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The SP Josephson Array Voltage Standard
Abstract

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The Swedish unit for DC voltage is realized from the frequency unit and the internationally accepted value for the Josephson constant $K_{190}$ by means of a Josephson array voltage standard. The uncertainty in the realization at the 1 volt level is $\pm 5$ nV ($k=2$).

Keywords: Superconductivity, Josephson array, voltage standard, voltage unit
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Preface

The SP Josephson array voltage standard is mainly built-up of commercially available equipment as gunn oscillator, digital voltmeter, computer etc. The most important part however, the different Josephson arrays we are using, have been provided by NIST (National Institute of Standards and Technology in USA) and PTB (Physikalisch-Technische Bundesanstalt in Germany). We would like to thank the following persons who have given contributions to our work in setting up the SP Josephson array voltage standard or for participating in the Josephson intercomparisons.

Dr. Clark A. Hamilton for the array provided by NIST and also for the very useful NIST-volt software.

Dr. Jürgen Niemeyer for providing the PTB arrays and for encouragement and helpful advices.

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At SP;

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Borås, November 1995

Gunnar Eklund and Hannu Pajander
Summary

The SP Josephson array voltage standard has since 1993 been used to realize the Swedish unit of DC voltage.

This report describes the voltage standard and how the Josephson voltage is used to determine the voltage of primary voltage standards as Weston cells.

The Josephson voltage is known with a very high accuracy but different sources of errors reduces the accuracy by which an unknown voltage can be determined. These error sources are investigated and summarized in an error budget. The total uncertainty at the 1 volt level is ±5 nV (k=2) for a Weston cell calibration.

The voltage realized by the SP Josephson array voltage standard has been compared to the voltage realized by the formerly used single junction Josephson voltage standard by means of the maintained Swedish voltage unit. The comparison showed an agreement within the uncertainty of the voltage realized by the single junction standard.

Josephson intercomparisons with two other national laboratories, Danish Institute of Fundamental Metrology in Denmark and Physikalisch-Technische Bundesanstalt in Germany have been performed and are reported. Both Josephson intercomparisons showed an agreement which is within the uncertainty of the realized Josephson voltages.
1 Introduction

This report gives a description of the Josephson array voltage standard in present use at SP. A short recapitulation of the physics and technique behind the Josephson voltage standard is given and how the SP Josephson array voltage standard is used to determine a traceable "Josephson voltage" of different types of voltage standards.

A discussion of different error sources which influence the accuracy for the Josephson calibration of a voltage standard and a complete error budget is given.

Measurement results including a comparison of the Swedish voltage given by the formerly used single junction Josephson standard and the voltage given by the Josephson array voltage standard are reported.

An important test of a Josephson voltage standard is to compare the realized voltage with the voltage realized by another Josephson voltage standard. Two such intercomparisons have been performed and are reported.
2 Theoretic background

2.1 Introduction

In this chapter a short recapitulation of the physics and technique behind the Josephson voltage standard is given and the defined traceability of the "Josephson volt".

2.2 The Josephson voltage standard

In 1962 Brian Josephson discovered the Josephson effect. He derived an equation (2.1) for the current that would flow through a tunnel junction formed by a thin insulating barrier separating two superconductors.

\[ I = I_0 \sin \left( \frac{4\pi e V}{h} \right) \int V \, dt \]  

(2.1)

In this equation, \( I \) is the current through the junction, \( I_0 \) is a constant (the critical current) of the junction and \( V \) is the junction voltage. The ratio of the electron charge to Planck's constant (e/h) is a fundamental constant in the equation.

If we apply a dc voltage across the junction as in figure 2.1 equation 2.1 shows that the current will oscillate at a frequency given by equation 2.2.

![A Josephson junction with an applied dc voltage V.](image)

Figure 2.1. A Josephson junction with an applied dc voltage V.
\[ f = \frac{2eV}{h} \]  \hspace{1cm} (2.2)

Now, if we apply an ac current to the junction the junction oscillation can phase lock to the applied frequency. If we have a phase lock between the two oscillations, the junction voltage \( V \) is given by equation 2.3. This is the ac Josephson effect.

\[ V = \frac{hf}{2e} \]  \hspace{1cm} (2.3)

It is also possible for the junction to phase lock to harmonics of the frequency \( f \). This will result in a series of voltages given by equation 2.4, where \( n \) is an integer. Figure 2.2 shows the I-V curve if we apply a microwave radiation of frequency \( f \) to the junction. Induced constant voltage steps appear corresponding to equation 2.4.

\[ V = \frac{n hf}{2e} \hspace{1cm} (n = 1,2,3,...) \]  \hspace{1cm} (2.4)

---

**Figure 2.2.** Current - voltage curve for a Josephson junction with applied microwave radiation.
A Josephson junction as in figure 2.1 can be used as a voltage standard. The junction is a frequency to voltage converter. The voltage is determined from the frequency and the fundamental constants in equation 2.3. For a single junction Josephson standard a bias current corresponding to a step number of approximately 100 is used. With a frequency of 10 GHz it is possible to generate a voltage of a few mV. This voltage can be used in a voltage divider to reach the one volt level. The use of a single junction Josephson standard is difficult due to the complexity given by the use of a cryogenic voltage divider, cryogenic current comparator and a squid. The voltage divider limits the accuracy to perhaps ±0.05 ppm at the one volt level.

2.3 The Josephson array

It is possible but very difficult to connect many single Josephson standards in series to generate one volt. The reason is that each junction needs a separate bias current supply and individual adjustments. A series of 100 junctions would be extremely difficult to use. Another solution to this problem was suggested in 1977 [1] which started the development of the Josephson array.

In a single Josephson voltage standard the junction is used with a bias current and a high Josephson voltage step number. If no current is applied through the junction zero-current voltage steps will appear in the I-V characteristic of the junction if a number of physical parameters of the junction are within certain limits. A very low microwave power irradiating the junction is also necessary to generate zero-current voltage steps. Figure 2.3 shows the I-V curve of a Josephson junction with zero current voltage steps.

![Figure 2.3. Zero current Josephson voltage steps in the current - voltage curve for a Josephson junction with applied microwave radiation.](image-url)
The zero-current steps make it possible to connect a large number of Josephson junctions in series. There is no need for an individual bias current supply because of the zero-current. The number of junctions connected in series to produce one volt needs to be very high because to maintain the zero-current condition the step number of each junction must be low. A microwave frequency of 70 GHz corresponds to a step voltage of 145 µV and a step number of three or four for each Josephson junction will produce one volt if approximately 2000 junctions are connected in series.

