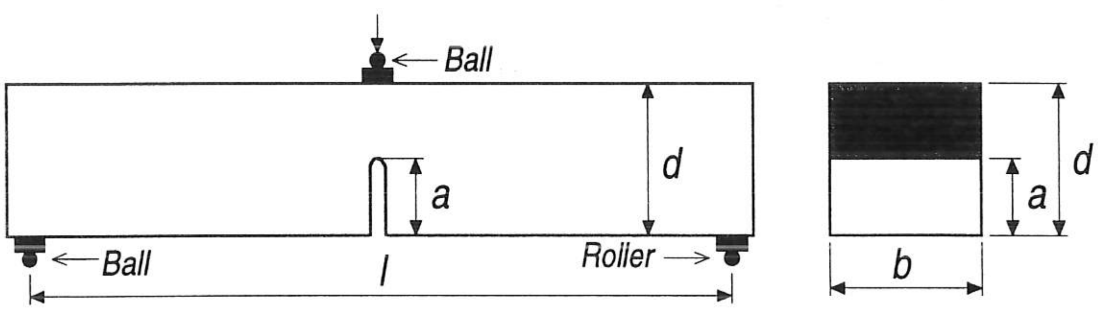


Figure 1 Determination of fracture energy by means of a three-point bend test.

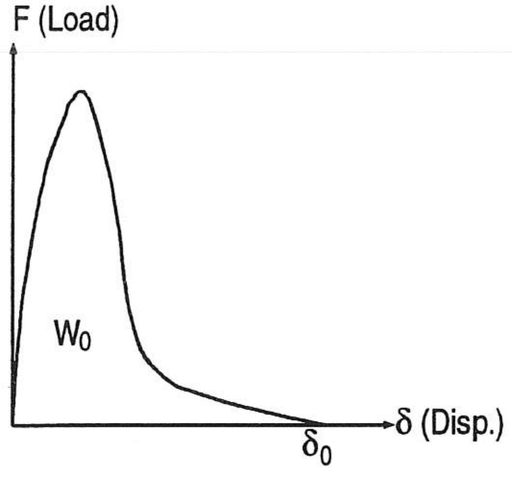


Figure 2 *Load-displacement curve.*

1.1.2 Wedge-splitting test

The wedge-splitting test is an alternative method for determination of the fracture energy. The principle of the wedge-splitting test is schematically presented in Figure 3. A groove and a notch are introduced into the specimen. The specimen is placed on a line support fixed to the lower plate of the testing machine, Figure 3a. Two massive steel loading devices equipped with roller or needle bearings on each side are placed on the top of the specimen, Figure 3b. A steel profile with two identical wedges is fixed at the upper plate of the testing machine. The actuator of the testing machine is moved so that the wedge enters between the bearings, which results in a horizontal splitting force component, Figure 3c.

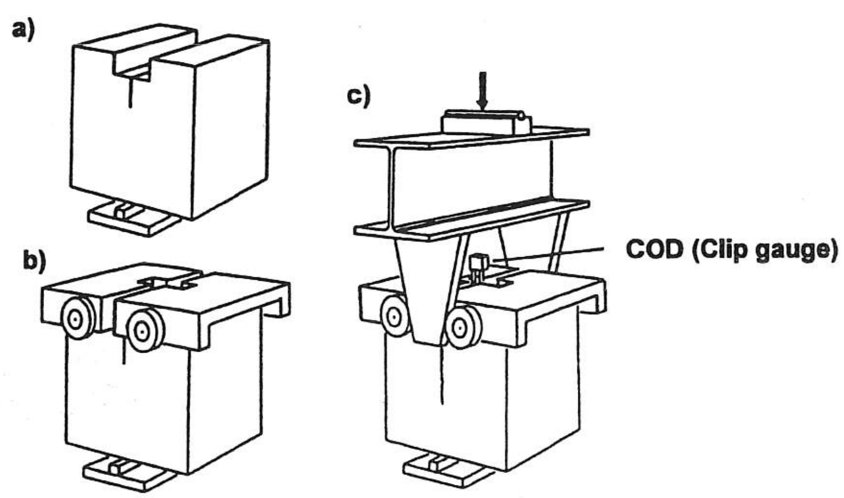


Figure 3 *Principle of the wedge-splitting test: (a) test specimen on a linear support; (b) placing of two loading devices with rollers; (c) the wedges are pressed between rollers in order to split the specimen into two halves.*

