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Analysis of Test Levels Applicable for Radiated Susceptibility Test of Built-In Equipment

Nordtest Project
No. 986-91
Abstract

This document describes different analysis methods that could be used for EMC predictions. The different methods are compared with each other in order to establish guidelines for when a particular method is preferable.

The interaction between an electromagnetic field and a wire is treated with a simple model derived from Faraday’s law, the transmission line theory and an integral approach solved by the method of moments.

Radiation from a wire is treated in a thorough way for the two-dimensional case. This treatment resulted in a computer code based on the method of moments. With the computer code it is possible to analyse the reduction that is achievable with the introduction of one or several ground planes. The code is also used to predict the crosstalk between wires and the reduction that is achieved by placing a ground plane between the wires.

Coupling of electromagnetic energy through apertures and slots is also included. The coupling through slots is treated in a thorough way for the two-dimensional problem. A computer code for the calculation of the field distribution in the slot as well as the attenuation was developed. The computer code is capable to analyse slots in thin ground planes, multiple slots in thin ground planes, slots in thick ground planes and slots with a cross section made up of cascaded rectangular sections. By examples and measurements it is shown that the attenuation in a slot, for the TE-polarisation, can be increased significantly by introducing a corrugation (choke) with a depth of a quarter of a wavelength.

Keywords: Aperture, Corrugation, Crosstalk, EMC analysis, Radiated emission, Radiated susceptibility, Radiating wire, Slot

SP
SP Rapport 1993:21
ISSN 0284-5172
Borås 1993

Swedish National Testing and Research Institute
SP Report 1993:21

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Preface

This project has been initiated by SP and has been supported by Nordtest, a joint Nordic body with the function of promoting development in the field of technical testing.
Summary

The purpose of this work has been to investigate the correlation between EMC-requirements for a whole system and requirements for a sub system. The investigation has been carried out in a theoretical way in the sense that appropriate susceptibility levels for built-in equipment have been calculated from test levels on a whole system.

Simple models that could be used for the prediction of emission as well as susceptibility levels are described and evaluated. Comparison between predicted levels and levels given by EMC standards are presented in order to highlight especially important problem areas.

In the first part of the document the interaction between an electromagnetic field and a wire is treated. Several different techniques to predict the induced currents and voltages on a transmission line caused by an incident electromagnetic field are given. The techniques included are a simple model based on Faraday's law, the transmission line model and an integral approach which is solved by the method of moments [1]. These techniques are compared with each other in order to establish guidelines for when a certain technique is preferable.

As a natural continuation of the interaction between fields and wires, the radiation from a wire is treated. A thorough treatment of the radiation from an infinitely long wire, i.e. the 2D-problem, is included. The advantage of the simple 2D-model is that it is very illustrative to demonstrate different techniques to reduce the radiation from a wire. With results obtained by a developed computer code, based on the method of moments, the effects of adjacent ground planes of finite sizes are shown. These results can, for instance, be used in order to establish guidelines for the layout of traces on printed circuit boards.

Treatment of crosstalk between two wires, i.e. induced current in one wire caused by a current in the other wire, is also included. The model used for the crosstalk is also a 2D-model and the attenuation that could be achieved by placing a ground plane of finite size between the wires is shown.

In the last parts the coupling of electromagnetic fields through apertures and slots are treated. The coupling through a small aperture is treated by a simple model assuming the field distribution in the aperture to be constant.

The coupling of electromagnetic fields through slots is treated in a thorough way and this has resulted in a computer code that is described in [8]. With this computer code it is possible to calculate the field distribution and the attenuation in a slot. The slot can either be placed in a ground plane of zero thickness or in a ground plane of finite thickness. Even slots with a cross section made up of cascaded rectangular sections can be treated. With several examples it is shown that the attenuation in a slot, for the TE-polarisation, can be increased considerably by introducing a corrugation (often called a choke) of a quarter of a wavelength depth in the cross section. This is also shown by means of measurements.
1 Introduction

Traditionally the work in the area of electromagnetic compatibility (EMC) has mostly been performed by means of laboratory measurements. Such measurements are often very time consuming and expensive, especially if prototypes have to be made for testing in the early stage of a project. Special equipment and test facilities are also needed and consequently the tests have to be performed at special test houses by qualified personnel. Due to these facts the EMC issues are often addressed very late in a project and restricted to only one qualifying measurement. Such a policy can turn out to be unwise since the modifications that are needed, if the equipment does not comply with the EMC requirements, can be very expensive and difficult to apply.

If tests on prototypes cannot be performed (due to for instance the costs) many possible problems can be found by means of a simple analysis. The advantage of an analysis is that it can be performed early in a project, even before the hardware exists. However, it must be remembered that an analysis can never replace a measurement on the real equipment.

In order to calculate the response of a built-in equipment to the stress on the whole system, the problem has been divided into smaller sub problems. The first sub problem recognised is the interaction between an incident electromagnetic field and a wire. This situation simulates, for instance, the stress on an equipment caused by a distant lightning stroke. By this treatment it is also possible to compare radiated susceptibility levels with conducted susceptibility levels, both of which are often given in standards.

A similar sub problem is that of a radiating wire, which for instance could simulate a data cable. This problem is treated in a thorough way and the effects of an adjacent ground plane are shown. Especially the effects of finite sized ground planes and their relative placements to the wire are pointed out. A closely related problem is the crosstalk between cables (i.e. induced currents in a wire caused by the current in an adjacent wire) which also is included.

For shielded equipment interference signals can couple to the inside following two principal paths. The first path is that of a wire or a cable, discussed above. The second path is the leakage through holes and cracks in the casing. Since the coupling through holes and cracks is very important this problem is treated in a thorough way. Different techniques for the reduction of the leakage are also discussed.

1.1 Disposition of this document

This document is divided into two main parts. The first part treats the interaction between fields and cables. In this part the coupling from electromagnetic fields to cables, the radiation from current carrying cables and the crosstalk between cables are included. Several different modelling techniques are used for the analysis. The advantages and disadvantages of the different techniques are also addressed. Special attention is paid to the use of ground planes in order to minimise the radiation from cables (or traces on a printed circuit board). This analysis is performed on a two-dimensional model and the simulation is done by a developed computer code based on the method of moments [1].

The second part of this document deals with the leakage through apertures and slots. A simple model based on the assumption of uniform field distribution in the aperture is used for analysing small apertures. Slots are analysed by a method based on the method
of moments. This technique is extended in order to include slots in ground planes of finite thickness as well as slots with a cross section made up of cascaded rectangular sub sections. With the help of this analysis tool it is shown that it is possible to increase the attenuation in the slot considerably by the use of a quarter of a wavelength deep choke. This is also shown by measurements.
Strategy of EMC analysis

In performing an EMC analysis on a complex system it is essential to divide the complex problem into smaller, and manageable, sub problems. In doing the division it is important to recognise the most important coupling paths and analyse the influence of these paths first. This is not always an easy task and often experience is required. However, the following guidelines can often be followed.

If the equipment is small, compared to the wavelength of the interference signal, and it is connected to a long unshielded cable (or wire) the most important coupling path is probably the cable. In such a case the first things that should be taken into consideration are the coupling from an external field to the cable, i.e. radiated susceptibility, and inversely the radiation produced by the signal in the cable.

If, on the other hand, the equipment is shielded and has no external cables, or they are shielded if present, the coupling of electromagnetic energy from the outside to the inside, or inversely from the inside to the outside, probably takes place through an aperture or a slot. In this case it is important to look for the most important aperture or slot and analyse the influence of its presence.

In every analysis the "real" world has to be replaced by a more or less exact model. The simplifications that are needed to "transform" the real world to a simplified model can be performed in many different ways. The choices of appropriate simplifications are not an easy task and requires experience. Many times it is also difficult to judge if a certain model is good enough and in such a case measurements are often needed to verify the model. It should also be pointed out that a model that could be used in a certain domain (for instance a frequency range) is perhaps totally incorrect in another domain. It is also important to realise that a very simple model often successfully can be used for parameter studies. For instance, it is often helpful to study simple two-dimensional models and use the conclusions for more complicated three-dimensional cases.

Even if it is not possible to use analysis tools for the prediction of the susceptibility of and/or emission from an equipment, it is often possible to use the results from an analysis in order to choose the most efficient way to improve the performance, i.e. lower the emission or increase the susceptibility threshold.
3 Coupling of fields to wires and cables

3.1 Introduction

When performing radiated susceptibility tests on an equipment it is often found that the cause of EMC problems are the cables connected to the equipment under test (EUT). Even in daily life the importance of the coupling to cables can be recognised, for instance in a thunder storm where it often happens that telephones and alike become damaged. This is due to the induced transient on the telephone line caused by the coupling from the fields from a distant lightning stroke to the line.

The example of the induced transient by a lightning stroke is most conveniently considered in the time domain. However, most EMC standards for radiated susceptibility tests present the requirements as amplitudes versus frequency, i.e. in the frequency domain. Therefore, the analyses in this chapter are performed in the frequency domain. This will seldom cause any problem because it is possible to transform results obtained in the frequency domain to the time domain (or the other way round) by a Fourier transformation.

It is often sufficient to restrict the analysis to the case of an incident plane wave and a straight wire placed on a constant height over a ground plane (if present). The assumption of an incident plane wave is, for EMC analysis, no drawback since the radiated susceptibility standards formulate the requirements as field strengths for a plane wave. The restriction to straight wires with a uniform characteristic impedance (constant height) can be a limitation but has the advantage that the well known transmission line theory can be used. If the wire cannot be approximated with a straight wire placed at a constant height other methods than the transmission line theory have to be used.

Three different analysis methods are used in this chapter, a simple model obtained by the application of Faraday's law, transmission line theory and the method of moments.

The simple model deduced from Faraday's law can successfully be used for low frequencies, i.e. for frequencies where the wavelengths are large compared to the dimensions of the analysed cable. A more exact model is the transmission line model which is used for parameter studies by the use of the computer code NULINE [2]. An even more exact model that also takes the radiation from the cable into consideration is a model implemented in the computer code AWAS [3]. This code uses the method of moments [1] for solving an integral equation obtained from Maxwell's equations. With this code it is also possible to perform simulations on bent wires and wires placed on different heights over the ground plane.

3.2 Problem geometry

The problem is to determine the voltages and currents in a wire over a ground plane caused by an incident electromagnetic field. The geometry for the problem is shown in Fig. 3:1. By using the image theory [4] this problem can be shown to be equivalent to a two-wire transmission line situated in free space, this is shown in Fig. 3:2.
Figure 3:1. Terminated single-wire over a ground plane.

It should be noted that not only the wire and the termination impedances are imaged but also the incident field that excites the wire. In Fig. 3:1 and 3:2, $Z_0$ and $2Z_0$ respectively are the characteristic impedance's of the wire which can be written as:

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\varepsilon}} \ln\left(\frac{2h}{a}\right)$$  \hspace{1cm} (3:1)

Figure 3:2. Terminated two-wire transmission line, image of Fig. 3:1.

Since the two problems shown in Fig. 3:1 and Fig. 3:2 respectively are equivalent it is only necessary to concentrate on one of them.
3.3 Comparison between conducted and radiated susceptibility limits by calculation of induced voltage caused by an incident plane wave

Many EMC-standards give limits for conducted susceptibility as well as limits for radiated susceptibility. It could therefore be interesting to compare the two cases.

One example of a standard that describes both conducted (CS) and radiated (RS) susceptibility levels are the military standards MIL-STD-461 & 462, [5] and [6] respectively. For comparison the methods CS02 and RS03 are chosen.

CS02 states that the equipment under test (EUT) should not exhibit any malfunction, degradation of performance or deviation from specified indications when subjected to 1 $V_{\text{rms}}$ from a 50 ohm source applied to input terminals in the frequency range 50 kHz - 400 MHz.

RS03 states that the equipment under test (EUT) should not exhibit any malfunction, degradation of performance or deviation from specified indications when subjected to a radiated electric field with a field strength of 10 V/m (according to [5] the field strength varies with the frequency but is here for simplicity taken as 10 V/m in the same frequency range as for CS02).

When testing according to RS03 the EUT should be placed on a ground plane and the cables should be placed on a height of 5 cm above the same and terminated with a specified impedance (LISN). The impedance of the LISN is frequency dependent but is for higher frequencies (higher than a few megahertz) approaching 50 ohm. Since the test has to be conducted in a shielded room there is some practical limitations in the set-up of the EUT, one of which is the possible length of the cables.

For calculations it is necessary to assume a certain cable length and termination impedances at each end of the wire. Following the discussions above it is believed that a realistic wire length is 2 meters and the termination impedance at one of the ends (at the LISN) is equal to 50 ohm. For calculation of the worst case the impedance of the other end (at the EUT) is assumed to be infinity (open circuit). With these assumptions the configuration for calculation of the induced voltage at the EUT, caused by an incident field, will look like the one shown in Fig. 3.3.

![Figure 3.3](image)

Figure 3.3. Configuration for calculation of induced voltage at RS03 test.

The test should be carried out for both horizontal and vertical polarisation but since only the worst case is interesting the horizontal polarisation is chosen for the calculations (see for instance figure 3.9 to 3.12).
The calculated open circuit voltage at the EUT for an incident field with a field strength of 1 V/m is shown in figure 3.4.

![Graph showing open circuit voltage vs frequency](image)

Figure 3.4. Open circuit voltage at EUT caused by an incident field 1 V/m.

For extrapolation to the RS03 test level the values in Fig. 3.4 should be multiplied by a factor of ten (calculations are done with 1 V/m and the RS03 limit is 10 V/m).

It can be seen from Fig. 3.4 (if the values are multiplied by ten) and comparison with the CS02 limit, 1 V_{\text{rms}}, that the case of CS02 is a more severe test for lower frequencies and RS03 is more severe for higher frequencies (approx. above 100 MHz). This is valid under the assumption that the problem is arising from induced currents on the cabling and that the equipment is sensitive to common mode currents.

From this example it could be seen that the severity of different tests, here conducted and radiated susceptibility tests, is often overlapping. It can also be concluded that if the geometry or the load conditions for the calculation is altered the frequency where the overlapping starts will also be altered.

### 3.4 Simple coupling model for low frequencies

A simple model for the induced current in a wire caused by an incident electromagnetic field is derived from Faraday's law. The idea is that the induced voltage in an open loop can be written as the time derivative of the net magnetic flux penetrating the loop (Faraday's law). The magnetic flux penetrating the loop is equal to the integral of the normal magnetic flux density over the loop area. If the frequency is low (the wavelength is small compared with the dimension of the loop) the flux density can be considered as constant over the loop and consequently the magnetic flux can be calculated as the magnetic flux density times the loop area. Finally the voltage can be calculated as the derivative of the magnetic flux density times the loop area, the details follows:
Induced voltage in a loop:

\[ V_{\text{loop}}(t) = -\frac{d\Phi}{dt} \]  \hspace{1cm} (3.2)

where:
- \( \Phi = \{ \text{Magnetic flux penetrating the loop} \} = \int \varrho_n \, dS = \{ \text{The magnetic flux} \}
- \varrho_n, \text{ is constant over the loop (low freq. approx.)} = \varrho_n A = \mu \chi_n A

and:
- \( \mu = \text{Inductivity of the medium in the loop, free space:} \mu = 4\pi \times 10^{-7}, \text{[H/m]} \)
- \( \chi_n = \text{the component of the magnetic intensity normal to the loop, [A/m]} \)
- \( A = \text{area of the loop, [m}^2\text{]} \)

Hence the induced voltage can be written as:

\[ V_{\text{loop}}(t) = -\mu A \frac{d\chi_n}{dt}, \text{[V]} \]

If the magnetic intensity is assumed to be time harmonic, i.e. \( \chi_n = \sqrt{2} H_n \cos(\omega t) \) where \( \omega \) is the angular frequency [rad/s], the induced current can be written as:

\[ V_{\text{loop}}(t) = \mu A \sqrt{2} H_n \cos(\omega t) \]

and the magnitude as:

\[ V_{\text{loop}}(\omega) = \mu A \omega H_n \]  \hspace{1cm} (3.3)

where \( V_{\text{loop}} \) and \( H_n \) represent the effective (rms.) values.

The fact that the magnetic intensity was assumed to be time harmonic is no limitation because any signal can be written as a sum of time harmonic signals, i.e. Fourier expansion.

If, again, the frequency is low the current in the loop can be considered as constant and can be calculated as the induced voltage divided by the total impedance in the loop.

![Diagram](image)

Figure 3.5. Configuration used for a comparison between the simple low-frequency model and the transmission line theory represented by the code NULINE [2]. Z1 and Z2 equals 50 ohm.
In order to determine how far up in frequency Eq. (3:3) can be considered as valid, the configuration in Fig. 3:5 is considered.

The incident field in Fig. 3:5 is a plane wave which means that the ratio between the electric and the magnetic field intensities is equal to the free space intrinsic impedance $(120\pi)$ i.e. $H = \frac{F}{120\pi}$ [A/m]. For an incident wave with unit electric field intensity the magnetic field intensity becomes $H = \frac{1}{120\pi}$ [A/m]. If the angle between the direction of the incident field and the plane of the loop is $\alpha$, the normal component of the magnetic intensity will be $H_n = H \sin(\alpha)$, for the field in Fig. 3:5: $H_n = H \sin(45) = \frac{1}{\sqrt{2}} \frac{1}{120\pi}$ [A/m].

Since the field is reflected in the ground plane, the area that should be used in Eq. (3:3) is twice the physical area of the loop. The current in the loop, which is equal to the voltage divided by the total impedance, can finally be written as:

$$I(f) = \mu A 2\pi f H_n \frac{1}{Z_1 + Z_2} = \mu 2A_p 2\pi f \frac{1}{\sqrt{2} 120\pi} \frac{1}{Z_1 + Z_2} = \{Z1=Z2=50; A=0.1; \mu=4\pi 10^{-7}\} = 2.9610^{-11} f [A]$$

Eq. (3:4) is plotted in Fig. 3:6 together with the result obtained by the transmission line theory (the code NULINE).

![Graph](image)

**Figure 3:6.** Induced voltage over the load Z2 in Fig. 3:5 a comparison between equation (3:4) and transmission line theory represented by the code NULINE [2].

From Fig. 3:6 it can be concluded that Eq. (3:4) holds for frequencies up to where the wavelength is greater than the largest dimension of the loop times 30, with reasonable accuracy, i.e. 10 MHz in Fig. 3:6. This can also be stated as: for Eq. (3:3) to be valid, with reasonable accuracy, the frequency, f, should be less than $10^7/D$ where D is the largest dimension of the loop.

It can be shown that this fact also holds for other incidence angles.
3.5 Coupling analysis by the transmission line theory

The basic assumption that must be valid when using transmission line theory is that the distance between the conductor and the return path (e.g. the distance between a single wire and ground plane) must be small compared to the wavelength. When this condition is met it is possible to define a unique voltage between the conductor and the return conductor (which could be the ground plane).

Referring to Fig. 3:1 (and 3:2) the electromagnetic field excites the wire over the entire length resulting in a continuous distribution of voltage sources along the wire. The voltage sources are shown in Fig. 3:7. In general the voltage sources are functions of the position along the wire, e.g. if the direction of the incident field is not normal to the wire. In addition to the sources along the wire it is necessary to account for two lumped voltage sources at each end of the wire (lumped because the distance between the wire and the ground is small compared with the wavelength). These lumped sources are due to the interaction between the field and the vertical conductors of the loads.

The infinitesimal voltage sources of Fig. 3:7 are equal to the total tangential (to the wire) electric field at the wire location but with the wire removed. The total field is equal to the sum of the incident and the ground plane reflected field. Thus the excitation field will be dependent not only on the incidence angle but also on the polarisation of the incident field.

Figure 3:7. Continuous distribution of voltage sources along the wire.

In order to calculate the voltage or current at either termination (Z₁ or Z₂) it is necessary to integrate over all of the sources along the wire. This is most conveniently done with a computer. A simple code usable on a personal computer (PC) which performs this task is the code NULINE [2]. The code gives the voltage or the current not only at the terminations but at an arbitrary position along the wire either in the time domain or in the frequency domain. The theory discussed above is treated in a very thorough manner in the manual as well.

3.5.1 Parameter studies for the load current

In order to show how different angles of incidence and polarisation will affect the response of the wire, the following pages will be devoted for such comparisons. The calculations are done with the NULINE code and the geometry is shown in Fig. 3:8.

For all calculations the following is valid: Length = 1 m
Height = 10 cm
Radius = 1 mm
Figure 3:8. Geometry used for parameter study.
3.5.1.1 Horizontal polarisation, variation of $\Psi$

Referring to Fig. 3:8 the following values are used:

- both load impedances are matched (equals 318 $\Omega$)
- $\Phi$ angle is equal to 90 degrees
- $\Psi$ angle is varied as 30, 60 and 90 degrees
- horizontal polarisation (electric field vector parallel to the ground)

![Graph showing induced current in load, $\Phi = 90$, $\Psi = 30, 60, 90$.](image)

**Figure 3:9.** Induced current in load, $\Phi = 90$, $\Psi = 30, 60, 90$.

From Fig. 3:9 it could be seen that the worst case (i.e. the highest current in the load) is obtained when the incident wave is coming right from above the line ($\Psi = 90$). It could also be seen that the induced current is proportional to sine of the incidence angle $\Psi$. 
3.5.1.2 Horizontal polarisation, variation of Phi

Referring to Fig. 3:8 the following values are used:

- both load impedances are matched (equals 318 Ω)
- Phi angle is varied as 30, 60 and 90 degrees
- Psi angle is equal to 90 degrees
- horizontal polarisation (electric field vector parallel to the ground)

Figure 3:10. Induced current in load, Phi = 30, 60, 90, Psi = 90.

The induced current is behaving in the same way as in the previous example, i.e. the current is proportional to sine of the incidence angle, in this case, Phi.
3.5.1.3 Vertical polarisation, variation of \( \Psi \)

Referring to Fig. 3:8 the following values are used:

- both load impedances are matched (equals 318 \( \Omega \))
- \( \Phi \) angle is equal to 0 degrees
- \( \Psi \) angle is varied as 30, 60 and 90 degrees
- vertical polarisation (magnetic field vector parallel to the ground)

Figure 3:11. Induced current in load, \( \Phi = 0, \Psi = 30, 60, 90 \).

From Fig. 3:11 it could be seen that when \( \Psi \) becomes smaller the contributions of the vertical raisers at the loads will no longer be negligible compared to the contribution of the distributed sources along the wire. That is why the current minimums for higher frequencies are vanishing.
3.5.1.4 Vertical polarisation, variation of Phi

Referring to Fig. 3:8 the following values are used:

- both load impedances are matched (equals 318 Ω)
- Phi angle is varied as 0, 30 and 60 degrees
- Psi angle is equal to 90 degrees
- vertical polarisation (magnetic field vector parallel to the ground)

![Graph showing induced current in load, Phi = 0, 30, 60, Psi = 90.](image)

Figure 3:12. Induced current in load, Phi = 0, 30, 60, Psi = 90.