A Josephson series array voltage standard requires a circuit design which can deliver nearly uniform microwave power to all junctions in the array and the junction parameters must be kept within certain limits during the fabricating process [2], [3], [4], [5]. However, during the 1980s the development of useful Josephson series arrays started at National Institute of Standards and Technology (NIST) and Physikalisch-Technische Bundesanstalt (PTB). Today not only 1 volt arrays but also arrays capable of producing voltages over 10 volts have been developed. A 10 volt array consists of 20000 or more Josephson junctions.

![Josephson array diagram](image)

Figure 2.4. A typical Josephson array design. The superconducting parts are made of lead and/or niobium.
2.4 Traceability

The traceability for the Josephson volt is from the known microwave frequency irradiating the Josephson junctions and the values of Planck's constant \( h \) and the elementary charge \( e \). Equation 2.4 can be written as equation 2.5.

\[
V_J = \frac{n f}{K_{J-90}} \quad (n = 1, 2, 3, \ldots) \tag{2.5}
\]

\[
K_{J-90} = \frac{2e}{h} \tag{2.6}
\]

\[
K_{J-90} = 483 \, 597.9 \, \text{GHz} / V
\]

The Josephson constant \( K_{J-90} \) is determined from the fundamental constants \( h \) and \( e \). The numerical value of \( K_{J-90} \) is known with an accuracy of ±0.4 ppm \( (k=1) \). This value has been accepted internationally and is used by all laboratories around the world. The uncertainty of \( K_{J-90} \) is omitted when the Josephson volt is used in normal calibrations. It should however be remembered, that when the Josephson volt is used as the SI volt the above uncertainty for \( K_{J-90} \) must be added to the total uncertainty.
3 The SP Josephson array voltage standard

3.1 Introduction

In this chapter we give a description of the Josephson array voltage standard in use at SP and how measurements are performed.

3.2 Description of the Josephson array voltage standard

The Josephson standard consists of five main parts. Figure 3.1 shows the Josephson standard and the used equipment is listed in Table 3.1. The cryogenic part consists of a liquid helium dewar and a cryoprobe. The Josephson array is mounted on the lower end of the cryoprobe and immersed below the liquid helium surface during measurements. The array is connected to the room temperature equipment with three pairs of wires and a waveguide.

The microwave radiation needed for the Josephson array to produce voltage steps is generated in the microwave part of the standard. A Gunn oscillator with a tunable output frequency of 70-74 GHz is stabilized in a phase locking loop. The stability of the locked output frequency is better than ±70 Hz. The microwave counter is locked to the calibrated frequency standard giving the traceability for the Josephson voltage. The microwave power is delivered to the Josephson array using the waveguide in the cryoprobe.

A current bias source (DC source) together with an oscilloscope and a digital voltmeter (DVM) is the next part of the Josephson standard. The DC source is connected to the array with a pair of wires. The output from the DC source is the sum of dc offset and a trianglewave sweep. The oscilloscope measures the bias current on the x-axis and the Josephson array voltage via a pair of wires, on the y-axis. All these connections to the array are made via filters to avoid interference. The digital voltmeter (DVM) measures the Josephson array voltage. The DVM is used to choose a voltage step close to the voltage of the standard to be calibrated.
Figure 3.1. The SP Josephson array voltage standard.
The voltage standard to be compared with the Josephson voltage is connected to the measurement part of the Josephson standard. This part consists of a reversing switch for elimination of thermal voltages and a nV-meter (resolution 1 nV) as null detector. The nV-meter is the DVM connected in this position during calibration measurements. A pair of wires is connected to the Josephson array via filters.

The remaining part is a computer that is reading the frequency counter and the nV-meter. The computer is storing all measurement data and performing the calculations. Table 3.1 gives the equipment used in the SP Josephson array voltage standard and Figure 3.2 shows a photograph of the apparatus.

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<td>Mixer</td>
<td>Hughes</td>
<td>43355H-1220</td>
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<td>Reversing switch</td>
<td>Guildline</td>
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<td>Digital voltmeter</td>
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<td>Keithley</td>
<td>182</td>
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<td>Cryoprobe</td>
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<tr>
<td>Dewar</td>
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<td>MSB60 2&quot;</td>
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Table 3.1. The SP Josephson array voltage standard equipment.
Figure 3.2. A photograph of the SP Josephson array voltage standard. The liquid helium dewar with cryoprobe can be seen to the right and the computer to the left. The rack in the middle contains most of the electronics.

3.3 **Comparison of a voltage standard with the Josephson array voltage standard**

The traceability for the realized Josephson voltage is given from the microwave frequency. When comparing a voltage standard with the Josephson array voltage standard the first step is to calibrate the external frequency standard for the microwave counter.

The next step is to adjust the current source, the microwave frequency and the power to obtain a stable Josephson voltage step at a level close to the voltage of the standard to be measured. Figure 3.3 shows the current - voltage characteristic of a Josephson array. From this oscilloscope picture the working condition and different array parameters can be observed.
Figure 3.3. The current - voltage characteristic of a Josephson array.

Figure 3.4. Oscilloscope picture of Josephson steps when the Josephson array is irradiated with microwave radiation.
When the Josephson array is irradiated with microwave radiation the expansion (1000 times) of the curve shows voltage steps. The picture of the voltage steps on the oscilloscope makes it possible to adjust all parameters to obtain a stable Josephson voltage. Figure 3.4 shows the oscilloscope picture of Josephson steps.

A single Josephson voltage step is found by adjusting the bias current, the microwave power and the frequency. In this process it is important to find a step which is stable and close to the voltage of the standard to be calibrated. If the voltage generated by the Josephson standard has a resistive component the oscilloscope will show a sloping step and the voltage is not a pure Josephson voltage. If the reason is a broken array or bad connection it will be impossible to find a horizontal step. Figure 3.5 shows a good Josephson voltage step.