The test is performed in displacement-controlled mode. The load in the vertical direction, F_v, and the crack opening displacement, COD, are monitored during testing. The COD is measured by means of a transducer or a clip gauge at the level where the splitting force acts on the specimen, i.e. at the axles of the rollers. The horizontal component of the force acting on the bearings, F_s, can be calculated by means of equation 2.

$$F_s = \frac{F_v}{2\tan\alpha} \cdot \frac{1 - f\tan\alpha}{1 + f\cot\alpha} \approx \frac{F_v}{2(1 + f\cot\alpha)\tan\alpha} \qquad (2a)$$

$$F_s \approx \frac{F_v}{2\tan\alpha} \qquad f\tan\alpha << 1 \qquad (2b)$$

The area under the F_s - COD curve is the fracture energy of the specimen (W_0, Nm), Figure 4. The fracture energy of the material (G_F) is obtained by dividing the W_0 by the area of the ligament.

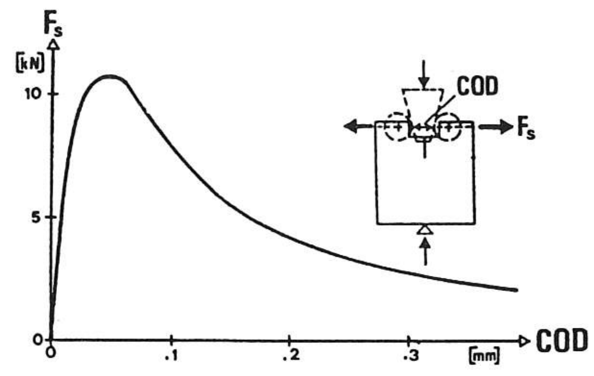


Figure 4 *The F_s - COD curve.*

1.2 Concrete mix

The fracture energies of three different concrete mixes with different water-cement ratios were determined using both methods. Each laboratory tested one concrete mix. The water-cement ratios which were used are 0,40 (SP), 0,45 (DTU) and 0,50 (SINTEF).

Each laboratory manufactured its own specimens from locally available materials in accordance with following requirements:

- the cement shall be ordinary Portland cement
- the aggregate shall be composed of 50% particles < 8 mm and 50% particles > 8 mm
- the maximum aggregate size shall be 16 mm, and
- the water content should be such that the slump of the mixes falls between 50 and 100 mm.

1.3 Tests and specimens

The fracture energy of each concrete mixes was determined by means of four bending tests and four wedge-splitting tests. The specimens were demoulded the day after casting and stored in water until the time of testing, 28 days after casting.

The required beam size, i.e. span, depth and width, was 0,8 *m*, 0,1*m* and 0,1 *m*. The required notch depth was equal to half the beam depth.

The required shape of the WST specimen was cubic with 0,1 *m* edge size. The required ligament depth and width were 0,05 *m* and 0,1 *m*, i.e. the same as the beam.

The loading rate was chosen so that the maximum load was reached within about 30 - 60 seconds.

2 Tests performed at the Swedish National Testing and Research Institute

2.1 Preparation of specimens

A total of 4 beams (1 *m* x 0,1 *m* x 0,1 *m*) , 7 cubes (0,1 *m* x 0,1 *m* x 0,1 *m*) and 4 cubes (0,15 *m* x 0,15 *m* x 0,15 *m*) were casted. The specimens were demoulded a day after casting and stored in water. Except for the 0,15 *m* cubes all specimens remained in water until the day of testing. All specimens were tested at either 28 or 29 days of age. Five days after casting, the 0,15 *m* cubes were taken out of the water and kept in a normal laboratory climate.

The concrete mix contained 401 kg/m^3 ordinary Portland cement (Slite Standard), 161 kg/m^3 water, 1828 kg/m^3 aggregates and 10 kg/m^3 plasticizer (Melcrete 35% dry substance). The aggregate batch was composed of 797 kg/m^3 sand and 1031 kg/m^3 crushed aggregates, i.e. the batch contained by weight 50% 0 - 8 mm particles and 50% 8 - 16 mm particles.

The batch volume was 0,1 m^3 and were mixed in a mixer with 0,25 m^3 capacity. The air content, the density and the slump of the mix were 1,4%, 2423 kg/m^3 and 85 *mm*.