From Fig. 3:12 it could be seen that the induced current is proportional to cosine of the incidence angle Phi.

3.6 Coupling analysis by the method of moments

The method of moments [1] is a mathematical method to reduce a functional equation (integral equation) to a matrix equation which can be solved on a computer. Depending on the problem the solution can be exact or approximate. Often the solution is in the form of an infinite summation and therefore approximate when solved on a computer. If there is no simplification done when the integral formulation of the problem is set up, the accuracy in the solution will only depend on the numerical process used by the computer (for instance truncation of infinite summation).

One approximation often used in calculations of scattering from wires is the so called thin wire approximation. In using the thin wire approximation it is assumed that the current is uniformly distributed over the wire perimeter and it is therefore possible to place a concentrated current on the wire axis instead of the distributed current on the surface. In doing so, but still satisfying the boundary condition on the wire surface (i.e. the total tangential electric field on the perfectly conducting wire surface equals zero), a relatively simple expression for the current distribution along the wire length as a function of the exciting incident field can be obtained.
There exist a number of computer codes that use the method of moments and the thin wire approximation for calculations on antennas as well as scatters. However, many of these codes are specialised in the sense that they are only suited for a particular set of problems (for instance one type of excitation). A simple but yet generally applicable computer code usable on a personal computer (PC) is the code "Analysis of Wire Antennas and Scatters", AWAS, [3]. With this code it is possible to analyse simple geometries consisting of loaded (distributed or lumped impedances) wires excited by either voltage generators or by an incident plane wave. It is only possible to calculate the response in the frequency domain with the code AWAS.

3.7 Comparison between the transmission line theory and the method of moments used for analysis of coupling to wires

The different results for the current in one of the terminations of the wire shown in Fig. 3:1 given by the transmission line theory and the method of moments are shown by an example.

The example is a wire with a length of 1 m and placed 0.1 m over a perfectly conducting ground plane. The wire is terminated with 50 ohms resistors at both ends. The excitation is an incident plane wave with the electric field vector parallel to the wire (\(\Phi = 90\)) and the angle of incidence (\(\Psi\)) is 45 degrees, see Fig. 3:13.

Calculations following the transmission line theory are performed with the computer code NULINE [2] and calculations based on the method of moments are done with the code AWAS [3].

Figure 3:13. Configuration used for comparison between transmission line theory and method of moments, \(Z_1\) and \(Z_2\) equals 50 ohm.
The calculated current in the termination $Z_2$ (same as in termination $Z_1$) caused by an incident field with the field strength 1 V/m is shown in Fig. 3:14.

![Line response graph](image)

Figure 3:14. Current in termination $Z_2$ caused by an incident field with field strength 1 V/m. The solid line represents transmission line theory calculated with NULINE [2] and the crosses joined with a dotted line represents the method of moments calculated with AWAS [3].

From the results shown in Fig 3:14 the following conclusions can be drawn:

- The agreement between transmission line theory and the method of moments is very good for frequencies below the resonance region.

- The current calculated with the method of moments is slightly lower than the current obtained with transmission line theory, this is due to radiation losses which are encountered for in the method of moments.

- The transmission line theory gives resonances at frequencies where the length of the wire is equal to multiples of the wavelength. The method of moments gives additional resonances slightly below the ones given by transmission line theory. This is due to the raisers at both ends which in the method of moments are modelled as wires with a length of 0,1 m each, so the total wire length is actually 1,2 m. For a physical wire with terminations, the model used by the method of moments is thought to be the most representative.

- The current at the resonances goes to zero for the transmission line theory (not very clear in the figure due to graphical resolution) but is finite for the method of moments. This is, again, due to radiation losses.

In summary it can be stated that the simpler (and more well-known) transmission line theory can be used up to the resonance region (even for higher frequencies if the
additional resonances, mentioned above, are not interesting) with good results. If the wire under consideration is not straight and/or not at constant height over the ground plane the method that should be used is the method of moments.

### 3.8 Shielded wires

For the case when the wire is shielded the problem can be divided into two sub problems. The first sub problem is to calculate the current distribution in the shield. This problem is treated with either transmission line theory or the method of moments, as discussed in the previous chapters. For low frequencies even the simple method of chapter 3.4 could be used.

The second sub problem is to calculate, from the current in the shield, the induced current in the wire. This can be solved by the knowledge of the transfer impedance of the shield.

The transfer impedance is defined by:

\[ dV(x, \omega) = Z_{t}(\omega) \cdot I_{l}(x, \omega) \cdot dx \]  \hspace{1cm} (3:5)

where:
- \( Z_{t}(\omega) \) = transfer impedance, [ohm/m]
- \( V(x, \omega) \) = induced voltage on the inside of the shield, [V]
- \( I_{l}(x, \omega) \) = current on the outside of the shield, [A]
- \( x \) = the length co-ordinate of the shield (without loss of generality here assumed to be the Cartesian x direction)
- \( d = \) denotes derivative (length and voltage increment respectively)

The interpretation of Eq. (3:5) is that the inner conductor can be viewed as being excited by a distribution of voltage sources along the wire, i.e. the same situation as in Fig. 3:7 with \( dV(x) \) exchanged to \( dV(x, \omega) \).

Referring to the situation in Fig. 3:7, which is treated for instance with the code NULINE, it is seen that if the driving voltage sources \( dV(x) \) is exchanged to \( Z_{t}I_{l}(x) \) the response of the shielded cable could be calculated in the same way. This is valid under the assumption that the lumped voltage sources at the wire ends, which are due to the vertical risers at the loads, are negligible in comparison with the response of the distributed sources along the wire (the shielded inner wire is only excited by the distributed voltage sources along the shield).

As mentioned in chapter 3.5 the infinitesimal voltage sources \( dV(x) \) is equal to the total tangential electric field at the location of the wire but with the wire removed. Thus if an incident field, which gives the same total field distribution at the wire location as the calculated current distribution on the shield, the current on the inner conductor could easily be calculated. An example when such an incident field could be found would be the case of a short shielded cable for which the current can be thought as constant over the entire length. In this case the incident field should be a horizontal polarised field with a field strength which makes the total field strength (incident plus reflected) equal to \( Z_{t}I_{l} \).
The transfer impedance of the shield is often unknown, it is therefore necessary to assume some values for the calculations. In table 3:1 some sort of worst case values for different types of shielded cables are given. In using the values in table 3:1 the transfer impedance is given by: \( Z_T = R + j\omega L \).

<table>
<thead>
<tr>
<th></th>
<th>R [ohm/m]</th>
<th>L [H/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single shield</td>
<td>4\times10^{-3}</td>
<td>3\times10^{-9}</td>
</tr>
<tr>
<td>Double shield</td>
<td>2\times10^{-3}</td>
<td>3\times10^{-10}</td>
</tr>
<tr>
<td>Triple shield</td>
<td>1.5\times10^{-3}</td>
<td>3\times10^{-11}</td>
</tr>
</tbody>
</table>

Table 3:1. Worst case transfer impedance for braided shields.
4 Radiation from wires

4.1 Introduction

Also when performing radiated emission tests on an equipment, the causes of EMC problems are often found to be the cables connected to the EUT. Even if the EUT is small and only contains a single printed circuit board the emission from the traces on the circuit board can radiate sufficiently enough for the EUT to fail to comply with the emission limits.

For prediction purposes even the traces on a circuit board can be viewed as radiating wires, either in free space or in the vicinity of a ground plane.

When the radiation from wires (and traces on circuit boards) are to be computed fairly complicated equations have to be solved. This can, for instance, be done on a computer by transforming the (integral) equations to matrix equations by the use of the method of moments [1]. This approach is used by the computer code AWAS [3], already discussed and used in the preceding chapters. With the code AWAS it is possible to calculate the radiation from an assembly of wires excited by a voltage source. Thus, the code calculates the radiation for a three-dimensional model.

Even if the real world is three-dimensional many helpful conclusions can be drawn from the simulation of two-dimensional models. For instance if a current carrying wire is thought as being infinitely long the radiation from the wire can be computed quite easily and the effects of finite sized ground planes in the vicinity can be studied. Such studies are important in the understanding of the radiating mechanism and how to reduce the radiated emission from wires and circuit board traces.

4.2 Radiated emission from a data cable

Many civilian EMC standards require (for instance FCC and VDE) that the radiated emission from an EUT should be measured on an open area test site. The distance from the EUT should be, for instance, 10 meters and the measuring antenna should be scanned in the vertical plane from 1 to 4 meters above the ground plane. In order to show how difficult it could be to fulfill the requirement of such a standard an example is considered.

The example is a simple two-wire line connecting a voltage source of strength 1 V at one end with a resistor load of 50 ohm at the other end. The voltage source is generating a sinusoidal signal with the fundamental frequency 40 MHz. The dimensions and relative placement, to the ground plane, are shown in Fig. 4:1. The radiated emission at a distance of 10 meters from the EUT and at the heights 1, 2, 3 and 4 meters have been calculated with the code AWAS [3]. The results are presented in table 4:1. The levels could, for instance, be compared with the limit given by CISPR which could be as low as 30 dBµV/m at the frequency 40 MHz. The conclusion is that even a simple two-wire line with these modest dimensions and fed by a voltage source with as low amplitude as 1 V can produce radiated levels sufficiently high for violating EMC standards.
Figure 4:1. Cable over a ground plane. L=0.5 m; h =1 m; d=0.02 m. Voltage source, 1V, in one end and 50 ohm resistance in the other end.

<table>
<thead>
<tr>
<th>Height of observation point [m]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiated emission level [dBμV/m] (distance 10m)</td>
<td>26</td>
<td>32</td>
<td>34</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 4:1. Radiated emission levels at a distance of 10 meters from the cable shown in Fig. 4:1. Calculated at heights 1, 2, 3 and 4 meters with the code AWAS [3]. (CISPR limit as low as 30 dBμV/m)

4.3 Radiation from wires, 2D-case

As already mentioned the two-dimensional case of a radiating wire can often be very illustrative and helpful in understanding the radiation mechanism. The study of radiation in the 2D-case can also be helpful in evaluating different techniques to reduce the radiation from a wire. As seen from the example in the preceding chapter this can be crucial in order to achieve the goal of compliance with an EMC standard. Therefore, the radiation from a two-dimensional wire, i.e. an infinitely long wire, placed in free space as well as placed near a ground plane are treated in the succeeding chapters.

4.3.1 Wire in free space

For a thin wire the radiation can be obtained as the radiation from a line source, i.e. a filament of current. If the wire is thick the radiation can be found as the summation of line sources placed around the perimeter of the wire. In this chapter we only deals with thin wires which can be considered as line sources. It should be pointed out that this approximation is very good for most cases, at least if the wire is thin compared to the wavelength.
A two-dimensional line source with the strength $I_0$ is shown in Fig. 4:2. The line source has an infinite extent in the $z$-direction and the current is constant in the $z$-direction.

![Diagram of a line source](image)

Figure 4:2. A line source of strength $I_0$.

The equation for the radiation from a line source can be found in many standard textbooks on electromagnetic field theory, for instance [4, ch. 5-6]. The electric field from a $z$-directed line source of electric current will only be $z$-directed and can be determined by:

$$E_z = \frac{-k^2 I_0}{4\pi \varepsilon} H_0^{(2)}(kr)$$  \hspace{1cm} (4:1)

where $H_0^{(2)}$ is the Hankel function of the second kind and order zero. The argument for the Hankel function is the wave number times the distance from the line source to the observation point, i.e. $r = \sqrt{x^2 + y^2}$. Eq. (4:1) describes outward travelling cylindrical waves.

In Fig. 4:3 the normalised amplitude of Eq. (4:1) as a function of the radial distance $r$ is shown.

![Graph of normalised amplitude](image)

Figure 4:3. Normalised amplitude of Eq. (4:1).
From Fig. 4:3 it can be seen that the amplitude of the radiated field from a line source, for large distances, decays as $\frac{1}{\sqrt{r}}$, i.e. 10 dB per decade distance.

### 4.3.2 Radiation from a wire over an infinite ground plane

The radiation from a current carrying wire can be reduced by introducing a ground plane in the vicinity of the wire. This technique is often used when the layout of a printed circuit board is done.

The radiation reduction is easily understood by considering the case of an infinite ground plane in which case the image theory [4] can be used. The current can for this case be imaged in the infinitely large ground plane and the resulting equivalent problem is the original current plus an image current flowing in the opposite direction and the ground plane removed, this is shown in Fig. 4:4. It should be noted that the equivalent problem is only valid for the upper (referring to Fig. 4:4) region. In the region below the ground plane we will have no fields (the ground plane is shielding the lower region).

![Diagram](image)

**Figure 4:4.** Wire over an infinite ground plane and the imaged equivalent problem.

If the distance between the wire (current) and the ground plane is small (compared to the wavelength) the real and imaged current will efficiently "cancel out" each other and the radiation will therefore be very small. By superposition the radiation from the wire over the ground plane can be determined as the sum of Eq. (4:1) for the real and the imaged current. If we are only interested in far-field radiation we can use the large argument formulas for the Hankel function and the usual far-field approximations for amplitude and phase terms, i.e. $r \approx \rho$ for amplitude factors and $r = \rho + \Delta L$ for phase factors, see Fig. 4:5.

![Diagram](image)

**Figure 4:5.** Definition of far-field parameters.
The expression for the normalised field in the far-field for a wire at a height \( h \) over the ground plane, as a function of the angle \( \theta \), is given in Eq. (4:2). The normalisation is against the radiated field from a single wire in free space and therefore Eq. (4:2) also gives the radiation reduction achieved by adding the ground plane.

\[
|E_r^{nw}| = \sqrt{2[1 - \cos(2kh\sin \theta)]} \tag{4:2}
\]

It should be noted that Eq. (4:2) is only valid for angles \( \theta \) in the range \( 0 \) to \( \pi \) radians, because of the ground plane.

The maximum value of Eq. (4:2) is, for small heights, obtained in the direction \( \theta = \pi/2 \). Since the maximum radiation is of interest in EMC analysis some sample values are given in Table 4:2. The radiation reduction, compared to a wire in free space, achieved by adding the ground plane is simply obtained by changing the sign of the radiation amplitude given in Decibels.

<table>
<thead>
<tr>
<th>Height of wire over ground plane ([\lambda])</th>
<th>0,001</th>
<th>0,005</th>
<th>0,01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation amplitude normalised to radiation from wire in free space ([\mathrm{dB}])</td>
<td>-38</td>
<td>-24</td>
<td>-18</td>
</tr>
</tbody>
</table>

Table 4:2. Normalised radiation amplitude for a line source of unit strength over a ground plane.

### 4.3.3 Radiation from a wire over a finite sized ground plane

In the preceding chapter it was seen that the radiation from a current carrying wire can be reduced by placing a ground plane in the vicinity of the wire. However, this was shown by using image theory which requires the ground plane to be infinitely large. In practical situations the ground plane is never, of course, infinitely large or could not even be approximated as such. If the ground plane has a finite extent (in \( x \)-direction in Fig. 4:4) the image theory could not be used and we have to use a more complex model. The derivation of the equations and how they can be solved by the method of moments is given in chapter 4.3.3.1.

![Figure 4:6. Geometry for a line source over a finite sized ground plane. \( d \) is the displacement, in \( x \)-direction, of the line source from the center of the ground plane.](image-url)
The most obvious effect of the finite ground plane is that we will also have fields below the ground plane. This is shown in Fig. 4:7 where the radiation pattern from a wire at a height of 0.01λ over a ground plane of width 1λ is shown. The outermost circle in the figure corresponds to the radiation from a single wire in free space. This means, for instance, that the radiation reduction in the θ=90° direction (maximum radiation) is about 17 dB, compared with the value of 18 dB for the infinite ground plane given in table 4:2.

![Figure 4:7. Radiation pattern for a wire over a finite ground plane. W=λ, d=0, h=0.01λ. 10 dB/div.](image)

In Fig. 4:8 the radiation pattern from a wire at a height of 0.01λ over a ground plane of width 1λ is shown. The wire is placed 0.1λ from the right edge of the ground plane, i.e. d=0.4λ. When comparing with Fig. 4:7 it can be seen that the radiation pattern is "tilted" but the maximum radiation remains at approximately the same level.

![Figure 4:8. Radiation pattern for a wire over a finite ground plane. W=λ, d=0.4λ, h=0.01λ. 10 dB/div.](image)

In EMC applications the radiation pattern is mostly of no interest, but the maximum radiation is. When comparing the results given in Fig. 4:7 - 4:8 with the results for a wire over an infinite ground plane, given in table 4:2, the radiation reduction seems to be about the same. The question now is how wide the ground plane has to be for a given reduction.
of the maximum radiation (compared to a single wire without ground plane). In Fig. 4:9 the reduction, compared to a wire in free space, of the maximum radiation as a function of the ground plane width is shown. The wire is centred and placed at different heights over the ground plane (0.001λ corresponds to 1 mm at 300 MHz).

Figure 4:9. Reduction of the maximum radiation as a function of the ground plane width. The wire is centred.

The radiation reduction is not just a function of the ground plane width but also (among other parameters) the height of the wire over the ground plane, this behaviour is shown in Fig. 4:10.

Figure 4:10. Reduction of the maximum radiation as a function of the height of the wire over the ground plane. The wire is centred.

Another parameter that will have influence on the radiation reduction is the placement of the wire relative to the ground plane centre in the x-direction, referring to Fig. 4:6. As
seen from figure 4:11 it is very important that the wire is placed straight over the ground plane in order to achieve a significant radiation reduction.

Figure 4:11. Reduction of the maximum radiation as a function of the displacement from the center. The wire is placed at a height of 0.001\(\lambda\) over a ground plane of width 0.01\(\lambda\).

Instead of sweeping one parameter at the time as in Fig. 4:9 - 4:11 it could be interesting to sweep all parameters at the same time, i.e. the same as sweeping the frequency. This is shown in Fig. 4:12 for a wire at a height of 1 mm over a ground plane of width 10 mm, the wire is centred.

Figure 4:12. Reduction of the maximum radiation as a function of the frequency. The wire is centred and placed at a height of 1 mm over a ground plane of width 10 mm.

As a summary the following guidelines are given:
The wire or the trace on a printed circuit board should be placed as near as possible to the ground plane or the return path (Fig. 4:9 - 10).

In order to achieve maximum radiation reduction it is essential that the wire or the trace on a printed circuit board is centered over the ground plane or the return path (Fig. 4:11).

The image theory could be used with reasonable accuracy even for finite sized ground planes, at least for the calculation of the maximum radiation (Fig. 4:9).

4.3.3.1 Wire over a finite sized ground plane - Theoretical treatment

The geometry for the problem is shown in Fig. 4:13. The task is to calculate the radiation pattern from the line source (current carrying wire) over the finite sized ground plane. In order to achieve the goal we have to calculate the current distribution in the ground plane first and then integrating the distribution in order to obtain the radiation pattern. To be able to solve the problem on a computer we use the method of moments [1] to transform the integral equations into matrix equations. The procedure is as follows.

![Wire over a finite sized ground plane](image)

Figure 4:13. Geometry for the problem of a current carrying wire over a ground plane.

The current in the wire is z-directed, i.e. $I = 2I_0$. The electric field from a current in free space, i.e. no ground plane present, will also be z-directed and can be written as, see Eq. 4:1:

$$E = 2E_z = \frac{-k^2 I_0}{4\omega \varepsilon} H_0^{(2)}(kp), \text{ field from line source in free space}$$

(4:3)

where $p$ is the distance from the line source to the observation point and $H_0^{(2)}$ is the Hankel function of second kind and order zero.

By using the equivalence principle [4, ch. 3-5] the perfectly conducting ground plane is replaced by its physical equivalent, i.e. electric current sources.
Figure 4.14. Equivalent problem to the problem shown in Figure 4.13.

All currents are now acting in free space and the total z-directed electric field at the observation point can be written as (summation of expressions like equation (4.3)):

\[ E_{z}^{\text{tot}} = -\frac{k^{2}}{4\pi \varepsilon_{0}} \left\{ I_{0} H_{0}^{(2)}(k\rho) + \int_{-W/2}^{W/2} I_{\text{strip}} H_{0}^{(2)}(kR) d\chi' \right\} \]  \hspace{1cm} (4.4)

where \( R \) is the distance from a current filament at the strip to the observation point.

The boundary condition on the ground plane strip is that total tangential electric field must vanish (perfect electric conductor), thus:

\[ E_{z}^{\text{tot}} \bigg|_{\text{strip}} = 0 = -\frac{k^{2}}{4\pi \varepsilon_{0}} \left\{ I_{0} H_{0}^{(2)}(k\rho) + \int_{-W/2}^{W/2} I_{\text{strip}} H_{0}^{(2)}(kR) d\chi' \right\} \bigg|_{\text{strip}} \]  \hspace{1cm} (4.5)

Letting the source, \( I_{0} \), be unity and rearranging the equation we get:

\[ H_{0}^{(2)}(k\rho) \bigg|_{\text{strip}} = -\frac{k}{2\pi \varepsilon_{0}} \int_{-W/2}^{W/2} I_{\text{strip}} H_{0}^{(2)}(kR) d\chi' \bigg|_{\text{strip}} \]  \hspace{1cm} (4.6)

Equation (4.6) is an integral equation for the unknown current distribution, \( I_{\text{strip}}(x') \), in the ground plane strip. In order to solve the integral equation (4.6) we use the method of moments. The first step in this method is to expand the unknown electric current, \( I_{\text{strip}}(x') \), in basis functions, thus:

\[ I_{\text{strip}}(x') = \sum_{n=1}^{N} a_{n} b_{n}(x') \]  \hspace{1cm} (4.7)

where \( a_{n} \) are coefficients and \( b_{n}(x') \) are basis functions.

Insertion of the series expression the integral equation can be written as:
\[ H_0^{(2)}(kp) \big|_{\text{strip}} = - \sum_{n=1}^{N} a_n \int_{-W/2}^{W/2} b_n(x') H_0^{(2)}(kR)\,dx' \bigg|_{\text{strip}} \]  

(4.8)

We can choose the basis functions as we wish and therefore the only unknowns in Eq. (4.8) are the coefficients \(a_n\). Since we only have one equation and \(N\) unknowns we have to create additional equations. This can be done by taking a suitable inner product of both sides of the equation with a set of weighting equations. Thus, defining a set of weighting functions, \(w_m(x)\), \(m = 1,2,\ldots,N\), and an inner product as:

\[ \langle f, g \rangle = \int_{-W/2}^{W/2} fg\,dx \]  

(4.9)

Taking the inner product of both sides of the integral equation (4.8) gives:

\[ \langle w_m, H_0^{(2)}(kp) \rangle_{\text{strip}} = - \sum_{n=1}^{N} a_n \left[ \int_{-W/2}^{W/2} b_n(x') H_0^{(2)}(kR)\,dx' \right]_{\text{strip}}, \quad m = 1..N \]  

(4.10)

We are also free to choose the weighting functions as we wish, for simplicity we choose them as Dirac pulses (i.e. point matching) thus, \(w_m(x) = \delta(x - x_m)\) where the points \(x_m\) have to be located on the ground plane. With this choice of weighting functions we avoid the integration associated with the inner product and Eq. (4.10) becomes:

\[ H_0^{(2)}(kp_m) = - \sum_{n=1}^{N} a_n \int_{-W/2}^{W/2} b_n(x') H_0^{(2)}(kR_m)\,dx', \quad m = 1..N \]  

(4.11)

\[
\begin{align*}
x_m &= (2m-1) \frac{\Delta x'}{2} - d - \frac{W}{2} \\
\Delta x' &= \frac{W}{N}, \quad \rho_m = \sqrt{x_m^2 + h^2} \\
R_m &= \left| x' + \frac{W}{2} - (2m-1) \frac{\Delta x'}{2} \right|
\end{align*}
\]  

(4.12)

where (see Fig. 4.15)
Figure 4.15. Point matching at the strip.