Figure 3.5. Oscilloscope picture of a good Josephson voltage step.
The digital voltmeter (DVM) measures the Josephson voltage. The DVM is used to choose a voltage step close to the voltage of the standard to be measured. After the desired Josephson step is found the microwave frequency is locked to the frequency standard and the voltage standard is calibrated from the Josephson voltage using the standard reversing technique to eliminate thermal voltages. The nV-meter is connected to measure the residual voltage between the Josephson standard and the voltage standard. The nV-meter is not working as an absolute null detector because the frequency can only be locked in steps corresponding to 70 nV and it is often difficult to obtain and stay on the nearest Josephson voltage step. We allow a voltage difference of a few steps (≤ 1 mV) but if the highest accuracy is required the microwave frequency is tuned corresponding to a voltage reading on the nV-meter to be less than 70 nV. The nV-meter has a resolution of 1 nV in this mode, corresponding to an accuracy of 1 ppm at 1 mV level. The calibration of the voltage standard and the linearity of the nV-meter with this accuracy can be obtained by using the Josephson voltage.

A microwave frequency of 70 GHz corresponds to a voltage difference between adjacent Josephson voltage steps of 145 μV. In order to be able to calculate the step number of the Josephson voltage generated by the array, the voltage has to be measured with an accuracy of ±50 μV. In the normal procedure of measurement the voltage of the standard to be measured is known with an accuracy of ±10 μV or better. From this knowledge it is easy to calculate the correct step number when the standard is compared with the Josephson voltage. If the voltage of the standard is unknown the digital voltmeter (DVM) can be used to measure the voltage with an accuracy of ±50 μV if the voltage standard is a zener reference standard, but not in the case of a Weston cell due to the loading effect from the digital voltmeter. The calculation of the step number can also be done from the measured Josephson voltage with the calibrated digital voltmeter connected to the array.

The input to the computer before a calibration is the frequency of the frequency standard and the known voltage of the voltage standard if the voltage is known with sufficient accuracy. Alternatively the voltage is measured by the digital voltmeter which is read by the computer. In the calibration process the computer is reading the microwave counter and the nV-meter. Twelve readings is taken in each polarity. Each reading consists of ten individual readings of the digital voltmeter. The two readings that are most deviating from the mean are excluded resulting in ten readings forming the final mean value. The minus polarity reading consists of twenty readings resulting in the same number of individual readings of the nV-meter in plus and minus polarity. The final value of the voltage standard is calculated and presented in a computer printout as in Figure 3.6.
Swedish National Testing and Research Institute  
Electrical Metrology  
Boras, Sweden

Reference Standard Calibration:  ID # SPOG 2/4 ---- 1994-09-08 14:21:4

Comment: PTB GJT 5/5

System Digital Voltmeter: Keithley-182  Microwave Freq. = 70.4596627 GHz

Initial Ref Voltage = 1.01822400 V  Samples Per Calibration Point: 10

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*** AVERAGES ***  
Reference Standard Voltage = +1.018223594

Figure 3.6. Computer printout of a Josephson calibration of a voltage standard.
3.4 The Josephson array voltage standard used as a potentiometer

The possibility to generate any voltage quasi continuously between 0 volt and the maximum voltage for the used array makes the Josephson array voltage standard useful as an extremely linear potentiometer. The voltage can be chosen in steps of approximately 150 µV in the whole range.

If the linearity of a voltmeter is to be calibrated the digital voltmeter (DVM) in Figure 3.1 is replaced with the voltmeter to be calibrated. The Josephson voltage is then adjusted to different voltages and the voltmeter is measuring the array voltage. If the voltmeter reading is within ±50 µV, the Josephson voltage can be calculated and the voltmeter error is given from the difference in voltmeter reading and the known Josephson voltage. Each voltage is calibrated relative to the zero step and this eliminates the problem with thermal voltages between the voltmeter and the Josephson array. This linearity calibration can be done as a manual calibration or with computer reading if the voltmeter has an IEEE bus. Figure 3.7 shows a calibration of a few voltages for a voltmeter with the possibility of computer reading.
System DVM Calibration: Chip# P74 C77 5/10 --- 1994-01-19 14:16:4

Comment: Linearity HP3458 no 01201 8 dig. NELC=25

System Digital Voltmeter: HP-3458 Microwave Freq. = 70.8018019 GHZ

Samples Per Calibration Point: 20

Total Offset Voltage: +0.796 µV

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</table>

Figure 3.7. Computer printout of a Josephson linearity calibration of a voltmeter.
4 Sources of error

4.1 Introduction

In this chapter we discuss different factors which influence the accuracy for the Josephson calibration of a voltage standard and how these error sources can be determined or estimated. A complete error budget is given.

4.2 Calibration of the frequency standard and microwave stability

The microwave source is phase locked to the frequency standard when the Josephson volt is generated by the Josephson array. The accuracy by which the Josephson volt is realized is determined by the accuracy of the frequency standard and the stability of the microwave frequency.

The calibration of the frequency standard is performed as a phase difference measurement against one of SPs primary frequency standards (cesium beam frequency standard). Figure 4.1 shows the calibration of the frequency standard.

![Diagram of calibration process](image)

Figure 4.1. Calibration of the frequency standard.
The frequency standard has a good short term stability. Therefore it is only necessary to calibrate the frequency standard once a week during measurements. The short term behaviour of the frequency standard is plotted in Figure 4.2.

![Graph showing frequency deviation from nominal 10 MHz](image)

**Figure 4.2.** Frequency standard calibration. The driftrate corresponds to +0.12 nV/24 hours and the error bars to ±0.1 nV.

With a total measurement time of 30 minutes an accuracy of ±1 part in $10^{10}$ is obtained. The uncertainty contribution from the calibration and the short term (one week) stability of the frequency standard is estimated to ±1 part in $10^9$ (k=2).

The stability of the microwave output and the accuracy of the measurement of the microwave frequency will contribute to the total error. This error contribution is estimated to ±1 part in $10^9$ (k=2), corresponding to ±70 Hz, at normal measurements. However, with extra care it is possible to achieve a microwave stability of a few Hz corresponding to a negligible error. Figure 4.3 shows the optimum microwave stability.
Figure 4.3. Stability of the microwave frequency which is ±4 Hz corresponding to less than ±0.1 nV for the Josephson voltage.

4.3 Leakage conductance of the cryoprobe

When the Josephson voltage standard is used for calibration the Josephson voltage (U_J) is connected to the voltage standard (U_s) with wires from the array, through the filter resistors, the reversing switch and the nV-meter. Any leakage conductance to the surroundings can cause current to flow in the wires, filter resistors, nV-meter and internal resistance of the voltage standard resulting in voltage drops. To avoid this source of error the influence of the leakage conductance has to be measured to ensure that it is acceptably low and to estimate its contribution to the total uncertainty. In Figure 4.4 the situation is described with indications of possible leakage conductances (G_L).
Figure 4.4. Possible leakage conductances ($G_L$) in the Josephson array voltage standard.