In order to establish the quality of the hardened concrete its compressive strength was also determined. This was done by means of 0,1 *m* cubes and 0,15 *m* cubes, which indicated a compressive strength of the order of 71,6 *MPa* (std = 0,6 *MPa*) and 75,7 *MPa* (std = 0,4 *MPa*), respectively.

The notches and the grooves were made by means of a diamond saw (thickness = 4 *mm*) a week before testing. The depth of the notches was approximately 50 *mm* in beams and 25 *mm* in cubes. The depth and the width of the grooves were approximately 25 *mm*.

2.2 Measurements

All tests corresponding to the measurements of the G_F were performed with a servo-hydraulic closed-loop testing machine. The load capacity of the machine was 100 *kN*. The load was measured on a scale of 10 *kN*.

The displacement was measured by means of Linear Variable Differential Transducers (LVDT), with ±2,5 mm measuring range.

In the bending tests, the load-point displacement was measured as the midpoint deflection of the beam axle. The deflection was measured by means of two LVDTs, which were placed at midpoint, on each side of the beam. The displacements corresponding to the load-displacement curves which are presented in this report are mean values of two LVDTs.

In the wedge-splitting tests the displacement, COD, was measured by means of a single LVDT. Since the measuring device was not a clip-gauge, an additional groove was made perpendicular to the main groove. The depth and the width of the additional groove were 15 *mm*. The LVDT was placed inside the groove at the level where the splitting force acts on the specimen, i.e. at same level as the axles of the rollers. The LVDT measured the displacements of the axles of the pair of rollers on one side of the notch relative to the displacements of the axles of the other pair of rollers on the other side of the notch.

All tests were performed in displacement-control, with LVDT measurements as the controlling parameter. The rate of displacement was chosen so that the maximum load was achieved within 30 - 60 seconds.

The weight of the specimens and size of the ligament have been measured.

2.3 Results of the beam tests, RILEM method

The load-displacement curves of the tests and corresponding measurements are presented in Figure 5 and Table 1.