The basis functions are now chosen as pulses:

$$b_n(x') = \begin{cases} 1, & (n-1)\Delta x' - \frac{W}{2} \leq x' \leq n\Delta x' - \frac{W}{2} \\ 0, & \text{elsewhere} \end{cases}$$  \hspace{1cm} (4.13)$$

$$\Rightarrow H_0^{(2)}(k\rho_m) = -\sum_{n=1}^{N} a_n \int_{(n-1)\Delta x' - W/2}^{n\Delta x' - W/2} H_0^{(2)}\left(k\sqrt{x'^2 + h^2} - (2m-1)\frac{\Delta x'}{2}\right) dx' \hspace{1cm} (4.14)$$

Equation (4.14) can also be written in matrix form as:

$$[E_m] = -[Z_{mm}] [a_n]$$  \hspace{1cm} (4.15)

where:

$$E_m = H_0^{(2)}(k\rho_m) = H_0^{(2)}(k\sqrt{x'^2 + h^2}) = H_0^{(2)}\left(k\sqrt{(2m-1)\frac{\Delta x'}{2} - d - \frac{W}{2}} + h^2\right)$$  \hspace{1cm} (4.16)

$$Z_{mn} = \int_{(n-1)\Delta x' - W/2}^{n\Delta x' - W/2} H_0^{(2)}\left(k\sqrt{x'^2 + h^2} - (2m-1)\frac{\Delta x'}{2}\right) dx' = \begin{cases} \frac{\Delta x' H_0^{(2)}(k\Delta x'|m-n|)}{\Delta x' \left[1 - j \frac{2}{\pi} \ln\left(\frac{1.781k\Delta x'}{4e}\right)\right]}, & m \neq n \\ \frac{\Delta x' H_0^{(2)}(2m-1)\Delta x' \Delta x'}{\Delta x' \left[1 - j \frac{2}{\pi} \ln\left(\frac{1.781k\Delta x'}{4e}\right)\right]}, & m = n \end{cases}$$  \hspace{1cm} (4.17)$$

The matrix components given by Eq. (4.16) and (4.17) can easily be calculated on a computer and finally we can determine the current coefficients, $a_n$, by matrix inversion. When the currents coefficients are found we also have the current distribution in the ground plane, as described by equation (4.7), and we can calculate the radiated fields.

In order to calculate the radiation pattern we have to integrate over the current distribution in the ground plane, the procedure will be as follows.
The total field is given by Eq. (4:4) which, for convenience, is repeated here:

$$E_z^{tot} = \frac{-k^2}{40\pi} \left[ H_0^{(2)}(kr) + \int_{-w/2}^{w/2} I_{strip}(x') H_0^{(2)}(kR) dx' \right]$$  \hspace{1cm} (4:18)

where the distances R and r are defined in Figure 4:16.

The location of the wire is defined as the reference point. In this way it is easy to compare the radiation from a single wire without a ground plane with the case of a wire with an adjacent ground plane (i.e., it is easy to see the reduction of the radiation that is possible to achieve with an adjacent ground plane). In the far field, i.e., where $r \to \infty$, we can use the large argument formula for the Hankel function and we use the usual approximations for the amplitude and phase factors, i.e., $R = r$ for amplitude factors and $R = r + \Delta L$ for phase factors (see Figure 4:16).

Using these approximations and normalising the total field to the field produced by a single wire in free space we obtain:

$$E_z^{tot} = \left\{1 + \int_{-w/2}^{w/2} I_{strip}(x') e^{-iR_0} dx'\right\}$$  \hspace{1cm} (4:19)

![Diagram](attachment:image.png)

**Figure 4:16. Definition of radiation parameters.**

By insertion of the series approximation for the current, Eq. (4:7), the radiation pattern given by equation (4:19) can easily be computed.

The equations presented in this chapter were implemented in a computer code that can be run under Windows. Example of results obtained by this computer code is presented in chapter 4.3.3.
4.3.4 Radiation from a wire in the vicinity of two finite sized ground planes

In chapter 4.3.3 it was seen that it is possible to reduce the radiation from a current carrying wire by introducing a ground plane in the vicinity of the wire. It was also seen that the reduction was most significant in the "shadow" region (i.e. below the ground plane). By using two ground planes instead of one, the radiation reduction can be increased even further. The geometry for a wire in the vicinity of two ground planes is shown in Fig. 4.17.

![Geometry for a wire in the vicinity of two finite sized ground planes.](image)

Figure 4:17. Geometry for a wire in the vicinity of two finite sized ground planes.

In Fig. 4.18 the radiation pattern for a wire placed between two ground planes both of widths \(1\lambda\) (\(W_1=W_2=1\lambda\)) is shown. The distance from the wire to both ground planes is 0.01\(\lambda\) (\(h_1=h_2=0.01\lambda\)) and the wire is centred with respect to both ground planes centre (\(d_1=d_2=0\)).

![Radiation pattern from a wire over a ground plane and a wire between two ground planes compared with the radiation from a wire in free space. The outermost circle corresponds to the radiation from a single wire in free space. 20 dB/div.](image)

Figure 4:18. Radiation pattern from a wire over a ground plane and a wire between two ground planes compared with the radiation from a wire in free space. The outermost circle corresponds to the radiation from a single wire in free space. 20 dB/div. \(W_1=1\lambda\), \(h_1=0.01\lambda\), \(d_1=0\), \(W_2=1\lambda\), \(h_2=0.01\lambda\), \(d_2=0\).
As seen from Fig. 4:18 the radiation reduction is considerable in all directions with the use of two ground planes. Even if one of the ground planes is relatively small and placed at a larger distance from the wire, the radiation reduction can be quite high, this is shown in Fig. 4:19.

\[ \begin{align*}
H_0^{(2)}(kR) \bigg|_{\text{strip1} \& \text{strip2}} &= - W^{1/2} \int_{-W^{1/2}}^{W^{1/2}} I_{\text{strip1}} H_0^{(2)}(kR_1) \, dx \\
&\quad - W^{3/2} \int_{-W^{3/2}}^{W^{3/2}} I_{\text{strip2}} H_0^{(2)}(kR_2) \, dx
\end{align*} \]

where the distances \( R_1 \) and \( R_2 \) represent the distances between a current filament at ground plane one and two respectively to the matching point.

The details in the solution of the integral equation are very much alike the details given in chapter 4.3.3.1 and will therefore not be given here. However, the equations were implemented in a computer code running under Windows and some example results are presented in chapter 4.3.4.
5 Crosstalk between wires

5.1 Introduction

Crosstalk is often used as the name for the interaction between wires, i.e. induced current in one wire caused by a current in another wire. The crosstalk is often analysed as either inductive or capacitive coupling between the wires. The advantage of this approach is that simple circuit models can be used for the prediction of the crosstalk. However, these models assume the wires to be short compared to the wavelength.

Another approach that can be taken is that of scattering analysis, the same method that was used for the radiating wires. For very long wires the two-dimensional model can be used and this model can also be used for parameter studies. For real three-dimensional problems with complicated wire layouts, commercial computer codes such as AWAS [3] can be used.

5.2 Simple two-dimensional model

The geometry for the two-dimensional problem of crosstalk between two wires is shown in Fig. 5:1.

\[ \begin{array}{c}
\text{d} \\
\leftrightarrow \\
I \bullet \quad \bullet I^{\text{ind}} \\
\leftrightarrow D \\
\end{array} \]

Figure 5:1. Geometry for crosstalk in the 2D-case.

From Eq. (4:1) we know the field from the line source, i.e. the source wire (I in Fig. 5:1), and if the receptor wire is perfectly conducting we also know that the total tangential electric field on the surface of the receptor wire has to vanish. Further, if the diameter, d, of the receptor wire is small compared to the wavelength, we can approximate the current distribution in the wire as a concentrated current filament situated at the centre of the wire (thin wire approx.). If we also assume the distance between the wires, D, to be large compared to the diameter of the receptor wire, d, we arrive at a quite simple formula for the induced current in the receptor wire (the current in the source wire is set at unity):

\[ I^{\text{ind}} = \frac{-H_0^{(2)}(kD)}{1 - j \frac{2}{\pi} \ln \left( \frac{\gamma d}{4} \right)} ; \quad \text{where } \gamma = 1.781 \quad (5:1) \]

The absolute value of Eq. (5:1) is plotted in Fig. 5:2.
Figure 5:2. Normalised induced current in receptor wire as a function of the distance between the two wires. Normalisation is against the current in the source wire.

From Fig. 5:2 it can be seen that the crosstalk can be reduced by separating the wires, and it is also clear that this is most efficient if the wires are close together.

5.3 Crosstalk between wires in the vicinity of a ground plane

Instead of using the simple model presented in chapter 5.2 (Eq. 5:1) the computer codes developed for the radiation from wires in the vicinity of one or two ground planes, chapter 4.3.3.1 and 4.3.4.1 respectively, can be used. In order to use these computer codes for crosstalk predictions, in the 2D-case, the ground plane dimension can be set equal to twice the wire diameter and the number of basis functions on the ground plane equal to two (in this way the two matching points will be separated by a distance equal to the wire diameter), see Fig. 5:3. The obtained current density then have to be multiplied by a factor equal to twice the diameter, d, (the ground plane width).

Original problem

Model using ground plane

Figure 5:3. Dimensions of ground plane when using the computer codes of ch. 4.3.3.1 and 4.3.4.1 to predict crosstalk.
For example if the crosstalk between a line source and a wire of diameter 0.001\( \lambda \) at a distance of 1\( \lambda \) is to be predicted the dimensions, referring to Fig. 4.6, have to be set to: \( W=0.002\lambda \), \( d=0 \), \( h=1\lambda \).

The advantage of using this model is that the program of chapter 4.3.4.1 can be used to study the effects of placing a ground plane between the two wires, see Fig. 5.4.

![Ground plane diagram](image)

**Figure 5.4.** Crosstalk between two wires with a ground plane in between.

In Fig. 5.5 the achieved attenuation when a ground plane of width 1\( \lambda \) is placed between two wires at a mutual distance of 1\( \lambda \) is shown. The attenuation is defined as the difference between the situation without a ground plane and the situation with a ground plane between the two wires.

![Attenuation graph](image)

**Figure 5.5.** Attenuation achieved by the placement of a ground plane of width 1\( \lambda \) between two wires at a distance of 1\( \lambda \) as a function of the distance D1 in Fig. 5.4. \( W=1\lambda \), \( D1+D2=1\lambda \), \( c=0 \), \( d=0.001\lambda \).

As seen from Fig. 5.5 it is possible to reduce the crosstalk significantly by the placement of a ground plane between the two wires.
6 Coupling through apertures

6.1 Introduction

Apertures are important since they offer a leakage path for electromagnetic energy from one region into another. In EMC-design it is often very helpful to be able to quantify the amount of leakage in order to compare with the contributions from other coupling paths. In many cases, however, it is obvious simply by looking at the system that the most important coupling path is an aperture. In such a system it is necessary to have the knowledge of the effects of the aperture in order to be able to determine the interaction between, e.g., the outer environment and the electronics placed behind the aperture.

In this chapter the amount of leakage of electromagnetic energy through apertures is treated. Since the theory in the general case is very complex the special case of an aperture in an infinite ground plane is treated. This approximation is, however, often a very good approximation also for practical systems, at least in a certain frequency range.

6.2 General treatment

The problem of an aperture in a ground plane is most conveniently handled by the use of the equivalence principle. The equivalence principle is based on the uniqueness theorem, and is in [4] expressed as follows:

"A field in a lossy region is uniquely specified by the sources within the region plus the tangential components of \( \mathbf{E} \) over the boundary, or the tangential components of \( \mathbf{H} \) over the boundary, or the former over a part of the boundary and the latter over the rest of the boundary."

To illustrate the use of the equivalence principle an aperture in an infinite ground plane illuminated by an incident plane wave, as shown in Fig. 6:1, is considered.

![Diagram](attachment:Figure_6.1.png)

**Figure 6.1.** Aperture in infinite ground plane excited by an incident plane wave.
According to the equivalence principle, as stated above, it is sufficient to know the tangential field over the boundary plus the sources within the enclosed region to know the field everywhere in the region. When applied to region A in Fig. 6:1 and also applying the theory of imaging [4] the equivalent problem shown in Fig. 6:2 can be defined.

\[ \hat{n} \]

Region A \[
2M_{ap}
\]
Region B

Figure 6:2. Equivalent problem for region A to the one shown in Fig. 6:1.

In Fig. 6:2 the equivalent magnetic current is \( M_{ap} = E_{ap} \times \hat{n} \), where \( E_{ap} \) is the electric aperture field in the original problem (Fig. 6:1) and \( \hat{n} \) is a unit vector normal to the ground plane pointing into region A.

The field produced by a magnetic current radiating into free space, as is the case shown in Fig. 6:2, can be obtained by [7]:

\[
E^A = - \nabla \times F
\]  \hspace{1cm} (6:1)

Where:

\[
F = \frac{1}{4\pi} \int_{s'} \frac{Me^{-jR}}{R} \, ds'
\]  \hspace{1cm} (6:2)

\[
M = 2M_{ap} = 2E_{ap} \times \hat{n}
\]

\( R \) = distance between source point and observation point (point on aperture and field observation point respectively)

\( s' \) = the area where the aperture was located

And the magnetic field, \( H^A \), can be determined by:

\[
H^A = - \frac{1}{j\omega \mu} ( \nabla \times E^A + M ) = \frac{1}{j\omega \mu} ( \nabla \times \nabla \times F - M )
\]  \hspace{1cm} (6:3)

It should be noted that the expressions for the \( E \) and \( H \) fields, equation (6:1) and (6:3) are valid only for the fields in region A.

From the above equations it is obvious that the field in the half space behind the ground plane in Fig. 6:1 (region A) can be determined by the knowledge of the field distribution in the aperture. So, the remaining problem is to determine the electric field in the aperture.
6.3 Electrically small apertures

If the aperture is small compared with the wavelength the aperture distribution can be approximated with a constant distribution over the aperture. Referring to Fig. 6.3 this means that the aperture dimensions $\Delta x$ and $\Delta y$ should be much less than the wavelength.

Referring to Fig. 6.3 with the ground plane in the xy-plane and the incident plane wave polarised in the y-direction and propagating in the z-direction the equivalent magnetic current will be:

$$2M_{sp} = 2M_{sp} \hat{x} = 2E_{inc} \times \hat{z} = 2E_0 \hat{y} \times \hat{z} = 2E_0 \hat{x}$$

(6.4)

Where: $E_0$ is the amplitude of the incident wave.

Figure 6.3. Aperture in infinitely large ground plane excited by plane wave polarised tangentially to the plane of the aperture.
Figure 6:4. Co-ordinate notation.

Notations used in Fig. 6:4:

\[ \mathbf{R} : \quad \text{vector from source point to observation point} \]
\[ r' : \quad \text{vector from origin to source point} \]
\[ r : \quad \text{vector from origin to observation point} \]

Primed symbols are associated with the source point.

For far-field observations, i.e. the observation point is at a large distance (compared with the wavelength) from the aperture (the source), the \( \mathbf{R} \) in equation 6:2 can be approximated with:

\[ \mathbf{R} = r - r' \cos(\psi) \quad \text{for phase variations} \]
\[ \mathbf{R} = r \quad \text{for amplitude variations} \]

Where the term \( r' \cos(\psi) \) can be calculated as (see Fig. 6:4).

\[ r' \cos(\psi) = r' \hat{\mathbf{\hat{\mathbf{r}}}'} = (\hat{x}' \mathbf{x'} + \hat{\mathbf{y}}')[(\mathbf{\hat{x}}' \sin(\theta) \cos(\phi) + \mathbf{\hat{\mathbf{y}}} \sin(\theta) \sin(\phi) + \mathbf{\hat{\mathbf{z}}} \cos(\theta))] = \]
\[ = x' \sin(\theta) \cos(\phi) + y' \sin(\theta) \sin(\phi) \]

And the electric vector potential, \( \mathbf{F} \), becomes.

\[ \mathbf{F} = \frac{\hat{\mathbf{\hat{x}}}}{2 \pi r} \int_{-\Delta x/2 - \Delta y/2}^{\Delta x/2 - \Delta y/2} \int_{-\Delta x/2 - \Delta y/2}^{\Delta x/2 - \Delta y/2} e^{-i2 \pi(r \cos(\psi)) / \lambda} \ dx' \ dy' \]

\[ = \frac{\hat{F}_x}{2 \pi r} \int_{-\Delta x/2 - \Delta y/2}^{\Delta x/2 - \Delta y/2} \int_{-\Delta x/2 - \Delta y/2}^{\Delta x/2 - \Delta y/2} e^{-i2 \pi r / \lambda} \ dx' \ dy' \]

\[ \text{(6.5)} \]
In the far-field region the radial components of the fields are negligible compared to the $\theta$ and $\phi$ components. Also in the far-field region the field components can be written as, see for instance [7]:

\[
\begin{align*}
E_r &= 0 \\
H_r &= 0 \\
E_\theta &= -jkF_\theta \\
H_\theta &= -\frac{E_\phi}{\eta} \\
E_\phi &= jkF_\theta \\
H_\phi &= \frac{E_\theta}{\eta}
\end{align*}
\] (6:6)

Where $\eta$ is the wave impedance in region A, free space : $\eta = 120\pi$ [ohm].

From equations (6:6) it is obvious that it is necessary to transform equation (6:5) to spherical co-ordinates:

\[
\begin{align*}
F_\theta &= F_z \cos\theta \cos\phi \\
F_\phi &= -F_z \sin\phi
\end{align*}
\] (6:7)

But still the integral of equation (6:5) is to be determined. The integral is easily solved for instance with the aid of standard mathematical handbooks. The resulting expressions for the electric fields in region A will be:

\[
\begin{align*}
E_r &= 0 \\
E_\theta &= j\frac{kE_\theta \Delta x \Delta y}{2\pi r} e^{-jk\rho} \sin\theta \cos\phi \frac{\sin X \sin Y}{X Y} \\
E_\phi &= j\frac{kE_\theta \Delta x \Delta y}{2\pi r} e^{-jk\rho} \cos\theta \cos\phi \frac{\sin X \sin Y}{X Y}
\end{align*}
\] (6:8)

Where:

\[
\begin{align*}
X &= \frac{\Delta x}{2} k \sin\theta \cos\phi \\
Y &= \frac{\Delta y}{2} k \sin\theta \sin\phi
\end{align*}
\]

The radiation intensity at a distance $r$ from the source is defined as the power density times the square of $r$, i.e. $U = r^2 W_{rd}$; where $W_{rd}$ is the power density. The power density is proportional to the square of the electric field intensity, i.e. $W_{rd} \sim E_r^2 + E_\phi^2 - \frac{1}{r^2}$, thus the radiation intensity is independent of the distance $r$. Using equations (6:8) in these definitions and taking the special cases of $\phi$ equals 0 and 90 degrees, the former is called the H-plane and the later the E-plane, gives the following expressions for the radiation intensity:

\[
U(\phi=0) = \text{(H-plane)} = \cos^2\theta \left[ \frac{k \Delta x}{\sin(\frac{\Delta x}{2} \sin\theta)} \right]^2
\]
\[ U(\phi=90) = \{E\text{-plane}\} = \left[ \frac{k\Delta y}{\sin(\theta/2 \sin(\theta))} \right]^2 \] (6.9)

Equations (6.9) are plotted in Fig. 6.5 and 6.6 respectively for different values of \( \Delta x \) and \( \Delta y \). The figures are plotted in polar co-ordinates, i.e. \( x = U(\theta)\cos(\theta) \) and \( y = U(\theta)\sin(\theta) \).

It can be seen that the radiation pattern will be more narrow for larger apertures. This will be true for apertures up to a certain size. If the size is increased further the radiation pattern will be divided into main and side lobes, i.e. several maximums in different directions. This is not shown in the figures because the plotted values are valid for a constant field distribution in the aperture, and this is true only for small apertures. Large apertures are treated in the next chapter.

Figure 6.5. Normalised radiation intensity for H-plane, \( \phi = 0 \).
Trace 1 : \( \Delta x = 0.1\lambda \), trace 2 : \( \Delta x = 0.5\lambda \), trace 3 : \( \Delta x = \lambda \).

Figure 6.6. Normalised radiation intensity for E-plane, \( \phi = 90 \).
Trace 1 : \( \Delta y = 0.1\lambda \), trace 2 : \( \Delta y = 0.5\lambda \), trace 3 : \( \Delta y = \lambda \).

The ratio of the incident power from region B to the total radiated power in region A is a measure of the attenuation in the aperture. The incident power from region B is equal to the incident power density times the aperture area, i.e. \( P_{\text{inc}} = \frac{E^2}{\eta} \Delta x \Delta y \). If the attenuation is abbreviated, \( L_{\text{ap}} \), we get:
\[ L_{ap} = \frac{P_{inc}}{P_{rad}} = \frac{E_0^2 \Delta x \Delta y}{\eta P_{rad}} \quad (6:10) \]

Where \( P_{rad} \) is the radiated power into region A. This power can be determined by integrating the power density in the far-field, given by \( \frac{1}{\eta} (E_\theta^2 + E_\phi^2) \), over the half space.

Using equations (6:8) \( P_{rad} \) can be written as:

\[ P_{rad} = \frac{1}{\eta} \left[ \frac{E_0^2 \Delta x \Delta y}{\lambda} \right] \int_0^{\pi/2} \int_0^{2\pi} \sin^2 \theta \sin^2 \phi \left( \frac{\sin X}{X} \right)^2 \left( \frac{\sin Y}{Y} \right)^2 \, d\theta \, d\phi \quad (6:11) \]

Where \( X \) and \( Y \) are the same as before.