The measurements of the influence from the leakage conductances are performed by connecting a floating voltage standard with a high resistance in series into the circuitry [3]. A voltmeter is measuring the voltage and the change of the voltage can be observed when different parts of the measurement circuitry is connected. The voltage will change due to the connected leakage conductance. The high series resistor ($1 \, \text{M}\Omega$) will magnify the voltage change compared with the normal measurement situation.

Figure 4.5 shows the measurement of the leakage conductance on the cryoprobe side of the Josephson voltage standard. Here we have possible leakage conductance in the insulation of the wires, the cryoprobe side of the reversing switch and the filter capacitors.

Figure 4.5. Measurement of leakage conductance on the cryoprobe side. The nV-meter is replaced by a short circuit. The array is disconnected leaving an open end.
The change in voltage is observed when the switch is in the positive position and in the negative position compared with the switch in the open position. Figure 4.6 shows the voltage before and after a change in switch position. The long time constant is due to the filter capacitances and the high series resistance (1 MΩ). This measurement is repeated with different ground connections. The maximum observed voltage change is used to calculate the error due to the leakage conductance.

![Graph showing voltage deviation over time](image)

Figure 4.6. Change in voltage due to leakage conductance in the cryoprobe. The voltage change is approximately 30 μV between the open position (0 V) and the positive (+) position (≈ -30 μV) of the reversing switch.

The observed change in voltage is 30 μV for a total lead resistance of 1 MΩ. However, in the normal connection the total lead resistance is 1.4 Ω which will reduce the change in voltage to

$$\Delta V = \frac{1.4 \Omega \times 30 \mu V}{1 \text{ MΩ}} \approx 0.05 \text{ nV} = \text{leakage conductance error}$$

A similar measurement of the leakage conductance is made on the voltage standard side. Figure 4.7 gives the measurement connection. In this case it is the voltage standard part of the reversing switch and the connecting leads to the voltage standard which contributes to the total leakage conductance.
Figure 4.7. Measurement of leakage conductance on the voltage standard side. The floating voltage is connected at the array connections and the nV- meter is replaced by a short circuit.

As in the previous measurement the change in voltage is observed when the switch is in the positive position and in the negative position compared with the switch in the open position. Figure 4.8 shows the voltage before and after a change in the switch position. This measurement is repeated with different ground connections. The maximum observed voltage change is used to calculate the error due to the leakage conductance.

Figure 4.8. The change in voltage due to leakage conductance on the voltage standard side. The voltage change is approximately 5 µV between the open position and the positive (+) position (between the two reversing spikes) of the reversing switch.
In this case the change in voltage is 5 μV for a total lead resistance of 10 MΩ. In the normal connection the total lead resistance is 0.1 Ω but the internal resistance is 1 kΩ for a zener reference standard and 500 Ω for a Weston cell. In the case of a zener reference standard the change in voltage is

\[
\Delta V = \frac{1 \text{ kΩ} \times 5 \text{ μV}}{10 \text{ MΩ}} = 0.5 \text{ nV} = \text{leakage conductance error}
\]

4.4 Thermal voltages in the reversing switch

When the Josephson voltage standard is used for calibration the measurements are made with the standard technique of polarity reversing to eliminate thermal voltages. However, with this method the changing thermal voltages in the reversing switch, due to different connections, is not eliminated and will contribute to the total error. It is therefore necessary to measure and estimate the maximum error contribution from these changing thermal voltages. Figure 4.9 shows the different thermal voltages in the reversing switch.

Figure 4.9. Thermal voltages between the Josephson array and the voltage standard with the reversing switch in the positive and the negative polarity.
The voltages in the positive polarity are $U_{SW1,4}$ and in negative polarity $U'_{SW1,4}$.
$U_{Tj}$ is the constant thermal voltage on the cryoprobe side and $U_{TS}$ is the constant thermal voltage on the voltage standard side.

The sum of the thermal voltages in the positive and the negative polarity can be written as $U_{SW1}$ and $U_{SW2}$ giving the simplified picture as in Figure 4.10.

Figure 4.10. Thermal voltages between the Josephson array and the voltage standard. The sum of the thermal voltages is $U_{SW1}$ and $U_{SW2}$.

From Figure 4.10 we obtain equations 4.1 and 4.2.

\[ U_{J1} + (U_{Tj} + U_{SW1}) + V_1 = (U_S + U_{TS}) \]  \hspace{1cm} (4.1)

\[ U_{J2} - (U_{Tj} + U_{SW2}) - V_2 = (U_S + U_{TS}) \]  \hspace{1cm} (4.2)

If $U_{SW1} \neq U_{SW2}$ we can write the difference as in equation 4.3.

\[ U_{SW2} = U_{SW1} + \Delta U_{SW} \]  \hspace{1cm} (4.3)
From equations 4.1, 4.2 and 4.3 we obtain equation 4.4.

\[ U_s + U_{TS} = \frac{U_{J1} + U_{J2} + V_1 - V_2 - \Delta U_{SW}}{2} \]  

(4.4)

\( \Delta U_{SW} \) is the change in thermal voltage in the reversing switch between positive and negative polarity. This thermal voltage can be less than 10 nV for a well designed switch. This statement however, should be confirmed.

An experimental determination of the thermal voltage \( \Delta U_{SW} \) as defined in equation 4.3 is based on Figure 4.11. The array connections and the voltage standard connections are shorted.

![Diagram of voltage circuit](image)

Figure 4.11. Determination of \( \Delta U_{SW} \) from the two voltmeter readings \( V_1' \) and \( V_2' \)

From figure 4.11 we obtain equations 4.5 and 4.6.

\[ U_{TJ} + V_1' + U_{SW1} - U_{TS} = 0 \]  

(4.5)

\[ U_{TJ} + V_2' + U_{SW2} + U_{TS} = 0 \]  

(4.6)
From equation 4.5 and 4.6 we obtain equation 4.7.

\[ 2U_{TS} + (V'_{2} - V'_{1}) + (U_{SW2} - U_{SW1}) = 0 \quad (4.7) \]

From equation 4.3 and equation 4.7 equation 4.8 will follow.

\[ 2U_{TS} + (V'_{2} - V'_{1}) + (\Delta U_{SW}) = 0 \quad (4.8) \]

The change in thermal voltage in the reversing switch between positive and negative polarity (\(\Delta U_{SW}\)) is determined from equation 4.8. If the thermal voltage \(U_{TS}\) is zero \(\Delta U_{SW}\) would be determined from the two voltmeter readings. The thermal voltage \(U_{TS}\) is the sum of the thermal voltages at the terminals of the standard and the measuring lead connection in the reversing switch. \(U_{TS}\) is not zero but should be very low, probably a few nV.