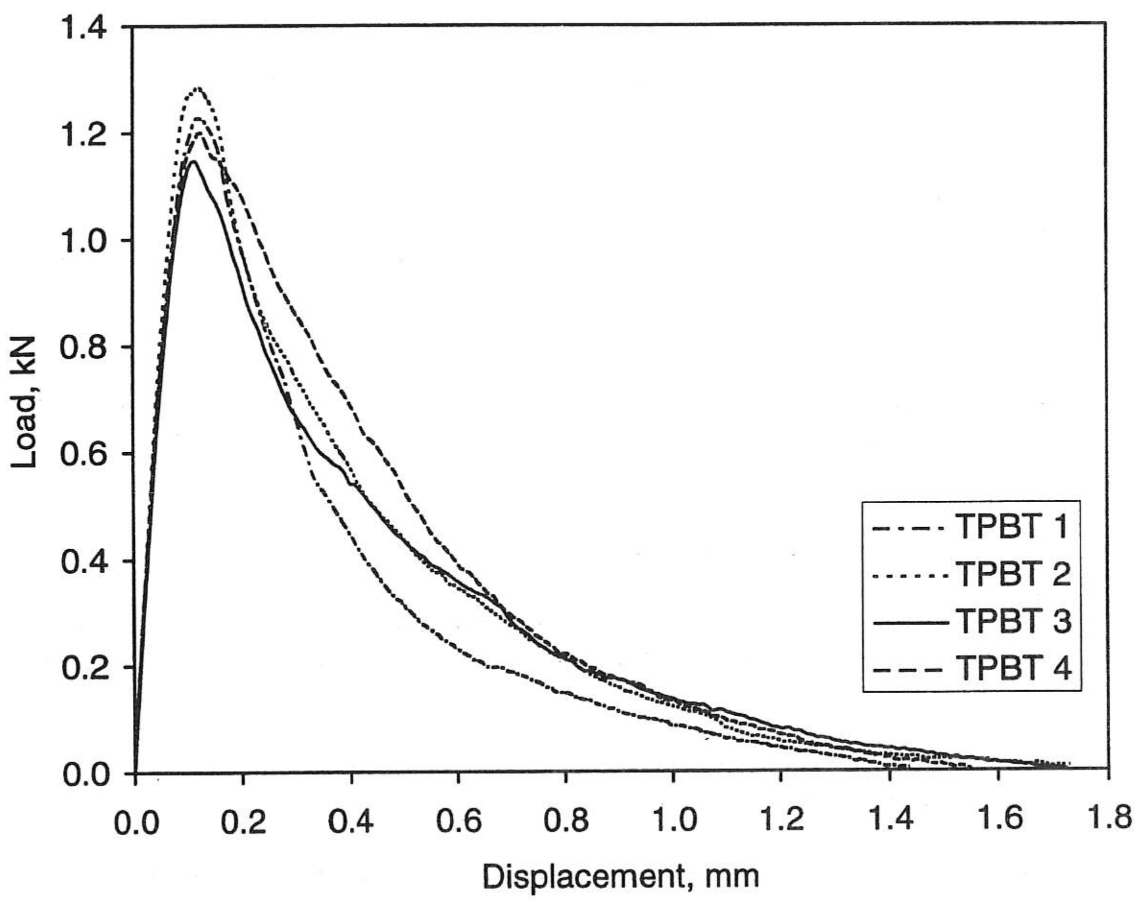


Figure 5 Load-displacement curve. TPBT according to the RILEM.

Table 1 *Test results of the three-point bend tests on notched beams. M is the total weight of the beam. The remaining parameters are defined in Figures 1 and 2.*

Specimen	M kg	b mm	$d\text{-}a$ mm	δ_0 mm	F_{max} N	W_0 Nmm	G_F N/m
TPBT1	24,271	101,8	53,2	1,434	1228	462,8	134,0
TPBT2	24,115	101,5	53,3	1,727	1284	549,9	159,8
TPBT3	24,022	101,4	51,6	1,723	1148	533,8	161,8
TPBT4	23,291	100,6	53,9	1,548	1200	595,0	160,0
					G_F (mean) ⇒		153,9
					Standard div. ⇒		13,3

The G_F is evaluated by means of equation 1. The weight of the beam between the supports, m_1, is *0,80xM*. The parts of the beam which are located outside of the supports also influence the fracture energy. The G_F values in Table 1 are calculated with regard to this effect using *0,75xM* as the weight of the beam between the supports. In the presented tests m_2 is equal to 0,24 kg which was accounted for when calculating the G_F.

In these tests the average work per unit area of the ligament, conducted by m_1 and m_2 was 0,35G_F.

2.4 Results of the wedge-splitting tests

The load displacement curves of the tests and corresponding measurements are presented in *Figure* 6 and Table 2.