The integral in equation (6:11) was evaluated for some aperture sizes and the corresponding aperture attenuation given by equation (6:10) is:

\[ \Delta x = \Delta y = 0.01 \lambda : \quad L_{ap} = 2400 = 34 \text{ dB} \]
\[ \Delta x = \Delta y = 0.1 \lambda : \quad L_{ap} = 24 = 14 \text{ dB} \]
\[ \Delta x = \Delta y = 0.2 \lambda : \quad L_{ap} = 6 = 8 \text{ dB} \]
\[ \Delta x = 0.1 \lambda ; \Delta y = 0.2 \lambda : \quad L_{ap} = 13 = 11 \text{ dB} \]
\[ \Delta x = 0.2 \lambda ; \Delta y = 0.1 \lambda : \quad L_{ap} = 13 = 11 \text{ dB} \]

From the examples above it can be seen that the attenuation, as defined by equation (6:10), is directly proportional to the inverse of the aperture area. It can also be concluded (the two last examples) that the attenuation is independent of the polarisation of the incoming wave, i.e. the aperture orientation. This can seem strange since the attenuation should be sensitive to the polarisation. However, the behaviour can be explained by the field distribution in the aperture which is assumed to be constant. This fact also shows that the model given in this chapter only should be used when the largest dimension of the aperture is small compared to the wavelength.
6.4 Electrically large apertures

When the aperture is large in terms of the wavelength the field distribution in the aperture cannot any longer be considered as constant. Even an aperture with a dimension of half a wavelength is considered as large. Depending on the wanted accuracy in the solution the problem can be solved in different ways. The simplest solution is to guess the field distribution in the aperture (this was done for the small aperture in the preceding chapter). If the only parameters of interest are far-field parameters the guess is not critical. If the field close to the aperture is of interest or if the wanted accuracy is very high the field distribution in the aperture has to be determined. This can be done by solving the integral equation for the problem by for instance the method of moments [1].

If the distribution in the aperture is guessed (or known by any method) the field behind the ground plane, i.e. region A in Fig. 6:1, can be determined by equations (6:1) - (6:3). The procedure to solve for the field distribution in the aperture is most simply handled by forming an equivalent problem with the equivalence principle. The application of the equivalence principle on an aperture in an infinitely ground plane is step by step:

A. Original problem. \( E^{\text{inc}} \) and \( H^{\text{inc}} \) are incident field.

B. Equivalent problem for the half space \( z>0 \). Equivalent sources are placed over the boundary plane \( z=0 \), where \( M = E \times \hat{z} \) and \( J = \hat{z} \times H \). Note that \( M=0 \) on the locations where the ground plane was placed because the tangential electric field has to be zero at the conductor.
C. Equivalent problem for the half space \( z > 0 \). A perfectly conducting plane is placed over the boundary plane \( z = 0 \). This can be done since the field in the half space \( z < 0 \) is of no interest (set equal to zero). The conductor will have the effect of "shorting out" the equivalent electric sources.

D. Equivalent problem for the half space \( z > 0 \). Imaging in the infinitely large and perfectly conducting plane. The equivalent source \( 2M_{ap} \), at the location of the aperture, as well as the incident and reflected fields are all acting in free space.
7 Coupling through slots

7.1 Introduction

A slot is defined as an aperture with one of the dimensions infinitely long. By definition a slot can never exist in reality but the slot can very often serve as an approximation of an aperture that is long in one direction, 3 - 4 wavelengths. The treatment in this part is thorough and the derived equations resulted in a computer code that is able to determine the field distribution in the slot [8].

7.2 Slot in a ground plane treated with the method of moments

The problem of an aperture in a ground plane is generally a 3D-problem, but if the aperture is large in one direction (3 - 4 wavelength) the problem can be viewed as a 2D-problem, i.e. a slot. When the problem is numerically solved on a computer the main advantage of treating the problem as a slot instead of an aperture is the achieved reduction in the number of computations needed.

If the above criterion of a dimension of at least 3 - 4 wavelength in one direction is fulfilled the agreement between the original 3D-problem and the 2D-problem is very good.

Since the problem has to be solved in different ways for different polarisation the problem is further divided into two sub problems. The first sub problem treated is the case of an incident plane wave polarised with the magnetic field vector parallel with the slot, i.e. TE to x. The second sub problem is the case of an incident plane wave polarised with the electric field vector parallel to the slot, i.e. TM to x.

7.2.1 TE-case

The geometry is shown in Fig. 7:1 and the incident field is polarised as $H^{inc} = \hat{x} H_0$. 
Figure 7.1. Slot in infinite ground plane excited by incident plane wave.

By using the equivalence principle [4] and the theory of imaging, the equivalent problems for regions \(z<0\) and \(z>0\) shown in Fig. 7.2 can be defined:

Figure 7.2. Equivalent problems (to Fig. 7.1) for regions \(z<0\) and \(z>0\).

The task is to determine the equivalent magnetic current \(M_{TE}(y')\) given the excitation of the incident plane wave. This is done with the method of moments and the technique is as follows.

The magnetic field produced by a plane sheet of x-directed magnetic current, \(M_{TE}(y')\), can be written as, for instance [7]:
\[ H_x(M_{TE}) = \frac{-k^2}{4\omega\mu} \int \limits_0^L M_{TE}(y') H_0^{(2)}(kR) dy' \]  

(7.1)

Where:
- Primed co-ordinates are associated with the source point.
- Index \( x \) of the magnetic field is denoting the direction of the field vector.
- \( R = \sqrt{(y-y')^2 + (z-z')^2} = \sqrt{(y-y')^2 + z^2} \) = distance from source to field point.
- \( k = \frac{2\pi}{\lambda} \)
- \( \omega = 2\pi f = 2\pi \frac{c}{\lambda}, \) [rad/s]
- \( \mu = 4\pi 10^{-7}, \) [H/m]
- \( H_0^{(2)} \) = Hankel function of zero order and second kind = 2D free space Green’s function.

The total field in the region \( z>0 \) can be written as the sum of the incident field, the reflected field and the field produced by the equivalent magnetic current sheet, i.e. the scattered field. The equivalent magnetic current is in this case \( 2M_{TE} \), see Fig. 7:2. Thus, the total magnetic field in the region \( z>0 \) is: \( H_x^{tot}|_{z>0} = H_x^{inc} + H_x^{ref} + H_x(2M_{TE}) \).

The total field in the region \( z<0 \) is the field produced by the equivalent magnetic current, -2\( M_{TE} \), i.e.: \( H_x^{tot}|_{z<0} = H_x(-2M_{TE}) \).

The boundary condition over the plane \( z=0; \ 0<y<L_y \), is that the tangential electric and magnetic field must be continuous, i.e. \( E_x|_{z<0, tan} = E_x|_{z>0, tan} \). The requirement of continuous tangential electric field is already satisfied by the use of the equivalent magnetic currents \( 2M_{TE} \) and \( -2M_{TE} \) for the regions \( z>0 \) and \( z<0 \) respectively.
The remaining requirement of continuous tangential magnetic field will result in the following equation for the magnetic current, \( M^{\text{TE}} \):

\[
2H_x^{\text{inc}} + H_x(2M^{\text{TE}}) = H_x(-2M^{\text{TE}}) \quad (7:2)
\]

\(2H_x^{\text{inc}} \) because \( H_x^{\text{inc}} = H_x^{\text{refl}} \) in the plane \( z = 0 \).

Where the magnetic field produced by the equivalent magnetic current, \( M^{\text{TE}} \), is given by equation (7:1). From equation (7:1) it also follows that \( H_x(-2M^{\text{TE}}) = -H_x(2M^{\text{TE}}) \) why the integral equation (7:2) for the magnetic current, \( M^{\text{TE}} \), can be written as:

\[
H_x^{\text{inc}} - \frac{k^2}{2\omega \mu} \int M^{\text{TE}}(y') H_o^{(2)}(kR) \, dy' = 0 \quad (7:3)
\]

Where \( R \) now is given by \( R = \sqrt{(y-y')^2} = |y - y'| \) and \( 0 < y, y' < Ly \).

Letting the incident field strength, \( H_x^{\text{inc}} \), be unity and rearranging, the equation will finally be:

\[
\frac{2\omega \mu}{k^2} = \int M^{\text{TE}}(y') \, H_o^{(2)}(kR) \, dy' \quad (7:4)
\]

The integral equation for the equivalent magnetic current, \( M^{\text{TE}}(y') \), is to be solved by the method of moments. In using the method of moments [1] the first step is to expand the unknown in a series of known basis functions, i.e., \( M^{\text{TE}}(y') = \sum_{n=1}^{N} a_n b_n(y') \).

Where:
\( a_n = \) unknown coefficients to be determined
\( b_n = \) known expansion (basis) functions

Insertion of the series for the magnetic current in the integral equation will result in:

\[
\frac{2\omega \mu}{k^2} = \sum_{n=1}^{N} a_n \int b_n(y') \, H_o^{(2)}(kR) \, dy' \quad (7:5)
\]

This is one equation for \( N \) unknowns (the \( a_n \)), thus we have to "create" additional \( N-1 \) linearly independent equations in order to solve for the coefficients, \( a_n \). This can be accomplished by defining a set of testing (or weighting) functions, \( w_m ; m=1..N \), and an inner product as:

\[
\langle f(y), g(y) \rangle = \int_{0}^{Ly} f(y) g(y) \, dy \quad (7:6)
\]
Taking the inner product (7.6) with each \( w_m \) of both sides of equation (7.5) gives:

\[
\langle w_m, \frac{k^2}{2\mu} \rangle = \sum_{n=1}^{N} a_n \langle w_m, \int_{0}^{L_y} b_n(y') H_{0,2}^{(2)}(kR) \, dy' \rangle
\]  

(7.7)

For \( m = 1 \ldots N \)

This is now \( N \) equations for the \( N \) unknowns, \( a_n \). In order to avoid the integration associated with the inner product as defined by equation (7.6) the testing functions, \( w_m \), are chosen as Dirac pulses, i.e., point matching.

\[
w_m = \delta(y - y_m) = \begin{cases} 1 & \text{when } y = y_m \\ 0 & \text{elsewhere} \end{cases}
\]  

(7.8)

With this choice of testing functions equation (7.7) becomes:

\[
\frac{2\mu k^2}{k^2} = \sum_{n=1}^{N} a_n \int_{0}^{L_y} b_n(y') H_{0,2}^{(2)}(kly_m - y') \, dy'
\]  

(7.9)

For \( m = 1 \ldots N \)

For simplicity the basis functions, \( b_n \), are now chosen as pulses according to:

\[
b_n(y') = \begin{cases} 1 & \text{when } (n-1)\Delta < y' < n\Delta \\ 0 & \text{elsewhere} \end{cases}
\]  

(7.10)

The \( y_m \) of equation (7.8) and the \( \Delta \) of equation (7.10) are defined in Fig. 7.3.

The \( \Delta \) and \( y_m \) of equation (7.8) and the \( \Delta \) of equation (7.10) are defined in Fig. 7.3.

\[
\Delta = \frac{L_y}{N}
\]

\[
y_m = (2m-1)\Delta / 2
\]

Figure 7.3. Definition of basis functions, \( b_n \), and the matching points, \( y_m \).
Insertion of the basis functions given by equation (7:10) in equation (7:9) finally gives:

\[
\frac{2\omega \mu}{k^2} = \sum_{n=1}^{N} a_n \int_{(n-1)\Delta}^{n\Delta} H_o^{(2)}(k' l' y_m - y') dy' \tag{7:11}
\]

For \( m = 1 \ldots N \)

When solved on a computer, equation (7:11) is most conveniently written in matrix form.

\[
[Y_{mn}] [a_n] = [E_m]
\]

With the solution: \([a_n] = [Y_{mn}]^{-1} [E_m]\) \tag{7:12}

Where the matrices are given by:

\[
[E_m] = \begin{bmatrix}
\frac{2\omega \mu}{k^2} & & & \\
\frac{2\omega \mu}{k^2} & \ddots & & \\
& \ddots & \ddots & \\
\frac{2\omega \mu}{k^2} & & & \\
\end{bmatrix}
\]

\[
[a_n] = \begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_N \\
\end{bmatrix}
\]

\[
[Y_{mn}] = \begin{bmatrix}
\int_{0}^{\Delta} H_o^{(2)}(k l_1 - y') dy' & \int_{0}^{\Delta} H_o^{(2)}(k l_1 - y') dy' & \ldots & \\
\int_{0}^{\Delta} H_o^{(2)}(k l_2 - y') dy' & \int_{0}^{\Delta} H_o^{(2)}(k l_2 - y') dy' & \ldots & \\
\vdots & \vdots & \ddots & \ldots \\
\int_{0}^{\Delta} H_o^{(2)}(k l_N - y') dy' & \int_{0}^{\Delta} H_o^{(2)}(k l_N - y') dy' & \ldots & \int_{(N-1)\Delta}^{N\Delta} H_o^{(2)}(k l_N - y') dy'
\end{bmatrix}
\]
One problem when evaluating the components of the "admittance" matrix, \( Y_{mn} \), is that the Hankel function goes to infinity when the argument is approaching zero, i.e. when \( m = n \). This singularity is, however, an integrable singularity and the problem can be overcome by using the small argument formula for the Hankel function. Since a numerical integration on a computer will take a "long" time the components of the admittance matrix has to be approximated. The approximations used here are the same approximations used by Harrington (Ch. 3.2 in [1]) and are as follows:

\[
Y_{mn} = \{ m=n \} = \Delta \left[ 1 - j \frac{2}{\pi} \ln \left( \frac{2\pi\Delta}{4e} \right) \right] \quad (7.13)
\]

\[
Y_{mn} = \{ m \neq n \} = \Delta H_0^{(2)} \left( k l (2m-1) \frac{\Delta}{2} - (2n-1) \frac{\Delta}{2} \right) \quad (7.14)
\]

Where:
\[ \gamma = 1.781 = \text{Euler's constant} \]
\[ e = 2.718... \]

By this the 2D-solution is concluded, i.e. \( M^{\text{TE}}(y') \) is determined.

A computer code usable on a PC that calculates \( M^{\text{TE}}(y') \) was developed. The code is capable of handling, in principle, an unlimited number of basis functions. But since the code is filling and inverting a matrix of size \( N \) by \( N \) (i.e. the admittance matrix \( Y_{mn} \)), very large \( N \) is not practical when the code is used on a PC. For instance the time taken for calculation of \( M^{\text{TE}}(y') \) when \( N \) equals 100 on a 386 based computer with a math co. processor is approximately 330 seconds. The time taken for calculation of \( M^{\text{TE}}(y') \) is roughly proportionally to \( N \) raised to three (=2.6), which means that a doubling of \( N \) gives a eight folded computation time. Fortunately the number of basis functions needed for a reasonably good accuracy is as low as about ten per wavelength. Thus if the dimension \( Ly \) is not to many wavelength the computation time is tolerable.

The convergency of the code is shown by an example where the dimension \( Ly \) is one wavelength and the number of basis functions are varied as 10, 20 and 30.

![Figure 7.4. Convergency of \( M^{\text{TE}}(y') \) for \( Ly = \lambda \). Solid line: N=10, Dashed line: N=20, Dotted line: N = 30.](image-url)
It should be pointed out that the figures showing the equivalent magnetic current, $M_{TE}^{TE}(y')$, in the slot also is a picture of the y-directed tangential electric field in the slot ($M=E \times \hat{n}$).

In Fig. 7.5 and 7.6 the shape of $M_{TE}^{TE}(y')$ for $Ly = \lambda/2$ and $Ly = 2\lambda$ respectively is shown. In both figures it can be seen that the field will have singularities close to the edges of the slot. The singularities will come closer to the edges when the number of basis functions, $N$, is increased (this is shown in Fig. 7.4).

![Figure 7.5](image1.png)  
**Figure 7.5.** $M_{TE}^{TE}(y')$ when $Ly = \lambda/2$ and $N = 50$.

![Figure 7.6](image2.png)  
**Figure 7.6.** $M_{TE}^{TE}(y')$ when $Ly = 2\lambda$ and $N = 100$. 
However, the interesting parameter is not (at least not always) the equivalent magnetic current (tangential electric field) in the slot but the field behind the ground plane. The field behind the ground plane can be calculated using equation (7:1) with \( M_{\text{TE}}^y(y) \) exchanged to \(-2M_{\text{TE}}^y(y')\). In the following the origin of the co-ordinate system is altered in order to be located in the middle of the slot, i.e. \( y' = -\frac{1}{2}L_y \) (see Fig. 7:7).

\[
H_x^{\text{tot}}|_{z=0} = \frac{k^2}{2\omega \mu} \int_{-\frac{1}{2}L_y}^{\frac{1}{2}L_y} M_{\text{TE}}^y(y') H_0^{(2)}(kR) \, dy'
\]

(7:15)

In the far field region \( (R > 2L_y/\lambda) \) \( kR \) is large and equation (7:15) can be simplified by the use of the large argument formula for the Hankel function.

\[
H_x^{\text{tot}}|_{z=0} = \frac{k^2}{2\omega \mu} \int_{-\frac{1}{2}L_y}^{\frac{1}{2}L_y} M_{\text{TE}}^y(y') \sqrt{\frac{2j}{\pi kR}} e^{-jkR} dy' =
\]

\[
= \frac{k^2 (1 + j)}{2\omega \sqrt{\pi k}} \int_{-\frac{1}{2}L_y}^{\frac{1}{2}L_y} M_{\text{TE}}^y(y') e^{jkR} dy' \]

(7:16)

Further the \( R \) in equation (7:16) can be approximated by (see Fig. 7:7):

\[
R = r - y'sin\theta \quad \text{for phase variations}
\]

\[
R \approx r \quad \text{for amplitude variations}
\]

Using these approximations in (7:16) the expression for the total magnetic field behind the ground plane (in the point determined by \( r \) and \( \theta \)) can be written as:

\[
H_x^{\text{tot}}|_{z=0} = \frac{k^2 (1 + j)}{2\omega \sqrt{\pi k}} e^{jkR} \int_{-\frac{1}{2}L_y}^{\frac{1}{2}L_y} M_{\text{TE}}^y(y') e^{jkysin\theta} dy' =
\]

\[
= \frac{k^2 (1 + j)}{2\omega \sqrt{\pi k}} e^{jkR} \sum_{n=1}^{N} a_n \int_{(n-1)\Delta L_y/2}^{n\Delta L_y/2} e^{jkysin\theta} dy' = \{ \Delta \text{ is small} \} =
\]

\[
= \frac{k^2 (1 + j)}{2\omega \sqrt{\pi k}} e^{jkR} \sum_{n=1}^{N} a_n \int_{(n-1)\Delta L_y/2}^{n\Delta L_y/2} e^{j\Delta k (n\Delta y - L_y/2)sin\theta} \]

(7:17)
Figure 7:7. Radiation into region $z<0$.

In figures 7:8 to 7:10 the normalised radiation pattern for the field behind the ground plane is shown, i.e. the absolute value of equation (7:17). The patterns are plotted in Decibels and are functions of the $\theta$ angle in Fig. 7:17.

As was mentioned in chapter 6.3 (small apertures) it can be seen that the radiation pattern will be divided into a main lobe and several side lobes when the slot width becomes large in terms of a wavelength (larger than one wavelength). The total number of lobes (including the main lobe) will be three for a slot width of two wavelengths, five for a width of three wavelengths, and so on.

Figure 7:8. Normalised $|H_x^{\text{tot}}|$ in Decibels as a function of $\theta$ when $Ly = \lambda/2$. $N = 20$. 
Figure 7.9. Normalised $|H_x|_{\text{eq}}$ in Decibels as a function of $\theta$ when $L_y = 2\lambda$. $N = 80$.

Figure 7.10. Normalised $|H_x|_{\text{eq}}$ in Decibels as a function of $\theta$ when $L_y = 3\lambda$. $N = 120$. 
7.2.2 TM-case

The geometry for the TM-case is also shown in Fig. 7:1 but with the incident plane wave rotated ninety degrees, i.e. \( H^{\text{inc}} = -\hat{y}H_0 \). In the same way as for the TE-case an equivalent problem (shown in Fig. 7:2 for TE-case) can be defined. The equivalent problem will be similar to the one shown in Fig. 7:2 but with the equivalent magnetic current rotated ninety degrees. To avoid confusion with the TE-case the magnetic current will here be denoted as \( M_{\text{TM}}(y') \) (directed along the negative y-axis).

The procedure for the TM-case is similar to the TE-case. The main difference between the two cases is the more complex expression for the magnetic field produced by the equivalent magnetic current in the TM-case.

![Diagram of TM-case](image)

Figure 7:11. Equivalent magnetic current \( M_{\text{TM}}(y') \).

The \( y \)-directed \( H \)-field produced by the magnetic current \( M_{\text{TM}}(y') \) (Fig. 7:11) can be written as, see for instance [7]: (\( M_{\text{TM}}(y') \) is directed along the negative y-axis).

\[
H_y(M_{\text{TM}}) = \frac{1}{4\omega\mu} \left( \frac{d^2}{dy^2} + k^2 \right) \int_0^L M_{\text{TE}}(y') H_0^{(2)}(k|y-y'|) dy'
\]  
(7:18)

where: \( H_0^{(2)} = \text{Hankel function of zero order and second kind} \)

As for the TE-case the tangential magnetic field in the slot has to be continuous and the resulting integro-differential equation will be:

\[
H^{\text{inc}} = \frac{1}{2\omega\mu} \left( \frac{d^2}{dy^2} + k^2 \right) \int_0^L M_{\text{TE}}(y') H_0^{(2)}(k|y-y'|) dy
\]  
(7:19)

This equation is to be solved with the method of moments and therefore the equivalent magnetic current, \( M_{\text{TM}} \), is expanded in a series of known basis functions, \( b_n(y') \).
\[ M^{TM}(y) = \sum_{n=1}^{N} a_n b_n(y) \]  
(7.20)

Insertion of equation (7.20) in (7.19) will result in:

\[ H^{inc} = \frac{1}{2\omega \mu} \sum_{n=1}^{N} a_n \left( \frac{d^2 f_n}{dy^2} + k^2 \right) \int_0^L b_n(y') H_0^{(2)}(k|y-y'|) dy' \]  
(7.21)

or

\[ H^{inc} = \frac{1}{2\omega \mu} \sum_{n=1}^{N} a_n \left( \frac{d^2 f_n}{dy^2} + k^2 f_n \right) \]  
(7.22)

where:

\[ f_n = f_n(y) = \int_0^L b_n(y') H_0^{(2)}(k|y-y'|) dy' \]  
(7.23)

Defining a set of weighting functions, \( W_m \) for \( m = 1..N \), and taking the inner product of both sides of equation (7.22) gives:

\[ \int_0^L H^{inc} W_m(y) dy = \frac{1}{2\omega \mu} \sum_{n=1}^{N} a_n \int_0^L \left( \frac{d^2 f_n}{dy^2} + k^2 f_n \right) W_m(y) dy \quad ; \quad m = 1..N \]  
(7.24)

In matrix form:

\[ [E_m] = [Y_{nn}] [a_n] \]  
(7.25)

Where the matrix components are defined as:

\[ E_m = \int_0^L H^{inc} W_m(y) dy \]  
(7.26)

\[ Y_{nn} = \frac{1}{2\omega \mu} \int_0^L \left( \frac{d^2 f_n}{dy^2} + k^2 f_n \right) W_n(y) dy \]  
(7.27)

And the equivalent magnetic current can finally be found by matrix inversion:

\[ [a_n] = [Y_{nn}]^{-1} [E_m] \]
Defining the weighting functions, \( W_m \), as the triangle functions, \( T_m \), shown in Fig. 7:12.

\[
\Delta \\
\begin{array}{c}
T_m(y) \\
\end{array}
\]

\[
\begin{array}{ccc}
l_y & y_m & y_{m+1} \\
y_{m-1} & y_m & y_m \\
0 & y_m & y \end{array}
\]

Figure 7:12. Triangle weighting functions, \( T_m \).

\[ y_m = m\Delta \]

In Fig. 7:12
\[ \Delta = \frac{L_y}{N+1} \]

With this choice of weighting functions the components of the "admittance" matrix can be written as:

\[
Y_{nn} = \frac{1}{2\omega \mu} \left\{ k^2 \int_{y_{n-1}}^{y_n} f_n T_m dy + \int_{y_{n-1}}^{y_n} \frac{d^2 f_n}{dy^2} T_m dy \right\} =
\]

\[
= \frac{1}{2\omega \mu} \left\{ k^2 \int_{y_{n-1}}^{y_n} f_n T_m dy + \left[ \frac{df_n}{dy} T_m \right]_{y_{n-1}}^{y_n} - \int_{y_{n-1}}^{y_n} \frac{df_n}{dy} \frac{dT_m}{dy} dy \right\} =
\]

\[
= \frac{1}{2\omega \mu} \left\{ k^2 \int_{y_{n-1}}^{y_n} f_n T_m dy - \frac{1}{\Delta} \int_{y_{n-1}}^{y_n} \frac{df_n}{dy} dy + \frac{1}{\Delta} \left[ \frac{df_n}{dy} \right]_{y_{n-1}}^{y_n} \right\} =
\]

\[
= \frac{1}{2\omega \mu} \left\{ k^2 \int_{y_{n-1}}^{y_n} f_n T_m dy + \frac{1}{\Delta} \left[ f_n(y_{n-1}) - 2f_n(y_n) + f_n(y_{n+1}) \right] \right\}
\]

(7:28)

Where \( f_n \) is defined by equation (7:25).