Equation 4.8 can be combined with equation 4.4 resulting in equation 4.9.

\[ 2(U_{S} + U_{TS}) = U_{J1} + U_{J2} + (V_{1} - V_{2}) + 2U_{TS} + (V'_{2} - V'_{1}) \quad (4.9) \]

Equation 4.10 follows from equation 4.9.

\[ U_{S} = \frac{U_{J1} + U_{J2} + (V_{1} - V_{2}) + (V'_{2} - V'_{1})}{2} \quad (4.10) \]

If the thermal voltage \(U_{TS}\) is the same when the Josephson voltage standard is used for calibration and in the observation of the change in voltmeter reading \((V'_{2} - V'_{1})\) when the reversing switch is changed between positive and negative polarity equation 4.10 is valid and we have compensated for all thermal voltages. \(U_{TS}\) is of course not the same in the two cases but the difference is estimated to be less than \(\pm 2\) nV if the measurement in Figure 4.11 is performed with the voltage standard connection shorted by the two measurement wires connected to the same voltage standard terminal.

The above analysis showed that a correction for or an estimate of the uncertainty due to the thermal voltages in the reversing switch can be determined experimentally. Figure 4.12 shows a determination of the change in voltmeter reading \((V'_{2} - V'_{1})\) when the reversing switch is changed between positive and negative polarity.
Figure 4.12. The change in voltmeter reading \((V_2' - V_1')\) when the reversing switch is changed between positive and negative polarity (measurement sequence positive - negative - positive). The voltage change is \(\pm 2\) mV.

A mean value of many similar measurements is

\[
(V_2' - V_1') = 2.3\text{ nV} \pm 1.2\text{ nV}
\]

It is possible to use this value as a correction in equation 4.10. From equation 4.10 it follows that the error due to \(V_2' - V_1'\) as determined in Figure 4.12 is reduced by a factor of two. At the same time we have the change in \(U_{RS}\) which is estimated to be less than \(\pm 2\) nV when a voltage standard is connected for calibration. This uncertainty should be added to the uncertainty and we have a total uncertainty of \((2^2 + (1.2/2)^2)^{1/2} = \pm 2.1\) nV if the above value for \(V_2' - V_1'\) is used in equation 4.10. If no correction is used the total uncertainty will be \((2^2 + ((2.3 + 1.2)/2)^2)^{1/2} = \pm 2.7\) nV.
4.5 Thermal voltage drift

A change or a drift of the thermal voltages during a measurement will result in an error. The measurement is a sequence of reversings (positive polarity - negative polarity - positive polarity). To estimate this source of possible error the change in the positive polarity voltage (change between first and second) has been observed for measurements on very stable Weston cells.

From a series of 50 measurements the observed mean difference between the first and the second positive polarity voltage was 3 nV ± 3 nV. Of the 50 observations 95 percent showed a change of less than 6 nV.

If we have linear drift of a thermal voltage the error will be small because of the reversing sequence. From equation 4.10 it follows that if we have a change in a thermal voltage in negative polarity the resulting error for $U_S$ will be reduced by a factor of two. From these considerations the contribution to the total error is estimated to ±3 nV ($k=2$).

4.6 Calibration of the nV-meter

When the Josephson voltage standard is compared with a voltage standard an nV-meter is connected as a "nulldetector". The nV-meter is not working as an ideal null detector due to the limitation in the resolution of the microwave locking and the difficulty in obtaining the nearest Josephson voltage step. The nV-meter is therefore used to measure the voltage difference between the Josephson voltage and the standard voltage. The voltage difference is held within a few Josephson steps which is less than ±1 mV.

The voltage standard and the linearity of the nV-meter are calibrated with the Josephson voltage. The nV-meter is connected to the array. The nV-meter is then used to measure the voltage of the first voltage steps on positive and negative polarity. The voltage difference between adjacent Josephson steps is known with a very high accuracy and is approximately 150 μV. The computer is reading the nV-meter and the microwave counter for each step and the voltmeter reading is compared with the known Josephson voltage. Figure 4.13 shows an nV-meter calibration.
Swedish National Testing and Research Institute  
Electrical Metrology  
Boras, Sweden

System DVM Calibration: Chip# PTB CPT 5/5 --- 1994-09-06 12:40:2

Comment: cal of k182 no 524298 NiCd battery

System Digital Voltmeter: Keithley-182  Microwave Freq. = 70.4701828 GHZ

Samples Per Calibration Point: 100

Total Offset Voltage: ZERO Assumed...

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<th>System Time</th>
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<th>Step Number</th>
<th>Array Voltage</th>
<th>Measured Voltage</th>
</tr>
</thead>
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</tr>
</tbody>
</table>

Figure 4.13. A part of a nV-meter calibration.
The problem with a possible long time thermal voltage drift is avoided because each calibration point is relative to the zerostep and the zerostep voltage reading is the mean value before and after each calibration point. No switches are changed during the calibration which could result in changing thermal voltages. Table 4.1 shows the calibration points for the nV-meter.

<table>
<thead>
<tr>
<th>Josephson step</th>
<th>Voltage (µV)</th>
<th>Max allowed nV-meter error (nV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+6</td>
<td>+868</td>
<td>± 5</td>
</tr>
<tr>
<td>+5</td>
<td>+724</td>
<td>± 5</td>
</tr>
<tr>
<td>+4</td>
<td>+579</td>
<td>± 5</td>
</tr>
<tr>
<td>+3</td>
<td>+434</td>
<td>± 5</td>
</tr>
<tr>
<td>+2</td>
<td>+289</td>
<td>± 5</td>
</tr>
<tr>
<td>+1</td>
<td>+145</td>
<td>± 5</td>
</tr>
<tr>
<td>-1</td>
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<td>± 5</td>
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<td>-289</td>
<td>± 5</td>
</tr>
<tr>
<td>-3</td>
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<td>± 5</td>
</tr>
<tr>
<td>-4</td>
<td>-579</td>
<td>± 5</td>
</tr>
<tr>
<td>-5</td>
<td>-724</td>
<td>± 5</td>
</tr>
<tr>
<td>-6</td>
<td>-868</td>
<td>± 5</td>
</tr>
</tbody>
</table>

Table 4.1. Calibration points for the nV-meter. Microwave frequency 70 GHz.