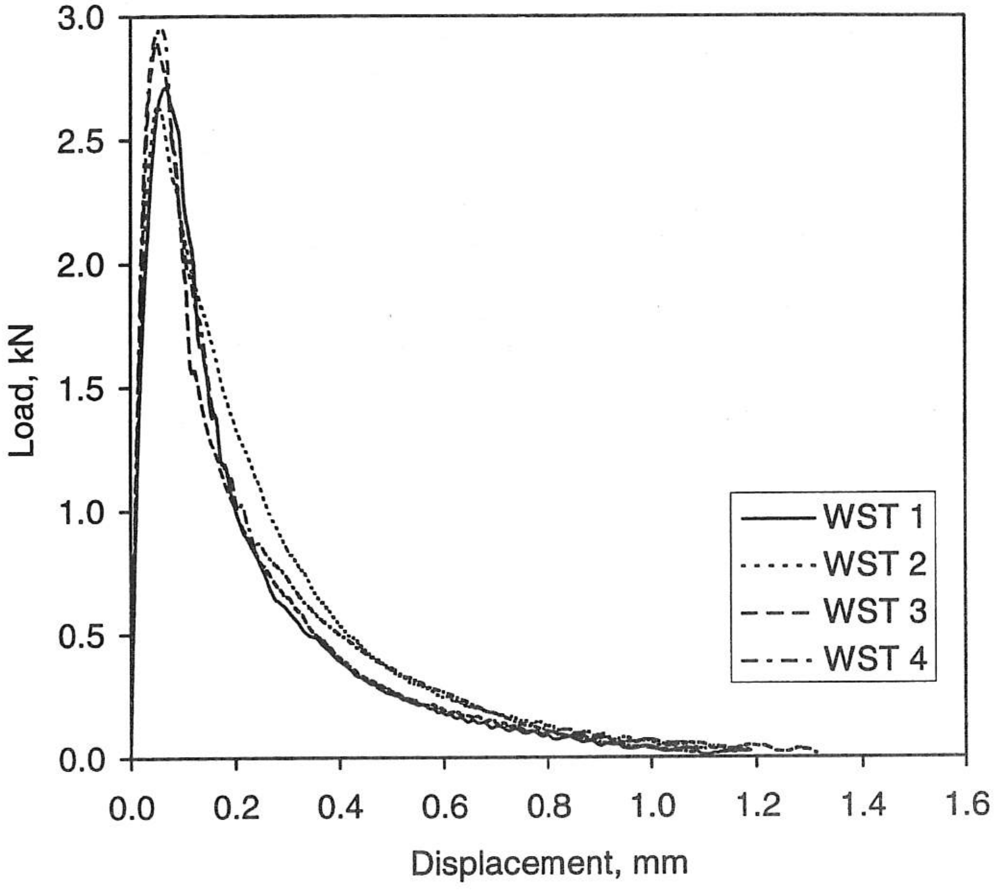


Figure 6 *Load-displacement curves (F_s - COD curves) of the wedge-splitting tests.*

F_s, the horizontal component of the vertical load, is calculated by means of equation 2a. The coefficient of friction (*f*) of the bearings has been determined. The coefficient is equal to 0,13.

The coefficient of friction has been determined by means of a vertical load - horizontal reaction diagram of the testing device, see Figure 7, and equation 2a. The test device was placed on a PTF (Poly Tetra Fluor Ethene) layer during testing.

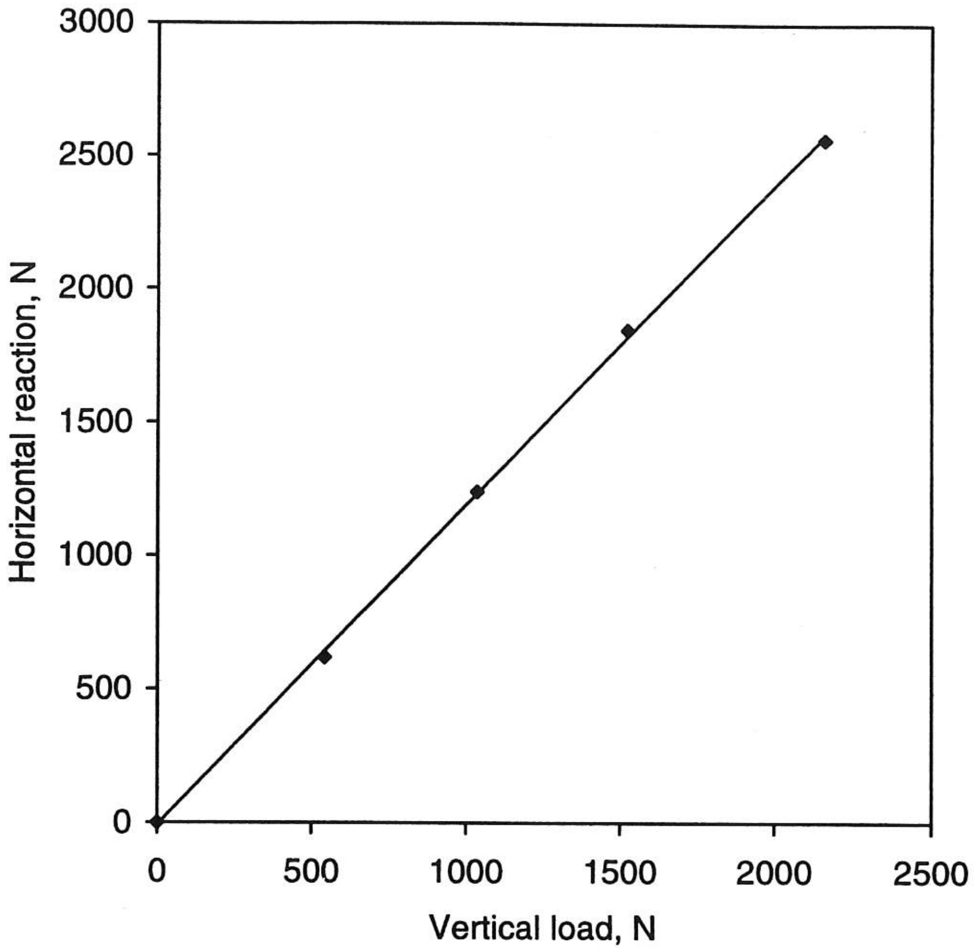


Figure 7 *Horizontal reaction versus vertical load.*

Table 2 *Test results of the wedge-splitting tests. m_2 is weight of the part of the loading device which is not attached to the testing machine. b and (d-a) are the width and the depth of the ligament respectively. The remaining parameters are according to the Figure 2.*