In order to be able to evaluate \( f_n \) it remains to choose the basis functions \( b_n \). The basis functions are chosen to be of the same type as the weighting functions, i.e. triangle
functions. The choice of equal type of basis and weighting functions is known as Galerkin's method.

\[ f_n(y) = \int_{y_n}^{y_{n+1}} T_n(y') H_0^{(2)}(k|y-y'|) dy' \]  
\[ (7:29) \]

The integrand in equation (7:29) is singular when the argument for the Hankel function is equal to zero. This happens for \( m = n \) and \( m = n \pm 1 \). For these cases the integral can be evaluated with the aid of the small argument formula for the Hankel function (same as was done for the TE-case). The result is:

\[ f_n(y_m) = \Delta \left[ 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma k \Delta}{4 e} \right) \right] ; \quad m = n \text{ and } m = n \pm 1 \]

Where:
\( \gamma = 1.781 = \text{Euler's constant} \)
\( e = 2.718... \)

When the integrand is non-singular, i.e. for \( m \neq n \) and \( m \neq n \pm 1 \), the \( f_n \) is approximated by:

\[ f_n(y_m) = \Delta H_0^{(2)}(k|y_m - y_n|) ; \quad m \neq n \text{ and } m \neq n \pm 1 \]

Using these approximations the "admittance" matrix components, \( Y_{mn} \), of equation (7:28) can finally be written as:

\[ m \neq n \text{ and } m \neq n \pm 1 \]

\[ Y_{mn} = \frac{1}{2 \omega \mu} \left\{ H_0^{(2)}(k \Delta|m-n-1|) - 2 \left[ 1 - \frac{(k \Delta)^2}{2} \right] H_0^{(2)}(k \Delta|m-n|) - H_0^{(2)}(k \Delta|m-n+1|) \right\} \]

\[ m = n \]

\[ Y_{nn} = \frac{1}{\omega \mu} \left\{ H_0^{(2)}(k \Delta) - \left[ 1 - \frac{(k \Delta)^2}{2} \right] \left[ 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma k \Delta}{4 e} \right) \right] \right\} \]

\[ m = n \pm 1 \]

\[ Y_{nm} = \frac{1}{2 \omega \mu} \left\{ H_0^{(2)}(2k \Delta) - 2 \left[ 1 - \frac{(k \Delta)^2}{2} \right] H_0^{(2)}(k \Delta) + 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma k \Delta}{4 e} \right) \right\} \]

And the components of the "excitation" matrix, \( E_m \), are approximated by:

\[ E_m = \Delta H_0^{(2)}(m \Delta) = \Delta \]
The computer code used for the TE-case was modified according to the above formulas in order to also compute the equivalent magnetic current in the slot for the TM-case. Since the components of the "admittance" matrix, $Y_{mn}$, are more complicated the time taken for filling the admittance matrix is longer for the TM-case. For instance the time taken for calculation of $M^T(y')$ when $N$ equals 100 is approximately 390 seconds which should be compared with 330 seconds for the TE-case.

Figure 7:13. Convergence of $M^T(y')$ for $Ly = \lambda$.

Figure 7:14. $M^T(y')$ for $Ly = \lambda/2$ and $N = 50$. 
Figure 7.15. $M_{TM}(y')$ for $L_y = 2\lambda$ and $N = 100$.

The total radiated field in the far-field behind the slot is very often of interest also for the TM-case. For the TM-case the magnetic field is not only $y$-directed, as expressed by equation (7.18), but also $z$-directed. The electric field, on the other hand, is only directed along the $x$-axis and can in the far-field be expressed as, [7]:

$$
E_{x}^{\text{tot}} \bigg|_{z \rightarrow 0} = \frac{k(1+j)}{2\sqrt{\pi kr}} e^{-\frac{jy}{\sqrt{2}}} \int_{-L_y/2}^{L_y/2} M_{TM}(y') e^{jy' \sin \theta} dy' 
$$  

(7.30)

where the same co-ordinate system as was used for the TE-case, Fig. 7.7, is used.

Insertion of the expression for the equivalent magnetic current, $M_{TM}(y')$, as defined by equation (7.20) gives:

$$
E_{x}^{\text{tot}} \bigg|_{z \rightarrow 0} = \frac{k(1+j)}{2\sqrt{\pi kr}} e^{-\frac{jy}{\sqrt{2}}} \sum_{n=1}^{N} a_{n} e^{j\Delta \sin \theta} \int_{(n-1)\Delta - L_y/2}^{(n+1)\Delta - L_y/2} T_{n}(y') e^{jy' \sin \theta} dy' \approx \{ \Delta \text{ is small} \} = 
$$

(7.31)

Also in the far field the electric and the magnetic field vectors are perpendicular to each other and the amplitude ratio is equal to the intrinsic impedance of the medium, in this case free space ($\eta = 120 \pi \text{ [ohm]}$). Thus the power density can be written as:

$$
D = |E_{x}^{\text{tot}}|^2 / \eta ; [\text{W/m}^2] 
$$

(7.32)
And the total radiated power into the region $z<0$ (see Fig. 7.7) can be written as:

$$P_{\text{rad}} = X\int_{-\pi/2}^{\pi/2} Drd\theta \quad [\text{W}] \quad (7:33)$$

where: $X$ is the dimension in the $x$-direction (infinitely long)

Using equation (7:31) and (7:32) in equation (7:33) gives:

$$P_{\text{rad}} = \frac{Xk}{2\pi\eta} \int_{-\pi/2}^{\pi/2} \left| \sum_{n=1}^{N} a_n \Delta e^{j(n\Delta - l_{1/2})m} \right|^2 d\theta \quad (7:34)$$

Using the same definition for the attenuation as was used in chapter 6.3 for the electrically small apertures the attenuation in the slot can be written as:

$$L_{\text{slot}} = \frac{P_{\text{inc}}}{P_{\text{rad}}} = \frac{|E_{x}^{\text{inc}}|^2 L_x X}{\eta P_{\text{rad}}} = \{E_{x}^{\text{inc}} \text{ was assumed to be } \eta \text{ in the calculations of } E_{x}^{\text{tot}}\} =$$

$$= \frac{\eta L_x X}{P_{\text{rad}}} \quad (7:35)$$

Where $P_{\text{rad}}$ is given by equation (7:34). Note that equation (7:35) is independent of the dimension in $x$-direction, $X$.

In figures 7:16 to 7:18 the normalised radiation pattern for the field behind the ground plane is shown, i.e. the absolute value of equation (7:31). The patterns are plotted in Decibels and are functions of the $\theta$ angle in Fig. 7:7.

In Fig. 7:19 the attenuation in a slot as a function of the slot width is shown. Calculations are according to equation (7:35) and the number of basis functions, $N$, used in the calculations is 50 for all widths.
Figure 7:16. Normalised $|E_x^{tot}|$ in Decibels as a function of $\theta$ when $Ly = \lambda/2$. $N=20$.

Figure 7:17. Normalised $|E_x^{tot}|$ in Decibels as a function of $\theta$ when $Ly = 2\lambda$. $N=80$. 
Figure 7:18. Normalised $|E_x|$ in Decibels as a function of $\theta$ when $L_y = 3\lambda$, $N = 120$.

Figure 7:19. Attenuation in a slot as a function of the width. Calculated with eq. (7:35).
7.2.3 Oblique incidence

In the two preceding chapters the case of a plane wave incident at an angle parallel to the normal of the slot were treated. In this chapter the theory is extended in order to include the case of an arbitrary angle of incidence as well. This is achieved by modifying the excitation matrix, $[E_m]$. 

The components of the excitation matrix, $[E_m]$, are equal to the sum of the incident and the reflected magnetic fields (multiplied by a constant factor) at the location of the matching points. Thus we have to determine the sum of the incident and the reflected magnetic fields as a function of the location in the slot, i.e. the $y$-co-ordinate.

The TE-case

The case of an incident field polarised as TE is shown in Fig. 7:20.

![Figure 7:20. Incident and reflected waves for the TE-case.](image)

According to Fig. 7:20 the sum of the incident and the reflected magnetic fields as a function of $y$ and $\alpha$ can be written as:

$$H_{\text{inc}} + H_{\text{refl}} = 2 \left[ \cos(ky\sin\alpha) + jsin(ky\sin\alpha) \right]$$

where the amplitude of the incident wave, $H_0$, is chosen to be unity.

Using this expression for the field at the location of the slot, i.e. at the matching points, $y_m$, the excitation matrix can be written as:

$$[E_m] = \frac{2\omega}{k^2} \begin{bmatrix} \cos(ky_1\sin\alpha) + jsin(ky_1\sin\alpha) \\ \cos(ky_2\sin\alpha) + jsin(ky_2\sin\alpha) \\ \ddots \\ \cos(ky_n\sin\alpha) + jsin(ky_n\sin\alpha) \end{bmatrix}$$
The TM-case

The case of an incident field polarised as TM is shown in Fig. 7:21.

![Diagram of incident and reflected waves for the TM-case](image)

Figure 7:21. Incident and reflected waves for the TM-case.

According to Fig. 7:21 the sum of the incident and the reflected magnetic fields as a function of $y$ and $\alpha$ can be written as:

$$H^{\text{inc}} + H^{\text{refl}} = -\gamma 2 \cos \alpha \left[ \cos(ky \sin \alpha) + jsin(ky \sin \alpha) \right]$$

where the amplitude of the incident wave, $H_0$, is chosen to be unity.

Using this expression for the field at the location of the slot, i.e. at the matching points, $y_m$, the excitation matrix can be written as:

$$[E_m] = \Delta \cos \alpha \begin{bmatrix}
\cos(ky_1 \sin \alpha) + jsin(ky_1 \sin \alpha) \\
\cos(ky_2 \sin \alpha) + jsin(ky_2 \sin \alpha) \\
.. \\
\cos(ky_N \sin \alpha) + jsin(ky_N \sin \alpha)
\end{bmatrix}$$

It should be noted that the expression for the $y_m$ is different for the TE and the TM-case, see chapter 7.2.1 and 7.2.2 respectively.

The modified excitation matrices for the TE and the TM-case were incorporated in the developed computer code. Some sample runs are shown in order to show the behaviour of the radiation pattern for different incidence angles.

It should be noted that for a positive incidence angle the maximum radiation will be for a negative angle. This is explained by the definition of the incidence angle, $\alpha$, and the radiation angle, $\theta$, see Fig. 7:20 (or 7:21) and 7:7 respectively.
Figure 7:22. Normalised $|H_x|_{\text{tot}}$ in Decibels as a function of $\theta$ for the TE-case. $Ly = 2\lambda$, $N = 80$ and $\alpha = 30$ degrees.

Figure 7:23. Normalised $|E_x|_{\text{tot}}$ in Decibels as a function of $\theta$ for the TM-case. $Ly = 2\lambda$, $N = 80$ and $\alpha = 30$ degrees.
7.2.4 Multiple slots in an infinite ground plane

In the preceding chapters the case of a slot in an infinite ground plane was treated. However, the problem of interest is very often the case of multiple slots in the same plane. This is, for instance, the case in a cabinet housing some sort of electronic equipment equipped with a ventilation lattice.

The problem of the ventilation lattice can be treated with an extension of the technique used in the preceding chapters.

To start with consider the geometry shown in Fig. 7:24.

![Diagram of multiple slots](image)

**Figure 7:24.** Multiple slot. Slots in x-direction referring to Fig. 7:1.

The problem of the multiple slot shown in Fig. 7:24 can be treated with the same technique as was used in chapters 7.2.1 and 7.2.2. The presence of the "metal bars" across the slot will have the effect of putting the equivalent magnetic current equals to zero on the locations of the metal bars. This can easily be accomplished in the equations by letting the corresponding $a_n$ (or the $b_n$) be zero. In the equations this is achieved by letting the corresponding column in the admittance matrix equals zero.

Thus, by modifying the admittance matrix, $[Y_{nn}]$, in chapter 7.2.1 (see Eq. 7:11) or chapter 7.2.2 the problem of Fig. 7:24 can be solved. It is an easy task to modify a computer code written according to the equations in chapter 7.2.1 or 7.2.2 in order to also solve the above problem.

Since the attenuation in a slot for TE-polarisation (E-field vector perpendicular to the slot axis) is practically independent of the slot width a bar in the slot will have very little effect on the attenuation in the slot. For TM-polarisation (E-field vector parallel to the slot axis), on the other hand, the attenuation is indeed a function of the slot width. This fact is shown in Fig. 7:19. This implies that the attenuation in a slot for the TM-polarisation should increase if a thin bar is placed in the slot.
For a practical situation the thin bar simulates a thin electrically conducting wire placed in the slot (parallel to the slot axis).

In order to show the effect of a thin wire placed in the slot for TM-polarisation some calculations were performed with a modified version of the computer code used in the preceding chapters.

Calculations were performed for a slot of width 0.2λ and the number of basis functions, N, was 101. The attenuation without wire was found to be 6.6 dB. In Fig. 7.25 the attenuation for a slot with one wire is shown. The attenuation is, of course, a function of the position of the wire in the slot. As it should, it is clear from Fig. 7.25 that the attenuation approaches the value for a slot without a wire when the wire approaches the edge of the slot.

![Graph showing attenuation vs. offset from center in wavelength](image)

Figure 7.25. Attenuation for a slot of width 0.2λ and with one wire.

Increasing the number of wires in the slot will further increase the attenuation. Some example calculations for a slot of width 0.2λ are shown in table 7.1.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6.6</td>
<td>No wires</td>
</tr>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>10.1</td>
<td>One wire</td>
</tr>
<tr>
<td>-0.05</td>
<td>0.05</td>
<td>-</td>
<td>12.0</td>
<td>Two wires</td>
</tr>
<tr>
<td>-0.05</td>
<td>0</td>
<td>0.05</td>
<td>14.8</td>
<td>Three wires</td>
</tr>
<tr>
<td>-0.01</td>
<td>0</td>
<td>0.01</td>
<td>13.2</td>
<td>Three wires</td>
</tr>
</tbody>
</table>

Table 7.1. Attenuation in a slot of width 0.2λ with and without wires.

In summary it can be concluded that the attenuation in a slot, for TM-polarisation, can be increased by placing one or more thin wires in the slot. Since the wires are thin they will have little, if any, effect on air flow which could be important if the slot is a ventilation opening.
Figure 7:26. Equivalent magnetic current in slot of width 0.2λ. Dotted line represents slot without wires and solid line represents slot with three wires. Wires are placed at offset -0.03λ; 0λ and 0.03λ from the centre line in the slot.

For a single slot, it was seen in the preceding chapters that the radiation pattern will be divided into a main lobe and several side lobes when the width of the slot becomes larger than about one wavelength. For multiple slots the radiation pattern can be divided into a main lobe and side lobes even if the widths of the individual slots are less than a wavelength. As an example the radiation pattern for the slots shown in Fig. 7:27 is shown in Fig. 7:28.

Figure 7:27. Three slots each of width Δ in a ground plane. Width Δ is 0.4λ.
Figure 7.28. Radiation pattern, i.e. normalised $|E_\theta|_{\text{dB}}$ in Decibels as a function of $\theta$, for the multiple slot shown in Fig. 7.27.

7.2.5 Large aperture in a ground plane, 3D-case

In this chapter the general case of a large (in terms of wavelength) rectangular aperture in an infinite ground plane is treated. The problem is solved by using the results from the 2D-problem treated in chapter 7.2.1 and 7.2.2. In order to justify the use of these results a discussion based on a Fourier transformation technique is used.

Figure 7.29. Geometry for the general case of an aperture in an infinite ground plane.

The 3D-problem shown in Fig. 7.29 is reduced to a spectrum of 2D-problems by a Fourier transformation in the x-co-ordinate. The source in each of the 2D-problems will then become an infinitely long (in x-direction) magnetic current sheet of the form:
\[ M(y') \tilde{M}(k_x) e^{ik_x x}; -L_y / 2 \leq y' \leq L_y / 2 \quad \text{(7.36)} \]

where:
\[ \tilde{M}(k_x) = \int_{-L_y / 2}^{L_y / 2} M(x') e^{-ik_x x'} dx' \quad \text{(7.37)} \]

\[ k_x = \sqrt{k^2 - k_y^2} \quad \text{(7.38)} \]

In equations (7.36) and (7.37) the \( M \) represents \( M^{TM} = M_y \) for TM-polarisation and \( M^{TE} = M_x \) for TE-polarisation.

Since the term \( \tilde{M}(k_x) e^{ik_x x} \) in equation (7.36) represents a constant in the 2D-case it can be suppressed. By doing so, the sources of the 2D-problems will be \( M(y') \) for all \( k_x \). This problem was, for \( k_z = 0 \) i.e. \( k_y = k \), solved in chapters 7.2.1 and 7.2.2. The 3D-solution can now be written as:

\[ M(y', x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(y') \tilde{M}(k_x) e^{ik_x x} dk_x \quad \text{(7.39)} \]

Since we, for the present problem, equally well could have performed the Fourier transformation in the y-co-ordinate it is concluded that the magnetic current distribution in the x- and y-directions can be determined in the same way, i.e. with the technique used in chapter 7.2.1 and 7.2.2. This also implies that it is not necessary to perform any Fourier transformation at all. The 3D-solution can simply be found by multiplying the distributions in the x- and y-directions. This means that for a rectangular aperture the equivalent magnetic current distribution can be found by multiplying the 2D-solutions for the TM- and TE-polarisation.
The procedure for determination of the equivalent magnetic current in the rectangular aperture shown in Fig. 7.29 for TE- and TM-polarisation will be:

**TE-polarisation**

- Determine the 2D-solution $M_x(y')$ for a slot oriented with slot axis along the x-axis and of width $L_y$, i.e. TE-solution of chapter 7.2.1

- Determine the 2D-solution $M_y(x')$ for a slot oriented with slot axis along the y-axis and of width $L_x$, i.e. TM-solution of chapter 7.2.2

- Multiply the two solutions to obtain $M_{xy}(x',y')$, i.e. $M_{xy}(x',y') = M_x(x')M_y(y')$

- Divide $M_{xy}(x',y')$ with the constant factor $120\pi$ ($=377$). This is necessary because $M_x(x')$ and $M_y(y')$ were both computed for an excitation of $|E|=120\pi$ [V/m]. The resulting $M_{xy}(x',y')$ will be the equivalent magnetic current for an excitation of $|E|=120\pi$ [V/m], i.e. $|H^{inc}|=1$ [A/m]

**TM-polarisation**

- Determine the 2D-solution $M_y(y')$ for a slot oriented with slot axis along the x-axis and of width $L_y$, i.e. TM-solution of chapter 7.2.2

- Determine the 2D-solution $M_x(x')$ for a slot oriented with slot axis along the y-axis and of width $L_x$, i.e. TE-solution of chapter 7.2.1

- Multiply the two solutions to obtain $M_{xy}(x',y')$, i.e. $M_{xy}(x',y') = M_x(x')M_y(y')$

- Divide $M_{xy}(x',y')$ with the constant factor $120\pi$ ($=377$). This is necessary because $M_x(x')$ and $M_y(y')$ were both computed for an excitation of $|E|=120\pi$ [V/m]. The resulting $M_{xy}(x',y')$ will be the equivalent magnetic current for an excitation of $|E|=120\pi$ [V/m], i.e. $|H^{inc}|=1$ [A/m]

Figure 7.30. Equivalent magnetic current distribution in quadratic aperture with side length $2\lambda$. 
Now when the 3D-distribution for the equivalent magnetic current in the rectangular aperture is determined the radiated field and the attenuation can be computed.

According to equations (6:7), (6:6) and (6:2) the radiated field in the far-field region can be written as:

\[
E_r = 0 \\
E_\theta = -jkF_\phi \\
E_\phi = jkF_\phi
\]  \hspace{1cm} (7:40)

where:
\[
F = \frac{1}{4\pi r} e^{-jr} \int \int _{\text{aperture}} M(x', y') e^{j\left(x'\sin\theta \cos\phi + y'\sin\theta \sin\phi\right)} dx' dy'
\]  \hspace{1cm} (7:41)

\(M\) in equation (7:41) represents twice the calculated equivalent magnetic current in the aperture, i.e. \(M=2M^{TM}\) for TM-polarisation and \(M=2M^{TE}\) for TE-polarisation.

**TE-polarisation**

\[
M(x', y') = \hat{x}M_x(x', y') \Rightarrow F = \hat{x}F_x
\]

\[
F_x = \frac{1}{4\pi r} e^{-jr} \int \int _{\text{aperture}} M_x(x', y') e^{j\left(x'\sin\theta \cos\phi + y'\sin\theta \sin\phi\right)} dx' dy'
\]

and

\[
E_r = 0 \\
E_\theta = jkF_x \sin \phi \\
E_\phi = jkF_x \cos \theta \cos \phi
\]

**TM-polarisation**

\[
M(x', y') = \hat{y}M_y(x', y') \Rightarrow F = \hat{y}F_y
\]

\[
F_y = \frac{1}{4\pi r} e^{-jr} \int \int _{\text{aperture}} M_y(x', y') e^{j\left(x'\sin\theta \cos\phi + y'\sin\theta \sin\phi\right)} dx' dy'
\]

and

\[
E_r = 0 \\
E_\theta = jkF_y \cos \phi \\
E_\phi = jkF_y \cos \theta \sin \phi
\]
7.3 Slot in a ground plane of finite thickness

In this chapter the more general case of a slot in a ground plane of finite thickness is treated. The problem is solved with an extension of the technique used in chapter 7.2.