Each calibration point is measured against the Josephson voltage five times and the mean value is used. The deviation between the nV-meter reading and the Josephson voltage for the calibration points are used to calculate a mean deviation between the nV-meter reading and the Josephson voltage. This mean deviation is a constant zero offset and the error for each calibration point is the difference from the mean deviation.

The calibration of the nV-meter should give an error which is within ±5 nV for each calibration point. If we introduce this error for V₁ and V₂ in equation 4.10 the resulting error will be ±5 nV which is used in the error budget.

If a Josephson measurement of the highest accuracy is required the microwave frequency is tuned corresponding to a voltage reading on the nV-meter to be less than 70 nV. In this case the nV-meter error is estimated to be within ±2 nV.
4.7 Noise sources

We have two main noise sources, the nV-meter and the voltage standard. The noise of the nV-meter can be observed if shorts are replacing the Josephson array and the voltage standard. The fluctuation of the nV-meter reading is due to the thermal voltage drift, stability and noise in the nV-meter. The short term fluctuation should be mainly from the nV-meter noise. Figure 4.14 shows the nV-meter reading when the Josephson array and the voltage standard are replaced by shorts.

![Figure 4.14. Measurement of the nV-meter noise.](image)

The error contribution from the nV-meter noise and stability are estimated from Figure 4.14 to ±5 nV. In the calibration process the computer is reading the nV-meter twenty times. Each reading consists of ten samples in positive polarity and twenty samples in negative polarity. By this we have the same number of samples in positive and negative polarity and the same nV-meter reading time. The reduced error due to the large number of readings is ±5 nV/√20 = ±1.2 nV (k=2).

The noise contribution from a Weston cell can be observed if two Weston cells are replacing the two shorts in the above measurement. The increase in noise from the measurement with two shorts is the contribution from the two Weston cells and the measurement result can be seen in Figure 4.15 below.

The total error contribution from the nV-meter noise and the noise from the two Weston cells is estimated from Figure 4.15 to ±6 nV. The error contribution from one Weston cell is estimated not to exceed ±1 nV (k=2).
Figure 4.15. Measurement of the noise given by two Weston cells and the nV-meter.

Figure 4.16. Measurement of the noise given by one Weston cell, one zener reference standard (Fluke 732A) and the nV-meter.
A similar measurement was performed with one Weston cell and one zener reference standard. Figure 4.16 shows the result. The total noise is ±15 nV. The noise contribution from the zener reference standard is estimated to ±10 nV (k=2). The noise of the 10 volt output from a Fluke 732A zener reference standard is ±350 nV which is, in relative values, three to four times the noise of the 1.018 volt output.

4.8 Error budget

In this chapter we summarize the different sources of error for the Josephson calibration of a voltage standard and give a total error budget for three different cases, calibration of a Weston cell and of a commercial zener reference standard at the 1 V and the 10 V levels. The error budgets are based on [8].

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<thead>
<tr>
<th>Error source</th>
<th>Error (type)</th>
<th>Contribution to RSS calculation</th>
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</thead>
<tbody>
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<td>(The errors are given in nV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calibration of frequency standard</td>
<td>± 1.0 (k=2)</td>
<td>1.0/2</td>
</tr>
<tr>
<td>Microwave frequency stability</td>
<td>± 1.0 (k=2)</td>
<td>1.0/2</td>
</tr>
<tr>
<td>Leakage conductance</td>
<td>± 0.25 (max)</td>
<td>0.25/√3</td>
</tr>
<tr>
<td>Uncompensated thermal voltages in reversing switch</td>
<td>± 2.7 (max)</td>
<td>2.7/√3</td>
</tr>
<tr>
<td>(Correction used for uncompensated thermal voltages)</td>
<td>±2.1 (max)</td>
<td>(2.1/√3)</td>
</tr>
<tr>
<td>Drift of thermal voltages during measurements</td>
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<td>3.0/2</td>
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<tr>
<td>Calibration of nV-meter</td>
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<td>5.0/√3</td>
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<tr>
<td>(Calibration of nV-meter)</td>
<td>±2.0 (max)</td>
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<td>Noise from nV-meter</td>
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<td>1.2/√3</td>
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<td>Noise from the voltage standard to be calibrated</td>
<td>± 1.0 (k=2)</td>
<td>1.0/2</td>
</tr>
</tbody>
</table>

| Total error (Root - Sum - Square) (k=2)                                      | ± 7.6 nV      |
| (Total error (Root - Sum - Square) (k=2))                                   | (± 5.0 nV)    |

Table 4.2. Error budget for calibration of a Weston cell, voltage 1.018 volt. The value within parenthesis is for a Josephson measurement with the highest accuracy. In this measurement the microwave frequency is tuned corresponding to a voltage reading on the nV-meter to be less than 70 nV and a correction for uncompensated thermal voltages in the reversing switch is used.
<table>
<thead>
<tr>
<th>Error source</th>
<th>Error (type)</th>
<th>Contribution to RSS calculation</th>
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<td>Calibration of frequency standard</td>
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<tr>
<td>Microwave frequency stability</td>
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<td>Leakage conductance</td>
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<td>Uncompensated thermal voltages in reversing switch</td>
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<td>Noise from the voltage standard to be calibrated</td>
<td>± 10.0 (k=2)</td>
<td>10.0/2</td>
</tr>
<tr>
<td>Total error (Root - Sum - Square) (k=2)</td>
<td>± 11.1 nV</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3. Error budget for calibration of a Fluke 732A zener reference standard, voltage level 1 volt.