Specimen	m_2 kg	b mm	d-a mm	δ_0 mm	F_{max} N	W_0 Nmm	G_F N/m
WST1	5,368	99,4	53,5	1,183	2476	610,4	142,9
WST2	5,368	99,6	52,4	1,187	2635	693,1	161,5
WST3	5,368	99,3	53,6	1,314	2892	603,4	144,5
WST4	5,368	100,0	52,4	1,082	2949	656,8	151,4
					G_F (mean) ⇒		150,1
					Standard div. ⇒		8,5

The G_F values presented in Table 1 include m_2. In the WST the contribution of m_2 to the fracture energy is calculated in the same way as the TPBT. The G_F is calculated by means of following equation:

$$G_F = \frac{W_0 + 2\alpha m_2 g \delta_0}{b(d-a)} \qquad \alpha = \frac{F_s}{F_v} \qquad (3)$$

W_0 is the area under the F_s - *COD* curve. $b(d\text{-}a)$ is the area of the ligament. b and d are the width and the depth of the specimen respectively. a includes the depth of the groove and the notch.

In these tests the average work per unit area of the ligament conducted by m_2 was $0{,}19G_F$.

3 Concluding remarks

The G_F of the present concrete is 153,9 N/m (std = 13,3 N/m) and 150,1 N/m (std = 8,5 N/m) according to the TPBT and WST respectively. According to these tests, both methods show similar results.

Our experiences during this project are as follows:

- The requirements regarding the stiffness and performance of the testing machine are more severe in the WST than the TPBT.

- WST specimens are smaller and lighter, which is advantageous with regard to storage and handling.

- The grooves in the WST specimens in these tests were saw cut, which requires approximately 15 minutes additional preparation time for each specimen.

Appendix E, statistical analysis

This appendix describes the procedure of the calculation of significance level on the basis of random sampling hypothesis, Box et. al. 1978.

Table E1 shows the results of the fracture energy measurements which are reported by the laboratories. The total number of tests were 12 and 13 in the case of TPBT (method A) and WST (method B) respectively. Since in this type of analysis requires an equal number of samples a fictive sample is added to the column corresponding to method A. The raw number of this sample is 9, which was originally empty. The value of this sample is the mean value of raws 6, 7, and 8. This measure has no influence on the mean value. Furthermore, its influence on the standard error is negligible.

Table E1. Results of fracture energy measurements, original results.

Number	Laboratory	G_F, N/m	
		Method A TPBT	Method B WST
1	DTU	108,7	58,7
2	DTU	116,7	72,8
3	DTU	110,0	78,4
4	DTU	137,8	83,0
5	DTU	131,5	80,1
6	SINTEF	127,1	101,4
7	SINTEF	104,6	81,2
8	SINTEF	71,7	92,6
9	SINTEF	101,1	91,5
10	SP	134,0	142,9
11	SP	159,8	161,5
12	SP	161,8	144,5
13	SP	160,0	151,3

Number of samples $n_A = 13$, $n_B = 13$

Degree of freedom $v = n_A + n_B - 2 = 13 + 13 - 2 = 24$

Sum = $\Sigma y_A = 1624{,}8$
Average = $Y_A = 125{,}0$
Difference = $Y_A - Y_B = 21{,}9$

Sum = $\Sigma y_B = 1339{,}9$
Average = $Y_B = 103{,}1$

$S_A = \Sigma(y_A)^2 - (\Sigma y_A)^2/n_A = 8468{,}0$

$S_B = \Sigma(y_B)^2 - (\Sigma y_B)^2/n_B = 14191{,}7$

Pooled estimate of σ^2 $\quad s^2 = (S_A + S_B)/(n_A + n_B - 2) = 944{,}2$

Estimated variance of $Y_A - Y_B$ $\quad s^2(1/n_A + 1/n_B) = 145{,}3$

Estimated standard error of $Y_A - Y_B$ $\quad s = 12{,}1$