The geometry for the problem of a slot in a ground plane of finite thickness is shown in Fig. 7:31.

![Diagram of slot in a ground plane](image)

Figure 7:31. Slot of width \( w \) in a ground plane of thickness \( t \).

The problem of the slot in a ground plane of finite thickness shown in Fig. 7:31 is solved by using the equivalence principle. First the problem is divided into three different regions in which the total field expressions can be obtained. Then the regions are matched together by using the boundary conditions for the fields. In this way two coupled integral equations are obtained which can be solved by the method of moments. The procedure is as follows:
A: Dividing the problem into three different regions

The problem region, which is the hole space, is divided into three different regions as shown in Fig. 7:32.

![Division of the problem into three different regions](image)

Figure 7:32. Division of the problem into three different regions.

The regions are:

Region A: The half space \( z \geq 0 \)
Region B: \( -t \leq z \leq 0, \ 0 \leq y \leq w, \ \text{all} \ x \)
Region C: The half space \( z \leq -t \)

The incoming field (exciting field) is assumed to be in region A.

B1: Equivalent problem for region A

With the aid of the equivalence principle the equivalent problem shown in Fig. 7:33 is constructed for the fields in region A.

![Equivalent problem for fields in region A](image)

Figure 7:33. Equivalent problem for fields in region A.
where the equivalent magnetic current is defined as: \( \mathbf{M}^A = \mathbf{E}^A \times \hat{z} \) and is located at the position of the slot.

**B2: Equivalent problem for region B**

With the aid of the equivalence principle the equivalent problem shown in Fig. 7:34 is constructed for the fields in region B.

![Diagram of equivalent problem for region B](image)

Figure 7:34. Equivalent problem for fields in region B.

where the equivalent currents \( \mathbf{M}^A \) and \( \mathbf{M}^B \) are defined under B1 and B3 respectively. Region B is now a closed two-dimensional box with conducting sides.

**B3: Equivalent problem for region C**

With the aid of the equivalence principle the equivalent problem shown in Fig. 7:35 is constructed for the fields in region C.

![Diagram of equivalent problem for region C](image)

Figure 7:35. Equivalent problem for fields in region C.

where the equivalent magnetic current is defined as: \( \mathbf{M}^B = -\mathbf{E}^C \times \hat{z} \) and is located at the position of the slot (at \( z = -t \)).
C: Matching of regions together

The choice of \( M^A \) for region A and \(-M^A\) for region B, \(-M^B\) for region B and \( M^B \) for region C will ensure that the tangential electric field at the region boundaries is continuous. The remaining boundary condition of continuous tangential magnetic field at the region boundaries will result in the following relations:

\[
z = 0 : \left[ H_A^{inc} + H_A^{off} + H_A(2M^A) = H_B(-M^A) + H_B(-M^B) \right]_{\text{un}}
\]

\[
\Rightarrow \left[ 2H_A^{inc} + 2H_A(M^A) = H_B(-M^A) + H_B(-M^B) \right]_{\text{un}}
\]

\[
z = -t : \left[ H_C(2M^B) = H_B(-M^A) + H_B(-M^B) \right]_{\text{un}}
\]

\[
\Rightarrow \left[ 2H_C(M^B) = H_B(-M^A) + H_B(-M^B) \right]_{\text{un}}
\]

where: \( H_A(M^A) \) is the magnetic field in region A produced by \( M^A \)
\( H_B(-M^A) \) is the magnetic field in region B produced by \(-M^A\)
\( H_B(-M^B) \) is the magnetic field in region B produced by \(-M^B\)
\( H_C(M^B) \) is the magnetic field in region C produced by \( M^B \)

Equations (7:42) are coupled integral equations for the unknown magnetic currents \( M^A \) and \( M^B \), i.e. the tangential electric field in the slots at \( z=0 \) and \( z=-t \) respectively.

D: Field expressions for the regions

In order to solve equations (7:42) we need the field expressions for the different regions. Since the expressions are different for different polarisations the problem is divided into two sub problems, one for TM-polarisation and one for TE-polarisation. The incoming field is defined as TM when the electric field vector is parallel with the slot axis and as TE when the magnetic field vector is parallel with the slot axis.

It should be noted that any polarisation can be constructed as a superposition of TM and TE-polarisations.

TM-case: \( E^{inc} = \hat{\lambda}E^{inc}, \ H^{inc} = -\hat{\gamma}H^{inc}, \ M^A = -\hat{\gamma}M^A, \ M^B = \hat{\gamma}M^B \)

TE-case: \( E^{inc} = \hat{\gamma}E^{inc}, \ H^{inc} = \hat{\lambda}H^{inc}, \ M^A = \hat{\lambda}M^A, \ M^B = -\hat{\lambda}M^B \)
Region A, TM-case, $z=0$

The total tangential magnetic field in region A (Fig. 7:33) at the plane $z=0$ can be written as:

$$\left[ 2H_A^{inc} + 2H_A(M^4) \right]_{z=0} = \hat{y} \left\{ -2H^{inc} + \frac{1}{2\omega \mu} \left( \frac{d^2}{dy'^2} + k^2 \right) \int_0^w M^4 H_0^{(2)}(k|y-y'|) dy' \right\}$$  \hspace{1cm} (7.43)

Region A, TE-case, $z=0$

The total tangential magnetic field in region A (Fig. 7:33) at the plane $z=0$ can be written as:

$$\left[ 2H_A^{inc} + 2H_A(M^4) \right]_{z=0} = \hat{x} \left\{ 2H^{inc} - \frac{k^2}{2\omega \mu} \int_0^w M^4 H_0^{(2)}(k|y-y'|) dy' \right\}$$  \hspace{1cm} (7.44)

Region B, TM-case, $z=0$, $0<y<w$

The total tangential magnetic field in region B (Fig. 7:34) at the strip $z=0$, $0<y<w$ can be written as (see appendix A and B):

$$\left[ H_b(-M^4) + H_b(-M^6) \right]_{z=0} = \hat{y} \left\{ \frac{j2}{\omega \mu} \left( \frac{d^2}{dy'^2} + k_b^2 \right) \sum_{m} \frac{\sin \left( \frac{inx}{w} \right)}{wk_m \tan(k_m t)} \int_0^w M^4 \sin \left( \frac{inx}{w} \right) dy' \right\} +$$

$$+ \hat{y} \left\{ \frac{j2}{\omega \mu} \left( \frac{d^2}{dy'^2} + k_b^2 \right) \sum_{m} \frac{\sin \left( \frac{inx}{w} \right)}{wk_m \sin(k_m t)} \int_0^w M^6 \sin \left( \frac{inx}{w} \right) dy' \right\}$$

$$\hspace{1cm} (7.45)$$
Region B. TE-case, z=0, 0<y<w 

The total tangential magnetic field in region B (Fig. 7.34) at the strip z=0, 0<y<w can be written as (see appendix C and D):

\[
\begin{align*}
[H_y(-M^A) + H_y(-M^B)]_{\text{ann}} &= \hat{x} \left\{ -\frac{jk_y^2}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i \pi y}{w} \right)}{w k_i \tan(k_i t)} \int_0^w M^A \cos \left( \frac{i \pi y'}{w} \right) \, dy' \right\} \\
&\quad - \hat{y} \left\{ \frac{j}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i \pi y}{w} \right)}{w k_i \sin(k_i t)} \int_0^w M^B \cos \left( \frac{i \pi y'}{w} \right) \, dy' \right\}
\end{align*}
\]

(7.46)

Region B. TM-case, z=-t, 0<y<w 

The total tangential magnetic field in region B (Fig. 7.34) at the strip z=-t, 0<y<w can be written as (see appendix A and B):

\[
\begin{align*}
[H_y(-M^A) + H_y(-M^B)]_{\text{ann}} &= \hat{y} \left\{ \frac{j}{\omega \mu} \left( \frac{\alpha_i \cos \left( \frac{i \pi y}{w} \right)}{w k_i \sin(k_i t)} \right) \sum_{i=0}^{\infty} \sin \left( \frac{i \pi y}{w} \right) \int_0^w M^A \sin \left( \frac{i \pi y'}{w} \right) \, dy' \right\} \\
&\quad + \hat{y} \left\{ \frac{j}{\omega \mu} \left( \frac{\alpha_i \cos \left( \frac{i \pi y}{w} \right)}{w k_i \sin(k_i t)} \right) \sum_{i=0}^{\infty} \sin \left( \frac{i \pi y}{w} \right) \int_0^w M^B \sin \left( \frac{i \pi y'}{w} \right) \, dy' \right\}
\end{align*}
\]

(7.47)
Region B, TE-case, $z=-t$, $0<y<w$

The total tangential magnetic field in region B (Fig. 7:34) at the strip $z=-t$, $0<y<w$ can be written as (see appendix C and D):

$$\left[ H_y(-M^A) + H_y(-M^B) \right]_{\text{tan}} = \hat{x} \left\{ -\frac{j k_B^2}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_n \cos\left(\frac{in\gamma}{w}\right)}{\omega k_n \sin(k_n t)} \int_0^w M^A \cos\left(\frac{in\gamma'}{w}\right) dy' \right\} - $$

$$-\hat{x} \left\{ \frac{j k_B^2}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_n \cos\left(\frac{in\gamma}{w}\right)}{\omega k_n \tan(k_n t)} \int_0^w M^B \cos\left(\frac{in\gamma'}{w}\right) dy' \right\}$$

(7:48)

Region C, TM-case, $z=1$

The total tangential magnetic field in region C (Fig. 7:35) at the plane $z=1$ can be written as:

$$\left[ 2H_C(M^B) \right]_{\text{tan}} = -\hat{y} \frac{1}{2 \omega \mu} \left( \frac{d^2}{dy^2} + k^2 \right) \int_0^w M^B H_0^{(2)}(k|y-y'|) dy'$$

(7:49)

Region C, TE-case, $z=1$

The total tangential magnetic field in region C (Fig. 7:35) at the plane $z=1$ can be written as:

$$\left[ 2H_C(M^B) \right]_{\text{tan}} = \hat{y} \frac{k^2}{2 \omega \mu} \int_0^w M^B H_0^{(2)}(k|y-y'|) dy'$$

(7:50)
E: Method of moment formulation

The coupled integral equations (7.42), with the operator functions given by equations (7.43) - (7.50), are to be solved with the method of moments. The first step in this procedure is to expand the unknown functions in known basis functions, thus:

\[ M^A = \sum_{n=1}^{N} a_{An} b_{An}(y') \]
\[ M^B = \sum_{n=1}^{N} a_{Bn} b_{Bn}(y') \]

where \( a_{An} \) are coefficients and \( b_{An} \) are basis functions.

TM-case

For the TM-case equations (7.42) state: (7.43) = (7.45) and (7.47) = (7.49).

Insertion of the expression for the unknown magnetic currents, \( M^A \) and \( M^B \), given by equations (7.51) gives:

\[
2H^{int} = \frac{1}{2 \omega \mu} \sum_{n=1}^{N} a_{An} \left( \frac{d^2 f_n}{dy'^2} + k^2 f_n \right) - \frac{j2}{\omega \mu} \sum_{n=1}^{N} a_{Bn} \sum_{i=1}^{N} \left( \frac{d^2 g_{ni}}{dy'^2} + k_{ni}^2 g_{ni} \right) - \frac{j2}{\omega \mu} \sum_{n=1}^{N} a_{Bn} \sum_{i=1}^{N} \left( \frac{d^2 h_{ni}}{dy'^2} + k_{ni}^2 h_{ni} \right) = 0
\]

where:

\[ f_n = \int_{0}^{y_0} b_{An}(y') H^{(2)}_0(k|y-y'|)dy' \]
\[ g_{ni} = \frac{\sin\left(\frac{in\pi y}{w} \right)}{wk_n \tan(k_n t)} \int_{0}^{y_0} b_{Bn}(y') \sin\left(\frac{in\pi y}{w} \right)dy' \]
\[ h_{ni} = \frac{\sin\left(\frac{in\pi y}{w} \right)}{wk_n \sin(k_n t)} \int_{0}^{y_0} b_{Bn}(y') \sin\left(\frac{in\pi y}{w} \right)dy' \]

(7.52)
and (7.47) = (7.49):

\[
0 = \frac{j2}{\omega_k} \sum_{n=1}^{N} a_n \sum_{i=1}^{N} \left( \frac{d^2 p_{ni}}{d\psi^2} + k^2 p_{ni} \right) + \frac{1}{2\omega_k} \sum_{n=1}^{N} a_n \left( \frac{d^2 q_{n}}{d\psi^2} + k^2 q_{n} \right) + \frac{j2}{\omega_k} \sum_{n=1}^{N} a_n \sum_{i=1}^{N} \left( \frac{d^2 r_{ni}}{d\psi^2} + k^2 r_{ni} \right) = \\
= \frac{j2}{\omega_k} \sum_{n=1}^{N} a_n \sum_{i=1}^{N} k^2 p_{ni} + \frac{1}{2\omega_k} \sum_{n=1}^{N} a_n \left( \frac{d^2 q_{n}}{d\psi^2} + k^2 q_{n} \right) + \frac{j2}{\omega_k} \sum_{n=1}^{N} a_n \sum_{i=1}^{N} k^2 r_{ni}
\]

where:

\[
p_{ni} = \frac{\sin \left( \frac{ny}{w} \right)}{wk_n \sin (k_n \psi)} \int_{0}^{\psi_n} b_{ni}(y) \sin \left( \frac{ny}{w} \right) dy
\]

\[
q_{n} = \int_{0}^{\psi_n} b_{n}(y) H_0^{(2)}(k|y-y'|) dy
\]

\[
r_{ni} = \frac{\sin \left( \frac{ny}{w} \right)}{wk_n \tan (k_n \psi)} \int_{0}^{\psi_n} b_{ni}(y) \sin \left( \frac{ny}{w} \right) dy
\]

(7.53)

In order to transform equations (7.52) and (7.53) into matrix equations we define two sets of weighting functions, one set \( W_{Am} \) for the plane \( z=0 \) and one set \( W_{Bn} \) for the plane \( z=\alpha \), both sets are defined for \( m=1..N \). Taking the inner product, as defined in equation (7.54), of equation (7.52) with the set \( W_{Am} \) and equation (7.53) with the set \( W_{Bm} \) gives the matrix equation:

\[
\langle f(y), g(y) \rangle = \int_{0}^{\psi} f(y) g(y) dy
\]

(7.54)

\[
\begin{bmatrix}
Y^A \\
Y^C
\end{bmatrix}
= \begin{bmatrix}
Y^B \\
Y^D
\end{bmatrix}
\begin{bmatrix}
[a_A] \\
[a_B]
\end{bmatrix}
= \begin{bmatrix}
[E]
\end{bmatrix}
\]

(7.55)

with the solution:

\[
\begin{bmatrix}
[a_A] \\
[a_B]
\end{bmatrix} = \begin{bmatrix}
Y^A \\
Y^C
\end{bmatrix}^{-1} \begin{bmatrix}
[E]
\end{bmatrix}
\]

(7.56)
The matrix components in (7:55) and (7:56) are defined as:

\[
Y_{mn}^A = \frac{1}{2\omega \mu} \int_0^\infty \left( \frac{d^2 f_n}{dy^2} + k^2 f_n \right) W_{Am}(y)dy - \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 g_n W_{Am}(y)dy
\]

\[
Y_{mn}^B = -\frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 h_n W_{Am}(y)dy
\]

\[
Y_{mn}^C = \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 p_n W_{Bm}(y)dy
\]

\[
Y_{mn}^D = \frac{1}{2\omega \mu} \int_0^\infty \left( \frac{d^2 q_n}{dy^2} + k^2 q_n \right) W_{Bm}(y)dy + \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 r_n W_{Bm}(y)dy
\]

\[
E_m = 2 \int_0^\infty H^{inc} W_{Am}(y)dy
\]

(7:57)

**TE-case**

For the TE-case equations (7:42) state : (7:44) = (7:46) and (7:48) = (7:50).

Following the same procedure as for the TM-case will result in the following equations:

\[
\begin{bmatrix}
[a_n] \\
[a_s]
\end{bmatrix} =
\begin{bmatrix}
Y^A & Y^B \\
Y^C & Y^D
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\]

(7:58)

where the matrix components now are defined as:

\[
Y_{mn}^A = \frac{1}{2\omega \mu} \int_0^\infty k^2 f_n W_{Am}(y)dy - \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 g_n W_{Am}(y)dy
\]

\[
Y_{mn}^B = -\frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 h_n W_{Am}(y)dy
\]

\[
Y_{mn}^C = \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 p_n W_{Bm}(y)dy
\]

\[
Y_{mn}^D = \frac{1}{2\omega \mu} \int_0^\infty k^2 q_n W_{Bm}(y)dy + \frac{j}{\omega \mu} \sum_{i=0}^\infty \int_0^\infty k_i^2 r_n W_{Bm}(y)dy
\]

\[
E_m = 2 \int_0^\infty H^{inc} W_{Am}(y)dy
\]

(7:59)

and the functions \(f_n, g_{ni}, h_{ni}, p_{ni}, q_n\) and \(r_n\) are defined as:
\[ f_n = \int_0^w b_{an}(y') H_0^{(2)}(k|y-y'|) dy' \]

\[ g_{ni} = \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{wk_{ni} \tan(k_{ni}t)} \int_0^w b_{bn}(y') \cos\left(\frac{iny'}{w}\right) dy' \]

\[ h_{ni} = \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{wk_{ni} \sin(k_{ni}t)} \int_0^w b_{bn}(y') \cos\left(\frac{iny'}{w}\right) dy' \]

\[ p_{ni} = \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{wk_{ni} \sin(k_{ni}t)} \int_0^w b_{bn}(y') \cos\left(\frac{iny'}{w}\right) dy' \]

\[ q_n = \int_0^w b_{bn}(y') H_0^{(2)}(k|y-y'|) dy' \]

\[ r_{ni} = \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{wk_{ni} \tan(k_{ni}t)} \int_0^w b_{bn}(y') \cos\left(\frac{iny'}{w}\right) dy' \]

\[ \alpha_0 = 1, \quad \alpha_i = 2 ; i \geq 1 \]

**F: Calculation of the matrix components**

In order to solve equations (7.56) and (7.58) numerically we have to calculate the matrix components given in equations (7.57) and (7.59) respectively. But first we have to choose the weighting functions, \( W_{xm} \) and the basis functions \( b_{an} \). The choices used here are the same as were used in chapter 7.2.1 and 7.2.2 (TE and TM respectively).

For simplicity the choices for both polarisations are equal basis functions for \( M^A \) and \( M^B \) and similarly the weighting functions \( W_{Am} \) and \( W_{Bm} \) are chosen to be equal, thus:

\[ b_{an}(y') = b_{bn}(y') = b_n(y') \]

and

\[ W_{Am}(y) = W_{Bm}(y) = W_n(y) \]
TM-case

The basis functions $b_n$ as well as the weighting functions $W_m$ are for the TM-case chosen as triangle functions, see chapter 7.2.2. With this choice the matrix components of equations (7.57) become (see chapter 7.2.2):

\[
Y_{mn}^A = \frac{1}{2 \omega \mu} \left\{ k^2 \int_{\gamma_{m-1}}^{\gamma_m} f_n(y_m) \, dy + \frac{1}{\Delta} \left[ f_n(y_{m-1}) - 2f_n(y_m) + f_n(y_{m+1}) \right] \right\} - \frac{j2}{\omega \mu} \sum_{i=1}^{\infty} k_n^2 \int_{\gamma_{m-1}}^{\gamma_m} g_n \, T_m \, dy
\]

\[
Y_{mn}^B = -\frac{j2}{\omega \mu} \sum_{i=1}^{\infty} k_n^2 \int_{\gamma_{m-1}}^{\gamma_m} h_n \, T_m \, dy
\]

\[
Y_{mn}^C = \frac{j2}{\omega \mu} \sum_{i=1}^{\infty} k_n^2 \int_{\gamma_{m-1}}^{\gamma_m} p_n \, T_m \, dy
\]

\[
Y_{mn}^D = \frac{1}{2 \omega \mu} \left\{ k^2 \int_{\gamma_{m-1}}^{\gamma_m} q_n(y_m) \, dy + \frac{1}{\Delta} \left[ q_n(y_{m-1}) - 2q_n(y_m) + q_n(y_{m+1}) \right] \right\} + \frac{j2}{\omega \mu} \sum_{i=1}^{\infty} k_n^2 \int_{\gamma_{m-1}}^{\gamma_m} r_n \, T_m \, dy
\]

\[
E_m = 2 \int_{\gamma_{m-1}}^{\gamma_m} H_{inc} \, T_m \, dy
\]

(7.60)

Where $T_m$, $y_m$ and $\Delta$ are defined as in chapter 7.2.2, i.e. $y_m = m \Delta$, $\Delta = \frac{\omega}{N + 1}$.

The functions $f_n$, $g_m$, $h_m$, $p_m$, $q_m$ and $r_m$ in equation (7.60) are defined as in equations (7.52) and (7.53) but with the basis functions $b_{An}(y')$ and $b_{Bn}(y')$ exchanged to $T_n(y')$, where $T_n$ is the triangle function defined in chapter 7.2.2.