<table>
<thead>
<tr>
<th>Error source</th>
<th>Error (type)</th>
<th>Contribution to RSS calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration of frequency standard</td>
<td>± 10,0 (k=2)</td>
<td>1.0/2</td>
</tr>
<tr>
<td>Microwave frequency stability</td>
<td>± 10,0 (k=2)</td>
<td>1.0/2</td>
</tr>
<tr>
<td>Leakage conductance</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Uncompensated thermal voltages in reversing switch</td>
<td>± 2,7 (max)</td>
<td>2.7/√3</td>
</tr>
<tr>
<td>Drift of thermal voltages during measurements</td>
<td>± 3,0 (k=2)</td>
<td>3.0/2</td>
</tr>
<tr>
<td>Calibration of nV-meter</td>
<td>± 5,0 (max)</td>
<td>5.0/√3</td>
</tr>
<tr>
<td>Noise from nV-meter</td>
<td>± 1,2 (max)</td>
<td>1.2/√3</td>
</tr>
<tr>
<td>Noise from the voltage standard to be calibrated</td>
<td>± 350 (k=2)</td>
<td>350/2</td>
</tr>
<tr>
<td>Total error (Root - Sum - Square) (k=2)</td>
<td>± 0,36μV</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4. Error budget for calibration of a Fluke 732A zener reference standard, voltage level 10 volt. No error from leakage conductance due to the low internal resistance at the zener voltage standard 10 volt output.
5 Measurements

5.1 Introduction

In this chapter some measurement results and a method for measurement of the 10 volt level with a 6 volt array are reported. A comparison with the earlier used single junction Josephson standard is evaluated from the maintained Swedish voltage level by the primary Weston cells.

5.2 Measurements of Weston cells and zener reference standards

The Josephson array voltage standard can be used for measurements of Weston cells and zener reference standards. Figure 5.1 shows a series of measurements of one Weston cell and one zener reference standard and we can see the higher short time stability of the measured Weston cell voltage.

![Graph showing deviation from the mean value of measurements](image)

Figure 5.1. Comparison of Josephson measurements (1.018 V level) on one Weston cell and one zener reference standard showing the higher stability of the Weston cell voltage.
Weston cells are very sensitive for loadings but we have found that with a careful measurement technique and if the voltage difference between the Weston cell and the Josephson voltage is kept within a few Josephson steps ($\Delta V < 1\text{mV}$) no effect on the Weston cell voltage is observed. This is due to the high input resistance of the nV-meter.

Weston cells are more stable (if they are kept under ideal conditions) and exhibit less noise than zener reference standards. Of this reason we have found it more exciting to make Josephson measurements on Weston cells. Weston cell measurements are also of great value since the Josephson volt is directly transferred to the maintained voltage level of the primary Weston cell group.

5.3 **Comparison between the single junction Josephson standard and the Josephson array voltage standard.**

The first measurements with the Josephson array voltage standard were performed in March 1993. At that time we were not able to use the single junction Josephson standard because the squid had broken. The last measurement with the single junction was made a year earlier in February 1992. Because of this, the comparison between the two Josephson standards could only be performed by the maintained voltage level in the primary Weston cell group.

Today the primary group and secondary Weston cells are stored in four separate airbaths with four Weston cells in each airbath. Earlier one oilbath stored all Weston cells. In 1992 the work to transfer the Weston cells to the new airbaths was not finished and it was only in the first airbath (SPOG 1) the Weston cells had stabilized voltages due to the change in temperature.

The comparison was made with the four available stable Weston cells between February 1992 and March 1993. Figure 5.2 to 5.5 shows the comparison. A short time after (April 1993) the first measurements with the Josephson array voltage standard leakage problems in the polarity reversing switch in the airbath made it necessary to take the airbath apart. The change in voltage between the first and second measurement with the Josephson array standard is due to this disturbance. One cell (SPOG 1/1) has been unstable since then.
Figure 5.2. Measurements of Weston cell SPOG 1/1. Before 1993 (larger error bars) measurements with the single junction Josephson standard and after 1993 with the Josephson array voltage standard. The error bars are for k=1.

Figure 5.3. Measurements of Weston cell SPOG 1/2. Before 1993 (larger error bars) measurements with the single junction Josephson standard and after 1993 with the Josephson array voltage standard. The error bars are for k=1.
Figure 5.4. Measurements of Weston cell SPOG 1/3. Before 1993 (larger error bars) measurements with the single junction Josephson standard and after 1993 with the Josephson array voltage standard. The error bars are for k=1.

Figure 5.5. Measurements of Weston cell SPOG 1/4. Before 1993 (larger error bars) measurements with the single junction Josephson standard and after 1993 with the Josephson array voltage standard. The error bars are for k=1.
From Figure 5.2 to 5.5 we estimate that the agreement between the single junction Josephson standard and the Josephson array voltage standard is within ±50 nV which is the estimated error (k=1) for the single junction Josephson standard. The mean value for the difference between the last measurement with the single junction Josephson standard and the first measurement with the Josephson array voltage standard is

\[ V_{\text{array}} - V_{\text{single}} = +38 \text{ nV} \pm 19 \text{ nV}. \]

5.4 Measurements at the 10 volt level with a 6 volt array

Josephson arrays with 20 000 or more junctions have been developed [4], [5]. These arrays are capable of a maximum voltage of 3 to more than 10 volts. We have used arrays capable of 6 volts to measure the 10 volt output on zener reference standards. The 10 volt output is calculated from two measurements. The first measurement is the voltage of four Weston cells connected in series. The second measurement is the voltage difference between the zener reference standard 10 volt output and the voltage of the four Weston cells connected in series. Figure 5.6 and 5.7 illustrates the measurement method. Each primary airbaths contains four Weston cells which is the reason for the use of four Weston cells in this method.

![Diagram](Image)

Figure 5.6. The measurement of the four Weston cell voltage $U_\text{W}$. 
Figure 5.7. The measurement of the voltage difference $\Delta U$ between the zener reference standard 10 volt output and the voltage of the four Weston cells.

The 10 volt output of the zener reference standard is the sum of the two measurements.

$$U_Z = U_W + \Delta U \quad (5.1)$$

The estimated total error is the sum of the errors of the two measurements which are $\pm 12 \text{ nV } (U_W)$ and $\pm 360 \text{ nV } (\Delta U)$. The values are for $k=2$. The total error is $\pm 361 \text{ nV}$ if we use the Root - Sum - Square summation. This error is the same as if we were using a 10 volt array measurement and can be understood from the fact that the dominating source of error is the zener reference standard 10 volt output noise.

Figure 5.8 shows a comparison with a 10 volt system based on a voltage divider [9] used at SP for 10 volt calibrations. This measurement system is using the voltage of one Weston cell in the voltage divider. In the comparison this Weston cell voltage comes from Josephson measurements during the comparison. The 10 volt system error is based on contributions from the voltage divider and the Weston cell calibration in this comparison.