Since the basis functions $b_{An}$ and $b_{Bn}$ are chosen to be equal the following identities are valid:

\[
f_n = q_n, \quad g_n = r_n, \quad h_n = p_n\quad (7.61)
\]

The integrands involved in $f_n$ (and consequently $q_m$) are singular when the argument for the Hankel function is equal to zero. This happens for $m=n$ and $m=n \pm 1$. For these cases the integral can be evaluated with the aid of the small argument formula for the Hankel function. The result is:

\[
f_n(y_m) = \Delta \left[ 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma \Delta}{4e} \right) \right] ; \quad m = n \quad \text{and} \quad m = n \pm 1
\]

(7.62)

Where: \[\gamma = 1,781 = \text{Euler's constant}\]
\[e = 2,718\ldots\]
When the integrands are non-singular, i.e. for \( m \neq n \) and \( m \neq n \pm 1 \), the \( f_n \) are approximated by:

\[
f_n(y_m) = \Delta H_0^{(2)}(k|y_m - y_n|) \quad m \neq n \text{ and } m \neq n \pm 1
\]  

(7.63)

In evaluating \( g_{ni} \), \( h_{ni} \), \( p_{ni} \) and \( r_{ni} \), the same integral has to be evaluated. The integrals are denoted by \( I_{ni} \) and are defined by:

\[
I_{ni} = \int_{y_{ni}}^{y_{ni}'} T_n(y') \sin \left( \frac{in\Delta}{w} \right) dy'
\]

(7.64)

Thus:

\[
g_{ni} = \frac{\sin \left( \frac{in\Delta}{w} \right)}{wk_{ni} \tan(k_{ni}t)} I_{ni} \quad \text{and} \quad h_{ni} = \frac{\sin \left( \frac{in\Delta}{w} \right)}{wk_{ni} \sin(k_{ni}t)} I_{ni}
\]

The integrals \( I_{ni} \) can be evaluated analytically and are found to be:

\[
I_{ni} = \frac{4}{\Delta} \left( \frac{w}{in} \right)^2 \sin \left( \frac{in\Delta}{w} \right) \sin^2 \left( \frac{in\Delta}{2w} \right)
\]

(7.65)

Using the approximations (7.62) and (7.63) the components of the \( Y^A \) matrix can be written as:

\[
ym^A = \frac{1}{2\omega i \mu} \left\{ H_0^{(2)}(k\Delta|m-n-1|) - \left[ 1 - \frac{(k\Delta)^2}{2} \right] H_0^{(2)}(k\Delta|m-n|) + H_0^{(2)}(k\Delta|m-n+1|) \right\} - \frac{j2}{\omega \mu w} \sum_{i=1}^{\infty} \frac{k_i l_{ni} l_{mi}}{\tan(k_{ni}t)}
\]

\[m \neq n \text{ and } m \neq n \pm 1\]

\[
ym = \frac{1}{\omega i \mu} \left\{ H_0^{(2)}(k\Delta) - \left[ 1 - \frac{(k\Delta)^2}{2} \right] \left[ 1 - j2 \frac{\Delta}{\pi} \ln \left( \frac{\gamma k\Delta}{4e} \right) \right] \right\} - \frac{j2}{\omega \mu w} \sum_{i=1}^{\infty} \frac{k_i l_{ni} l_{mi}}{\tan(k_{ni}t)}
\]

\[m = n\]
\[ m = n \pm 1 \]

\[ y_{mn}^A = \frac{1}{2\omega \mu} \left( H_0^{(2)}(2k\Delta) - 2 \left[ 1 - \frac{(k\Delta)^2}{2} \right] H_0^{(2)}(k\Delta) + 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma k\Delta}{4e} \right) \right) \]

\[ \frac{j2}{\omega \mu w} \sum_{n=1}^{\infty} I_{nl} \frac{I_{nl}}{\tan(k_n t)} \]

where \( I_{nl} \) are given by equation (7.65).

The expressions for the other matrices, \( Y^B, Y^C \) and \( Y^D \), will be similar. Finally the components of the excitation matrix \( E \) are approximated by:

\[ E_n = 2\Delta H^{inc}(m\Delta) = 2\Delta \]

Since we cannot evaluate infinite summations, as in the expression for the matrix components, we must truncate the summations.

The summations are:

\[ \sum_{n=1}^{\infty} \frac{k_n I_{nl} I_{nl}}{\tan(k_n t)} \quad (7.66) \]

The propagation constant in region B, \( k_B \), is defined as:

\[ k_B = \omega \sqrt{\varepsilon_B \mu_B} \quad , \quad k_B^2 = k_{ni}^2 + \left( \frac{i\pi}{w} \right)^2 ; \quad i = 1, \infty \quad (7.67) \]

If \( \varepsilon_B \) and \( \mu_B \) are assumed to be real constants, i.e. no losses are present, the propagation constant \( k_B \) will be real and positive. Thus the \( k_{ni} \) can be written as:

\[ k_{ni} = \sqrt{k_B^2 - \left( \frac{i\pi}{w} \right)^2} \quad \text{when} \quad k_B \geq \frac{i\pi}{w} \]

\[ k_{ni} = j \sqrt{\left( \frac{i\pi}{w} \right)^2 - k_B^2} \quad \text{when} \quad k_B \leq \frac{i\pi}{w} \quad (7.68) \]

When \( k_{ni} \) are real the corresponding modes are representing standing waves. When \( k_{ni} \) are imaginary the corresponding modes are representing evanescent fields.

At least all modes representing standing waves should be included in the summation (7.66), thus the index \( i \) should at least be increased until \( i = \frac{k_B w}{\pi} \).
When \( k_z \) are imaginary, say \( k_z = j\xi \) where \( \xi = \sqrt{\left(\frac{ln}{w}\right)^2 - k_B^2} \), equation (7.66) becomes:

\[
\sum_{i=1}^{\infty} \frac{\xi_{il} I_{il}}{\tanh(\xi l)}
\]

where:
- \( \xi \) is proportional to \( i \), thus increasing for increasing \( i \)
- \( \tanh(\xi l) \) is approaching unity for increasing \( i \)
- \( I_{il} \) and \( I_{in} \) are inversely proportional to the square of \( i \), thus decreasing for increasing \( i \)

In summary it can be concluded that the components of the summation is decreasing for increasing index \( i \), i.e. for large \( i \). Thus, only a limited number of evanescent modes needs to be included in the summation.

The components in the other matrices, \( Y^B, Y^C, Y^D \), will behave in a similar fashion (in some components \( \tanh \) should be replaced with \( \sinh \)).

**TE-case**

For the TE-case pulses are used as basis functions and Dirac pulses as testing functions, the same choice as in chapter 7.2.1.

Thus:

\[
b_n(y') = \begin{cases} 1 & (n-1)\Delta \leq y' \leq n\Delta \\
0 & \text{elsewhere} \end{cases}
\]

\[
W_m(y) = \delta(y - y_m) = \begin{cases} 1 & y = y_m \\
0 & \text{elsewhere} \end{cases}
\]

\[
y_m = \frac{(2m-1)\Delta}{2} , \quad \Delta = \frac{w}{N}
\]

Using the same approximations as were used for the TM-case, i.e. equations (7.62) and (7.63), the components of the \( Y^A \) matrix can be written as:
\[ Y_{mn}^A = \frac{k^2 \Delta}{2 \omega \mu} \left[ 1 - j \frac{2}{\pi} \ln \left( \frac{\gamma k \Delta}{4 \epsilon} \right) \right] - \frac{j k \mu}{\omega \mu \tan(k \theta t)} - \frac{j 4 k_B^2}{\omega \mu \pi} \sum_{m'} \cos \left( \frac{i \pi \Delta (2m - 1)}{2w} \right) \cos \left( \frac{i \pi \Delta (2n - 1)}{2w} \right) \sin \left( \frac{i \pi \Delta}{2w} \right) \]

\[ Y_{mn}^B = \frac{k^2 \Delta}{2 \omega \mu} \left( H_0^2 \left( k \Delta |m - n| \right) \right) - \frac{j k \mu}{\omega \mu \tan(k \theta t)} - \frac{j 4 k_B^2}{\omega \mu \pi} \sum_{m'} \cos \left( \frac{i \pi \Delta (2m - 1)}{2w} \right) \cos \left( \frac{i \pi \Delta (2n - 1)}{2w} \right) \sin \left( \frac{i \pi \Delta}{2w} \right) \]

The expressions for the other matrices, \( Y^B, Y^C \) and \( Y^D \), will be similar. Finally the components of the excitation matrix \( E \) are approximated by:

\[ E_m \approx 2 H^{inc} (y_m) = 2 \]

The infinite summations involved in the expressions for the matrix components are similar to the summations in the TM-case. Thus, only a limited number of modes has to be taken into account.

7.4 Numerical results for a slot in a ground plane of finite thickness

The theory presented in the preceding chapter was incorporated in a computer code and in this section a few sample runs are presented.

Since the thick ground plane is expected to have a considerable effect only for the TM-case (i.e. when the incident electric field vector is parallel with the slot axis) this is the only case shown here.

In Fig. 7.36 below it is seen, as expected, that the equivalent magnetic current is lower in amplitude on the "backside" of the ground plane, i.e. the amplitude of \( M^B \) is lower than the amplitude of \( M^A \).
Figure 7:36. Equivalent magnetic currents $M^A$ and $M^B$ in a slot of width $0.2\lambda$ placed in a ground plane of thickness $0.1\lambda$.

Figure 7:37. Attenuation in a slot of width $0.2\lambda$ as a function of the ground plane thickness, calculated for $N=20$ (number of basis functions).
In Fig. 7:37 and 7:38 the attenuation in a slot of width $0,2\lambda$ as a function of the ground plane thickness is shown. As seen in Fig. 7:38 the thickness of the ground plane has to be taken into account if the ground plane is "thick". Referring to the results shown in Fig. 7:37 the ground plane can be considered as "thick" when the thickness is greater than about one hundredth of the width.

### 7.5 Slot with a cross section made up of cascaded rectangular sections

In this chapter the theory presented in the preceding chapters is extended to also include the case of slots with a cross section made up of cascaded rectangular sections, i.e. corrugated slots. In the field of EMC the corrugation can serve as a model for the gasket groove often used when joining two parts of a structure. Corrugations can also be used for increasing the attenuation in a slot, if the dimensions of the corrugations are selected correctly.

The cross section of a slot with two corrugations is shown in figure 7:39.
Figure 7:39. Example of corrugated slot with two corrugations.

Using the configuration in Fig. 7:39 as an example the procedure for determination of the equivalent magnetic currents at the two slot faces (the openings in the ground plane) can be developed as described in the following. Even though the theory is developed based on the geometry shown in Fig. 7:39 the technique is general and can be used for any slot which has a cross section that could be divided into a number of rectangular sections.

As described in preceding chapters the problem is divided into several sub problems by the use of the equivalence principle. For the configuration in Fig. 7:39 seven regions are needed. The different regions are shown in Fig. 7:40. Referring to Fig. 7:40 the regions A and C are the same as in the case treated in the preceding chapters, regions Bk are almost the same as region B in the preceding chapters. The difference between the general region Bk in Fig. 7:40 and region B treated before is that the equivalent magnetic currents at the left and right sides of the regions not necessarily are defined over the hole width. The equivalent magnetic currents are defined at the boundary between any two different sub regions, shown as dotted lines in Fig. 7:40. Let the equivalent magnetic currents be named in a numbered sequence, i.e. referring to Fig. 7:40 the extreme left current is named as M⁰ and the extreme right current as M⁶. Thus, for a general sub region Bk the current at the left side is named Mₖ and the current at the right side as Mₖ⁺¹. A general sub region Bk is shown in Fig. 7:41.
Figure 7.40. Division of the problem in seven different regions.

Figure 7.41. General sub region Bk.

A slightly different sign convention compared with the case treated in the preceding chapters is used here. In this chapter all equivalent magnetic currents are defined as the cross product between the electric field vector, at the present boundary, and the unit vector in the positive z-direction. Thus,

\[ M^k = E^k \times \hat{z} \] where \( E^k \) is the electric field at the \( k \):th boundary

\[ \Rightarrow M^k = \hat{x}E^k \] for TE and \( M^k = -\hat{y}E^k \) for TM

(This sign convention only implies that \( M^B \) of the preceding chapters would here be of the opposite sign.)

Now, when the equivalent magnetic currents are defined and equivalent sub problems are formed in the same way as was done in preceding chapters the regions have to be linked together. This is done by matching the tangential fields at the region boundaries.
The results are:

\[ z = z_1 = 0 : \]
\[
\left[ 2H_A(M^1) + H_{B1}(M^1) - H_{B1}(M^2) = -2H^\text{loc} \right]_{\text{em}}
\]

\[ z = z_6 : \]
\[
\left[ -H_{B(k-1)}(M^{k-1}) + H_{B(k-1)}(M^k) + H_{Bk}(M^k) - H_{Bk}(M^{k+1}) = 0 \right]_{\text{em}}
\]

\[ z = z_6 = -(T1 + T2 + T3 + T4 + T5) : \]
\[
\left[ -H_{B5}(M^5) + H_{B5}(M^6) + 2H_C(M^6) = 0 \right]_{\text{em}}
\]

(7.69)

Equations (7.69) are integral equations for the unknown equivalent magnetic currents \( M^1 \) - \( M^6 \) (in the general case integral equations for \( M^1 \) - \( M^{k+1} \)). In equations (7.69) the field operators \( H_A \) and \( H_C \) are the same as in chapter 7.2, i.e.:

\( \text{TE-case:} \)
\[
H_A(\hat{\delta}M) = H_C(\hat{\delta}M) = -\hat{\delta} \frac{k^2}{4\omega\mu} \int_{\text{bound}} MH_0^{(2)}(k|y - y'|)dy'
\]

(7.70)

\( \text{TM-case:} \)
\[
H_A(-\hat{\gamma}M) = H_C(-\hat{\gamma}M) = -\hat{\gamma} \frac{1}{4\omega\mu} \left( \frac{d^2}{dy^2} + k^2 \right) \int_{\text{bound}} MH_0^{(2)}(k|y - y'|)dy'
\]

where the integrals have to be taken over the appropriate boundary, i.e. at \( z_1 \) and \( z_6 \) respectively.

In order to be able to solve the system of equations (7.69) we also have to know the field operators for a general sub region \( B_k \), i.e. we have to know the following operators (refer to Fig. 7.41 and equations (7.69)):

\[
H_{Bk}(M^k)_{z = z_k}, \ H_{Bk}(M^{k+1})_{z = z_k}, \ H_{Bk}(M^k)_{z = (k+1)}, \ H_{Bk}(M^{k+1})_{z = (k+1)}
\]

(7.71)

The operators in (7.71) represent the magnetic field at the left and right hand side boundaries of region \( B_k \) (which are common with adjacent regions) produced by the equivalent magnetic currents at these boundaries. The operators (7.71) are easily obtained from the results derived in appendices A-D, which are the results used in the preceding chapters. The only difference between the operators in (7.71) and the ones derived in appendices A-D is that the equivalent magnetic currents in (7.71) are defined only on a
limited part of the left and right hand side boundary. It is merely a question of adopting the integration intervals in the expressions obtained in appendices A-D to fit in with the geometry for the general sub region Bk, shown in Fig. 7.41.

The results for the TM-case are:

\[
H_{y,Bk}(M^k)\bigg|_{z=z_{k-1}} = -j2 \sum_{i=1}^{\infty} \frac{k_a \sin \left( \frac{iy}{w_k} \right)}{w_k \tan(k_a T_k)} \int_{y_k}^{y_{k+1}} M^k \sin \left( \frac{iyy}{w_k} \right) dy
\]

\[
H_{y,Bk}(M^{k+1})\bigg|_{z=z_{k-1}} = j2 \sum_{i=1}^{\infty} \frac{k_a \sin \left( \frac{iy}{w_k} \right)}{w_k \sin(k_a T_k)} \int_{y_k}^{y_{k+1}} M^{k+1} \sin \left( \frac{iyy}{w_k} \right) dy
\]

\[
H_{y,Bk}(M^{k+1})\bigg|_{z=z_{k}} = -j2 \sum_{i=1}^{\infty} \frac{k_a \sin \left( \frac{iy}{w_k} \right)}{w_k \sin(k_a T_k)} \int_{y_k}^{y_{k+1}} M^{k+1} \sin \left( \frac{iyy}{w_k} \right) dy
\]

\[
H_{y,Bk}(M^{k+1})\bigg|_{z=z_{k}} = j2 \sum_{i=1}^{\infty} \frac{k_a \sin \left( \frac{iy}{w_k} \right)}{w_k \tan(k_a T_k)} \int_{y_k}^{y_{k+1}} M^{k+1} \sin \left( \frac{iyy}{w_k} \right) dy
\]

(7.72)
and for the TE-case:

\[
H_{x,\theta k}(M^k)_{|z=k} = \frac{j}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w_k} \right)}{w_k k_n \tan(k_n T_k)} \int_{y_0}^{y_0^*} M^k \cos \left( \frac{i\pi y}{w_k} \right) dy'
\]

\[
H_{x,\theta k}(M^{k+1})_{|z=k} = -\frac{j}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w_k} \right)}{w_k k_n \sin(k_n T_k)} \int_{y_0}^{y_0^*} M^{k+1} \cos \left( \frac{i\pi y}{w_k} \right) dy'
\]

\[
H_{x,\theta k}(M^k)_{|z=k+1} = \frac{j}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w_k} \right)}{w_k k_n \sin(k_n T_k)} \int_{y_0}^{y_0^*} M^k \cos \left( \frac{i\pi y}{w_k} \right) dy'
\]

\[
H_{x,\theta k}(M^{k+1})_{|z=k+1} = -\frac{j}{\omega \mu} \sum_{n=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w_k} \right)}{w_k k_n \tan(k_n T_k)} \int_{y_0}^{y_0^*} M^{k+1} \cos \left( \frac{i\pi y}{w_k} \right) dy'
\]

(7.73)

It should be noted that local co-ordinates are used in the expressions for the field operators.

As in the preceding chapters the next step in the solution process is to expand the unknown magnetic currents in series. Since the various equivalent magnetic currents are extending over different lengths it is wise to expand the currents in different numbers of basis functions, but still in functions of the same type.

Thus, \( M^k = \sum_{n=1}^{N_k} a_{kn} b_{kn}(y') \) where \( N_k \) is the number of basis functions for \( M^k \).

Insertion of the series expressions for the equivalent magnetic currents in equations (7.69) and taking the inner product, as defined in equation (7.6), with weighting functions \( W_{km} \); \( m=1..N_k; k=1..6 \) will result in a matrix equation. The matrix equation for the geometry in Fig. 7.39 (and 7.40) will be:
\[
\begin{bmatrix}
[y_4^A + y_1^B] & -[y_2^B] \\
-[y_3^A] & [y_2^B + y_3^B] \\
[0] & -[y_2^B] \\
[0] & [y_3^B + y_5^B] \\
[0] & [0] \\
[0] & [0] \\
[0] & [-y_5^B] \\
[0] & [y_5^B + y_6^B] \\
\end{bmatrix}
\begin{bmatrix}
[a_1] \\
[a_2] \\
[a_3] \\
[a_4] \\
[a_5] \\
[a_6] \\
[a_7] \\
[a_8] \\
\end{bmatrix}
= -[E]
\]

(7.74)

where the notations used for the sub matrices are constructed according to: $Y_{\text{region name}}\text{ current No.}$

and the minus sign over the region name is denoting that the corresponding field expressions (see the following) should be valid for the left boundary in the region. Consequently the absence of minus sign means the right boundary. The components of the sub matrices in equation (7.74) are given by:

\[
Y_{\text{b}m}^{\beta k} = \int_{y_{\beta}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} dy, \quad Y_{\text{b}m}^{\beta k} = \int_{y_{\beta}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} dy
\]

\[
Y_{k+1}^{\text{b}m} = \int_{y_{k+1}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy, \quad Y_{k+1}^{\text{b}m} = \int_{y_{k+1}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy
\]

\[
Y_{k+2}^{\text{b}m} = \int_{y_{k+2}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy, \quad Y_{k+2}^{\text{b}m} = \int_{y_{k+2}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy
\]

\[
Y_{k+2}^{\text{b}m} = \int_{y_{k+2}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy, \quad Y_{k+2}^{\text{b}m} = \int_{y_{k+2}} y_{\text{a}m}^{\beta k} H_{Bk}(b_{\text{m}n})_{\beta} \right|_{k+1}^{k+2} W_{(k+1)m} dy
\]

(7.75)

$H_A$ and $H_B$ are defined in (7.70) and the $H_{Bk}$ are defined in (7.72) and (7.73) for TM and TE respectively.

By choosing the basis and weighting functions to be the same as those used in chapter 7.2 the matrix components in (7.75) can be determined in the same way as the matrix components in chapter 7.2. Of course the matrix equation (7.74) can be solved by direct inversion, but since the matrix is sparse it is more efficient to solve the equation using the following method.

The last row in the matrix (7.74) is:

\[-[y_5^{B5}] [a_5] + [y_6^{B5} + y_6^{C}] [a_6] = [0]\]

which solved for the coefficients $a_6$ gives:

\[[a_6] = [y_6^{B5} + y_6^{C}]^{-1} [y_5^{B5}] [a_5]\]

Insertion of this equation in the second equation from the bottom in the matrix, and solving for the coefficients $a_5$ gives:
\[
[a_i] = \left[ (Y_5^{bs} + Y_5^{bs}^{-1} [Y_6^{bs} + Y_6^{bs}]^{-1} [Y_5^{bs}])^{-1} [Y_4^{bs}] \right]^{[a_i]}
\]

insertion of this equation in the next equation, and continuing this process until the last
equation in the matrix is treated, finally gives:

\[
[a_i] = -[M_{i2}] [E]
\]

where:

\[
[M_{i2}] = \left[ (Y_i^{A} + Y_i^{B}) - [Y_i^{B}] [M_{i3}] [Y_i^{B}]^{-1} \right]
\]

\[
[M_{i3}] = \left[ (Y_i^{B(i-1)} + Y_i^{B}) - [Y_i^{B(i-1)}] [M_{i4}] [Y_i^{B}]^{-1} \right]
\]

........

\[
[M_{i(i+1)}] = \left[ (Y_i^{B(i-1)} + Y_i^{B}) - [Y_i^{B(i-1)}] [M_{i(i+1)(i+2)}] [Y_i^{B}]^{-1} \right]
\]

........

\[
[M_{i(i+1)}] = \left[ (Y_i^{bs} + Y_i^{C}) \right]^{-1}
\]

Thus, the procedure for a general slot with k B-regions will be:

1. Determine \([M_{(i+1)(i+2)}] = \left[ (Y_i^{bs} + Y_i^{C}) \right]^{-1}\)
2. Let \(i = k\)
3. Determine \([M_{i(i+1)}] = \left[ (Y_i^{B(i-1)} + Y_i^{B}) - [Y_i^{B(i-1)}] [M_{i(i+1)(i+2)}] [Y_i^{B}]^{-1} \right]\)
4. Let \(i = i - 1\)
5. Repeat from point 3 until \(i = 0\)
6. Determine \([a_i] = -[M_{i2}] [E]\)
7. Let \(i = 2\)
8. Determine \([a_i] = \left[ (Y_i^{B(i-1)} [Y_i^{(i-1)}] [a_{i-1}] \right]\)
9. Let \(i = i + 1\)
10. Repeat from point 8 until \(i = k + 2\)

This procedure is known as mode matching.

7.6 Corrugated slots, a method to increase the attenuation in a slot for the TE-polarisation

In this chapter it is shown that it is possible to increase the attenuation in a slot considerably, for the TE-case, by introducing a corrugation in the cross section. The corrugation has to be a quarter of a wavelength deep so the corresponding bandwidth will
be limited. However, it is also shown that the bandwidth can be increased by introducing more than one corrugation in the cross section.

The cross section for a slot with one corrugation is shown in Fig. 7:42.