The difference between the Josephson measurements and the 10 volt system measurements is within the estimated error of the 10 volt system of $\pm 0.05$ ppm ($k =1$).
Figure 5.8. Comparison between the 10 volt system and the Josephson 10 volt measurements with a 6 volt array. The error bars are for k=1.
6 Intercomparisons, conclusions and the future

6.1 Introduction

An important test of a Josephson voltage standard is to compare the realized voltage with the voltage realized by another Josephson voltage standard. Two such intercomparisons have been performed. The first intercomparison was with Danish Institute of Fundamental Metrology (DFM) in Denmark and the second with Physikalisch-Technische Bundesanstalt (PTB) in Germany. Both intercomparisons resulted in a compliance in voltage realization within a few nV which is a satisfactory result considering the error budgets given for the Josephson standards. Some conclusions and future developments are briefly discussed.

6.2 Intercomparison with DFM

The first intercomparison was carried out in January 1994 when Danish Institute of Fundamental Metrology, DFM, in Lyngby, Denmark, brought their standard to SP [10].

The DFM and the SP Josephson array voltage standards are equal with some minor differences. The DFM standard had a 1 V array manufactured by NIST, USA, and the null detector used was an EM nV-meter connected to a Hewlett-Packard digital voltmeter. On this occasion the SP standard had a 1 V array manufactured by PTB and as null detector a Keithley 182 nV-meter. Both standards were externally trigged from the same frequency standard.

The intercomparison was made by measuring alternately with both Josephson standards at the same Weston cell. A series of at least three measurements were performed with each standard of the cell before switching to the other standard. Measurements were made of totally three Weston cells and the results are reported in Figure 6.1 to 6.3.
Figure 6.1. This graph gives the result of measurements performed of the first Weston cell used in the SP - DFM intercomparison. The range in this series of measurements is 9 nV.

Figure 6.2. This graph gives the result of measurements performed of the second Weston cell used in the SP - DFM intercomparison. The range in this series of measurements is 15 nV.
Figure 6.3. This graph gives the result of measurements performed of the third Weston cell used in the SP - DFM intercomparison. The range in this series of measurements is 23 nV. After the first six measurements the Weston cell was probably disturbed and the following measurements shows the drift of the Weston cell voltage due to the disturbance.

A total number of 74 measurements were performed on the three Weston cells and the statistical calculation of these results gives a difference between the both Josephson standards of

\[ V_{SP} - V_{DFM} = -1.5 \text{ nV} \pm 0.3 \text{ nV}. \]

This result is by a comfortable margin within the stated error budget for the SP Josephson array voltage standard.

### 6.3 Intercomparison with PTB

The second intercomparison was carried out in April 1995 when we brought the SP Josephson array voltage standard to Physikalisch-Technische Bundesanstalt (PTB) in Germany.
The PTB and SP Josephson array voltage standards are different in construction but are in principle equal. The PTB standard had a 1 V array manufactured by PTB and the null detector used was a Keithley 182 nV-meter. At this occasion the SP standard had a 1 V array manufactured by PTB and as null detector a Keithley 182 nV-meter. The standards were trigged from different frequency standards.

The intercomparison was made by measuring alternately with both Josephson standards at the same zener reference standard. One measurement was performed with each Josephson standard on the zener reference standard before switching to the other Josephson standard. Measurements were made on two zener reference standards and the results are reported in Figure 6.4 to 6.6.

In the first series of measurements each Josephson standard was connected to the zener reference standard. In the following two series the measurements were performed with a common reversing switch to avoid to change the thermal voltages between the reversing switch and the zener reference standard.

![Figure 6.4. The first series of measurements at the intercomparison between PTB and SP. The intercomparison was performed with individual reversing switches. The range in this series of measurements was 20 nV. Compare the lower short term stability of the zener reference standard in this Figure with Figures 6.1 to 6.3 which have Weston cells as transfer standards in the intercomparison.](image)
Figure 6.5. The second series of measurements at the intercomparison between PTB and SP. The intercomparison was performed with a common reversing switch. The range in this series of measurements was 21 nV.

Figure 6.6. The third series of measurements at the intercomparison between PTB and SP. The intercomparison was performed with a common reversing switch. The range in this series of measurements was 22 nV.
A total number of 59 measurements was performed on the two zener reference standards.

The results give a difference between the SP and PTB Josephson voltage standards when the measurements were performed with individual reversing switches of

\[ V_{SP} - V_{PTB} = -3.9 \, \text{nV} \pm 3.0 \, \text{nV} \]

In the case of a common reversing switch the difference was

\[ V_{SP} - V_{PTB} = -1.8 \, \text{nV} \pm 2.4 \, \text{nV}. \]

These results are within the stated error budget for the SP Josephson array voltage standard.

### 6.4 Conclusions and future development

The intercomparisons have shown that the error budget has given a reasonable calculation for the uncertainty which is \( \pm 5 \, \text{nV}(k=2) \) for a Weston cell measurement and \( \pm 11 \, \text{nV}(k=2) \) for zener reference standard measurement \( (k=2) \) at the 1 V level. The uncertainty by which the voltage unit is realized is small enough for the needs of today and probably for the future.

One dominating part of the uncertainty calculation is the thermal voltages. If the uncertainty is to be reduced to \( \pm 1 \, \text{nV} \) we are forced to reduce the thermal voltages including the thermal voltages at the terminals of the voltage standard to be less than 1 nV. This problem is difficult to solve and as long as the noise of the zener reference standards is of the order of \( \pm 10 \, \text{nV} \) the reduction of the thermal voltages will not give a significant reduction of the total uncertainty.

The situation is different for Weston cell calibrations but Weston cells are not used today to transfer the voltage unit to dependent laboratories. However, the Swedish voltage unit is maintained by a group of Weston cells and we will continue the work on the reduction of the uncertainty for our Josephson array voltage standard calibration of the primary Weston cells.

If, in future, a low noise electronic voltage standard will be available at the 10 V level there would be a demand for a calibration with a relative uncertainty of 1 part in \( 10^9 \). It will be possible to reach this accuracy with our Josephson array voltage standard.
An interest for the future will be to use the Josephson array voltage standard as a potentiometer to test the linearity of digital voltmeters. The best instruments today are close to a stated linearity which only can be verified with a Josephson array voltage standard. Our experience of linearity measurements is today limited but our equipment is easy to use for linearity measurements of digital voltmeters. It is important to have an array capable of voltages over 10 V for linearity measurements. Today our maximum useful Josephson voltage is 7 V but we expect to have an array capable of higher voltages available in future. A 10 V array would also facilitate the determination of the 10 V outputs of zener reference standards.
7 References