![Figure 7:42. Corrugated slot. All dimensions in mm.](image)

The attenuation, as defined by equation (7:35), was computed for the slot in Fig. 7:42 by the method described in the preceding chapter, the result is shown in figure 7:43. As seen in Fig. 7:43 the attenuation is considerable in the frequency range corresponding to a quarter of a wavelength deep corrugation. It is also seen that the attenuation is considerable at a frequency corresponding to the quarter of a wavelength depth times three. In fact it can be shown that the attenuation will be high at every odd multiple of the frequency corresponding to the quarter of a wavelength depth. Henceforth the frequency that corresponds to a quarter of a wavelength deep corrugation will be called the resonance frequency.

![Figure 7:43. Attenuation as a function of the frequency (in GHz) for the configuration in Fig. 7:42.](image)

By changing the width of the corrugation, 3 mm in Fig. 7:42, the resonance frequency as well as the bandwidth change. The attenuation for a corrugation width of 1, 2 and 3 mm is shown in Fig. 7:44 (the other dimensions are kept the same as in Fig. 7:42). In Fig. 7:44 it can be seen that the resonance frequency as well as the bandwidth decrease as the width of the corrugation decreases.
Figure 7.44. Attenuation for the configuration shown in Fig. 7.42 but for different widths of the corrugation.

In order to verify the results obtained by the computer code also for the case of corrugated slots, a comparison with measured results was conducted. The comparison was made for a slot without corrugation and for a slot with one corrugation. The compared quantity was the transmission coefficient, which is the inverse of the attenuation. The geometries for the two slots are shown in Fig. 7.45 and the results in Fig. 7.46. A good agreement between measured and calculated values can be observed.

Figure 7.45. Geometries for comparison between measured and calculated values, dimensions in mm.
Figure 7.46. Results for a comparison between measured and calculated transmission coefficients.

One conclusion that can be drawn from Fig. 7.44 is that one way to increase the bandwidth for the system would be to cascade two corrugations with different widths. The configuration for two cascaded corrugations with the widths 1.5 and 3 mm is shown in Fig. 7.47, the depths are kept at 7.5 mm. The attenuation as a function of the frequency for the configuration in Fig. 7.47 is shown in Fig. 7.48. As expected the bandwidth for the system is increased, the bandwidth at the 20 dB level is approximately 3.5 GHz. It can also be seen that the resonance frequencies for the different corrugations are still present. Another observation that can be made is that the overall attenuation is increased because of the effects of the two corrugations.

Figure 7.47. Two corrugations with different widths, 1.5 and 3 mm respectively and the depth 7.5 mm.
Figure 7:48. Solid line represents attenuation for the configuration in Fig. 7:47, dotted line represents attenuation for an ordinary slot with the width 3 mm and the thickness 7.5 mm.

Another, and perhaps more natural, way to increase the bandwidth for the system would be to cascade two corrugations with the same width but with different depths. The configuration for such a system is shown in Fig. 7:49. The depths are chosen to be equal to a quarter of a wavelength at 8 and 10 GHz. The attenuation for the slot shown in Fig. 7:49 is shown in Fig. 7:50. The bandwidth at the 20 dB level is now almost 5 GHz.

Figure 7:49. Two corrugations with different depths, dimensions in mm.
Figure 7:50. Solid line represents attenuation for the configuration in Fig. 7:49, dotted line represents attenuation for an ordinary slot with the width 3 mm and the thickness 9 mm.

The bandwidth can be increased even further by adding more corrugations. In Fig. 7:51 the attenuation for a triple corrugation with the depths corresponding to a quarter of a wavelength at 8, 10 and 12 GHz is shown. In Fig. 7:52 the attenuation for a triple corrugation with the depths corresponding to a quarter of a wavelength at 8, 10 and 14 GHz is shown. For both cases, all corrugation widths are chosen to be 3 mm and the material thickness between corrugations are 1 mm, the openings in the ground plane are kept at 3 mm. The achieved bandwidths at the 20 dB level are approximately 7 and 8 GHz respectively (if the small dip around 9 GHz in Fig. 7:52 is disregarded).
Figure 7.51. Solid line represents attenuation for a triple corrugation with the depths corresponding to a quarter of a wavelength at 8, 10 and 12 GHz, the widths of all corrugations are 3 mm and the material thickness between corrugations is 1 mm, the opening in the ground plane is 3 mm. Dotted line represents attenuation for an ordinary slot with the width 3 mm and the thickness 13 mm.

Figure 7.52. Solid line represents attenuation for a triple corrugation with the depths corresponding to a quarter of a wavelength at 8, 10 and 14 GHz, the widths of all corrugations are 3 mm and the material thickness between corrugations is 1 mm, the opening in the ground plane is 3 mm. Dotted line represents attenuation for an ordinary slot with the width 3 mm and the thickness 13 mm.
The depths of two opposite grooves do not have to be equal as in the examples presented so far. By using different depths for every groove the bandwidth can be increased, compared with a system with the same number of grooves but with every two grooves equally deep. An example of a triple corrugation with different depths for all grooves is shown in Fig. 7:53 and the attenuation for the system is shown in Fig. 7:54. The bandwidth at the 20 dB level is now approximately 10 GHz.

Figure 7:53. A triple corrugation with different depths for all grooves. Dimensions in mm.

Figure 7:54. Solid line represents attenuation for the system shown in Fig. 7:53. Dotted line represents attenuation for an ordinary slot with the width 3 mm and the thickness 13 mm.
8 Sample case

In this chapter an example is treated in order to exemplify the use of the prediction models discussed in the preceding chapters.

The example consists of a "black box" connected to two terminated transmission lines, as shown in figure 8:1. One of the lines is used for transmitting a square wave from the box to a resistive load, this could for instance simulate a clock signal from a computer to a peripheral equipment. The second line is used for transmitting a DC-signal, produced by a sensor, to the box.

![Diagram of the example setup](image)

Figure 8:1. Configuration for sample case.

The parameters for the example are as follows:

- \( V_{\text{Box}} = 5 \ V_{\text{peak-peak}} \), filtered square wave, 5 MHz, internal impedance 100 Ω, voltage produced by the "black box"
- \( V_{\text{Sen}} = 60 \text{ mV} \), internal impedance 100 Ω, DC-voltage produced by the sensor
- \( Z_L = 100 \Omega \), impedance of the load
- \( Z_\Omega = 100 \Omega \), internal impedance of the "black box"
- \( L = 1 \text{ m} \), length of the transmission lines
- \( r = 0.1 \text{ mm} \), radius of transmission lines
- \( h = 0.1 \text{ m} \), height of the lines over the ground plane

For the present example we considered two EMC requirements, i.e. radiated emission and radiated susceptibility.

8.1 Radiated emission

If we assume the "black box" to be shielded the radiation will undoubtedly emanate from the wires connected to the box. It is only line 1 that is carrying a signal with a frequency content, therefore we concentrate on that line. The radiation from a wire over a ground plane can, for instance, be analysed with the method of moments program AWAS [3].

The clock signal on line 1 is a filtered square wave with the fundamental frequency 5 MHz. The unfiltered signal is assumed to be ideal and therefore we know from the Fourier analysis that the signal can be expressed as an infinite sum of sinusoidal signals with increasing frequency. The frequencies will be odd multiples of the fundamental
frequency and the amplitudes will be proportional to the inverse of the frequency. If the signal is filtered in a way so that all frequencies above 90 MHz are suppressed, the signal can be expressed as a finite sum of sinusoidal signals. Thus, the clock signal can be written as:

\[ V(i) = \frac{10}{\pi} \sum_{i=1}^{10} \frac{1}{i} \sin(2\pi f_s t) \text{; where } f_s = 5 \text{ MHz} \]

The clock signal is visualised in the frequency domain in Fig. 8.2.

![Graph showing the frequency domain of the clock signal.](image)

**Figure 8.2.** The clock signal on line 1 shown in the frequency domain.

To be able to compare with requirements given in many civilian standards, the radiated field is computed at a distance of 10 meters from the equipment. Further, many standards require the field to be monitored at heights between 1 and 4 meters, therefore the field values are calculated at heights in this range.

The amplitudes of the electric field, computed by OAS, at a distance of 10 meters and at different heights are shown in Fig. 8.3 and Fig. 8.4 for horizontal and vertical polarisation respectively.
Figure 8.3. Radiated emission levels at a distance of 10 meters from the equipment shown in Fig. 8.1. Calculated at the heights 1, 2, 3 and 4 meters with the code AWAS [3]. Wire height over the ground plane 0.1 m. Horizontal polarisation.

Figure 8.4. Radiated emission levels at a distance of 10 meters from the equipment shown in Fig. 8.1. Calculated at the heights 1, 2, 3 and 4 meters with the code AWAS [3]. Wire height over the ground plane 0.1 m. Vertical polarisation.
The radiation amplitudes given in Fig. 8:3 and 8:4 could, for instance, be compared with the requirements given in CISPR 22 for class B equipment which are as low as 30 dBμV/m, in the frequency range of interest. Thus, it is clear that we need to reduce the radiation with approximately 25-30 dB in order to comply with the requirements.

According to chapter 4, radiation from wires, the radiation from a wire over a ground plane can be reduced by placing the wire closer to the ground plane. This was examined, in chapter 4, for the two-dimensional case. It was also stated that results obtained for a two-dimensional problem can be used as guidelines for three-dimensional problems. So, let us see if the radiation could be sufficiently reduced by lowering the wire to the height 2 mm. The results are shown in Fig. 8:5 and Fig. 8:6.

**Figure 8:5.** Radiated emission levels at a distance of 10 meters from the equipment shown in Fig. 8:1, but with the wire lowered to a height of 2 mm. Calculated at the heights 1, 2, 3 and 4 meters with the code AWAS [3]. Horizontal polarisation.
Figure 8.6. Radiated emission levels at a distance of 10 meters from the equipment shown in Fig. 8.1, but with the wire lowered to a height of 2 mm. Calculated at the heights 1, 2, 3 and 4 meters with the code AWAS [3]. Vertical polarisation.

According to the results in Fig. 8.5 and 8.6 the step taken by lowering the wire to a height of 2 mm was sufficient to make the equipment comply with the radiated emission requirement of 30 dBμV/m at a distance of 10 m.

8.2 Radiated susceptibility

As was the case for the radiated emission, the wire connected to the box will be our main concern. The "black box" shall measure the signal produced by the sensor, which is a DC-signal with an amplitude of 60 mV. In order to get the correct result from the sensor the voltage induced on the wire, seen by the "black box", caused by an incident electromagnetic field should be sufficiently low. It is assumed that the induced voltage on the line is rectified in a way so that the voltage seen by the box is the peak value.

A realistic requirement for the radiated susceptibility level is 3 V/m in the frequency range 27 - 500 MHz, i.e. EN 50 082-1.

The induced voltage, seen by the box (peak value), as a function of the frequency of the interfering electromagnetic field is shown in Fig. 8.7. The electric field strength is 3 V/m and the results were obtained with the transmission line code NULINE [2].
Figure 8:7. Induced voltage, seen by the "black box", caused by an incident electromagnetic field. The electric field strength is 3 V/m. The response is calculated with NULINE [2]. Dotted line represents the voltage amplitude produced by the sensor, 60 mVDC.

As seen from Fig. 8:7 the "black box" will not see the true signal produced by the sensor when the system is exposed to an electromagnetic field with the field strength 3 V/m. In order to suppress the induced signal to a sufficiently low level the input terminal at the "black box" could be filtered. Another way would be to shield the transmission line.

To filter the signal a simple inductor with the inductance 0.1 mH connected in series with the input terminal is chosen, Fig. 8:8.

![Series inductor, 0.1 mH](image)

Figure 8:8. The input terminal with a simple 0.1 mH series inductor as a filter.

The induced voltage seen by the black box with the inductor installed is shown in Fig. 8:9.
Figure 8.9. Induced voltage, seen by the "black box", caused by an incident electromagnetic field. The electric field strength is 3 V/m. The response is calculated with NULINE [2]. The input terminal is equipped with a series inductor with the inductance 0.1 mH.

If an induced voltage of maximum 10 mV could be accepted the step taken by using an inductor with the inductance 0.1 mH as a filter is sufficient to make the equipment comply with the radiated susceptibility requirement of 3 V/m.
References


Appendix A \( H_\beta(-M^A) \bigg|_{\tan} \) TM-case

Derivation of field expression for the tangential magnetic field in region B produced by the magnetic current \(-M^A\), i.e. \( H_\beta(-M^A) \bigg|_{\tan} \).

For the TM-case:
\[-M^A = jM^A\]

The geometry for the problem is shown in Fig. A:1.

![Geometry for determination of the field in region B.](image)

Figure A:1. Geometry for determination of the field in region B.

Since all magnetic current is y-directed, a y-directed electric vector potential is sufficient for representing the field.

Thus:
\[ F = jF(y, z) \]

When the electric vector potential, \( F \), is determined the field quantities can be determined by the following equations, [7]:

\[ E_x = \frac{dF}{dz}, \quad E_y = 0, \quad E_z = -\frac{dF}{dx} = 0 \]

\[ H_x = -j\frac{1}{\omega \mu} \frac{d^2F}{dx dy} = 0, \quad H_y = -j\frac{1}{\omega \mu} \left( \frac{d^2}{dy^2} + k_b^2 \right) F, \quad H_z = -j\frac{1}{\omega \mu} \frac{d^2F}{dy dz} \quad \text{(A:1)} \]

Where the electric vector potential, \( F \), must satisfy the scalar wave equation:

\[ \nabla^2 F + k_b^2 F = 0 \quad \text{(A:2)} \]

with the following boundary conditions (follows from equations (A:1)):
\[
\frac{dF}{dz} = 0 \quad \text{at} : \quad y = 0, \ y = w, \ z = -t \\
\frac{dF}{dz} = M_A \quad \text{at} : \quad z = 0
\]  
(A:3)

A solution that satisfies (A:2) is assumed:

\[
F(y, z) = \sum_{i=1}^{\infty} A_i \cos[k_i(z + t)]\sin\left(\frac{iny}{w}\right)
\]  
(A:4)

where:

\[A_i\] are coefficients (yet unknown)

\[k_i^2 = k_b^2 - \left(\frac{i\pi}{w}\right)^2
\]

(A:5)

\[
\Rightarrow \frac{dF}{dz} = -\sum_{i=1}^{\infty} A_i k_i \sin[k_i(z + t)]\sin\left(\frac{iny}{w}\right)
\]

Which clearly satisfies the first three boundary conditions of (A:3). The remaining boundary condition requires:

\[
\left.\frac{dF}{dz}\right|_{z=0} = M_A = -\sum_{i=1}^{\infty} A_i k_i \sin(k_i t)\sin\left(\frac{iny}{w}\right)
\]  
(A:6)

The right hand side of (A:6) is recognised as a Fourier sine series, with the coefficients

\[-A_i k_i \sin(k_i t)\]. The Fourier coefficients can be determined as:

\[-A_i k_i \sin(k_i t) = \frac{2}{w} \int_{0}^{w} M_A \sin\left(\frac{iny}{w}\right)dy
\]

(A:7)
To sum up:

\[ F(y, z) = \sum_{i=1}^{\infty} A_i \cos[k_{\alpha t}(z + t)] \sin \left( \frac{iny}{w} \right) \]

\[ A_i = \frac{-2}{\omega k_{\alpha t} \sin(k_{\alpha t} t)} \int_0^w M^\alpha \sin \left( \frac{iny}{w} \right) dy' \]

\[ k_{\alpha t}^2 = k_{b t}^2 - \left( \frac{in\pi}{w} \right)^2 \]

The interesting field component is the tangential magnetic field at \( z=0 \) and \( z=-t \), i.e. \( H_y \) in equation (A.1).

\[ H_y|_{z=0} = -j \frac{\omega}{\mu} \left( \frac{d^2}{dy^2} + k_{b t}^2 \right) \sum_{i=1}^{\infty} A_i \cos(k_{\alpha t} t) \sin \left( \frac{iny}{w} \right) = \]

\[ = \frac{j2}{\omega \mu} \left( \frac{d^2}{dy^2} + k_{b t}^2 \right) \sum_{i=1}^{\infty} \frac{\sin \left( \frac{iny}{w} \right)}{wk_{\alpha t} \tan(k_{\alpha t} t)} \int_0^w M^\alpha \sin \left( \frac{iny'}{w} \right) dy' \]

and

\[ H_y|_{z=-t} = -j \frac{\omega}{\mu} \left( \frac{d^2}{dy^2} + k_{b t}^2 \right) \sum_{i=1}^{\infty} A_i \sin \left( \frac{iny}{w} \right) = \]

\[ = \frac{j2}{\omega \mu} \left( \frac{d^2}{dy^2} + k_{b t}^2 \right) \sum_{i=1}^{\infty} \frac{\sin \left( \frac{iny}{w} \right)}{wk_{\alpha t} \sin(k_{\alpha t} t)} \int_0^w M^\alpha \sin \left( \frac{iny'}{w} \right) dy' \]
Appendix B  \( H_b(-M^b) \bigg|_{\text{tan}} \) TM-case

Derivation of field expression for the tangential magnetic field in region B produced by the magnetic current -\( M^B \), i.e. \( H_b(-M^B) \bigg|_{\text{tan}} \).

For the TM-case:

\[-M^B = -\hat{j}M^B\]

The geometry for the problem is shown in Fig. B:1.

![Figure B:1. Geometry for determination of the field in region B.](image)

Following the same procedure as in appendix A but now with the following boundary conditions for the vector potential, \( F \):

\[
\frac{dF}{dz} = 0 \quad \text{at: } y = 0, \; y = w, \; z = 0
\]

\[
\frac{dF}{dz} = -M^B \quad \text{at: } z = -t
\]

leads to the following expression for the electric vector potential:

\[
F(y, z) = \sum_{l=1}^{\infty} A_l \cos(k_{ul}z) \sin \left( \frac{lny}{w} \right)
\]

\[
A_l = \frac{-2}{wk_{ul} \sin(k_{ul}t)} \int_0^w M^B \sin \left( \frac{lny'}{w} \right) dy'
\]

\[
k_{ul}^2 = k_p^2 - \left( \frac{ln}{w} \right)^2
\]
And the tangential magnetic field, \( H_y \), at \( z=0 \) and \( z=-t \) can be written as:

\[
H_y\big|_{z=0} = -\frac{j}{\omega \mu} \left( \frac{d^2}{dy^2} + k_b^2 \right) \sum_{i} A_i \sin \left( \frac{iny}{w} \right) = \\
= \frac{j2}{\omega \mu} \left( \frac{d^2}{dy^2} + k_b^2 \right) \sum_{i} \frac{\sin \left( \frac{iny}{w} \right)}{wk_i \sin(k_i t)} \int_{0}^{w} M^b \sin \left( \frac{iny'}{w} \right) dy'
\]

and

\[
H_y\big|_{z=-t} = -\frac{j}{\omega \mu} \left( \frac{d^2}{dy^2} + k_b^2 \right) \sum_{i} A_i \cos(k_i t) \sin \left( \frac{iny}{w} \right) = \\
= \frac{j2}{\omega \mu} \left( \frac{d^2}{dy^2} + k_b^2 \right) \sum_{i} \frac{\sin \left( \frac{iny}{w} \right)}{wk_i \tan(k_i t)} \int_{0}^{w} M^b \sin \left( \frac{iny'}{w} \right) dy'
\]
Appendix C  \( H_n(-M^A) \bigg|_{\text{tan}} \) TE-case

Derivation of field expression for the tangential magnetic field in region B produced by the magnetic current \(-M^A\), i.e. \( H_n(-M^A) \bigg|_{\text{tan}} \).

For the TE-case:
\[-M^A = -iM^A\]

The geometry for the problem is shown in Fig. C:1.

![Figure C:1. Geometry for determination of the field in region B.](image)

Since all magnetic current is x-directed, a x-directed electric vector potential is sufficient for representing the field.

Thus:
\[ F = iF(y, z) \]

When the electric vector potential, \( F \), is determined the field quantities can be determined by the following equations, [7]:

\[ E_x = 0, \quad E_y = -\frac{dF}{dz}, \quad E_z = \frac{dF}{dy} \]

\[ H_x = -j \frac{1}{\omega \mu} \left( \frac{d^2}{dx^2} + k_b^2 \right) F = -j \frac{k_b^2}{\omega \mu} F, \quad H_y = -j \frac{1}{\omega \mu} \frac{d^2F}{dxdy} = 0, \quad H_z = -j \frac{1}{\omega \mu} \frac{d^2F}{dxdy} = 0 \]  

(C:1)

Where the electric vector potential, \( F \), must satisfy the scalar wave equation:

\[ \nabla^2 F + k_b^2 F = 0 \]  

(C:2)

with the following boundary conditions (follows from equations (C:1)):
\[ \frac{dF}{dz} = 0 \quad \text{at} : z = -t \]
\[ \frac{dF}{dy} = 0 \quad \text{at} : y = 0, \ y = w \]
\[ \frac{dF}{dz} = -M^4 \quad \text{at} : z = 0 \] 

(C:3)

A solution that satisfies (C:2) is assumed:

\[ F(y, z) = \sum_{i=0}^{\infty} A_i \cos[k_n(z + t)] \cos \left( \frac{i\pi y}{w} \right) \] 

(C:4)

Where:

\[ A_i \] are coefficients (yet unknown)
\[ k_n^2 = k^2 - \left( \frac{i\pi}{w} \right)^2 \] 

(C:5)

\[ \Rightarrow \frac{dF}{dz} = -\sum_{i=0}^{\infty} A_i k_n \sin[k_n(z + t)] \cos \left( \frac{i\pi y}{w} \right) \]

and

\[ \Rightarrow \frac{dF}{dy} = -\sum_{i=0}^{\infty} A_i \frac{i\pi}{w} \cos[k_n(z + t)] \sin \left( \frac{i\pi y}{w} \right) \]

Which clearly satisfies the first three boundary conditions of (C:3). The remaining boundary condition requires:

\[ \left. \frac{dF}{dz} \right|_{z=0} = -M^4 = -\sum_{i=0}^{\infty} A_i k_n \sin(k_n t) \cos \left( \frac{i\pi y}{w} \right) \] 

(C:6)

The right hand side of (C:6) is recognised as a Fourier cosine series, with the coefficients

\[-A_i k_n \sin(k_n t)\]. The Fourier coefficients can be determined as:
\[ A_i k_n \sin(k_n t) = \frac{\alpha_i}{w} \int_0^w M^A \cos\left(\frac{iny}{w}\right)dy \]  
\[ \text{where: } \begin{cases} \alpha_i = 1, & i = 0 \\ \alpha_i = 2, & i \geq 1 \end{cases} \]  
\[ (C:7) \]

To sum up:

\[ F(y, z) = \sum_{i=0}^{\infty} A_i \cos(k_n(z+t)) \cos\left(\frac{iny}{w}\right) \]

\[ A_i = \frac{\alpha_i}{w k_n \sin(k_n t)} \int_0^w M^A \cos\left(\frac{iny}{w}\right)dy \]

\[ k_n^2 = k_n^2 - \left(\frac{\alpha_i}{w}\right)^2 \]

The interesting field component is the tangential magnetic field at \( z=0 \) and \( z=-t \), i.e. \( H_x \) in equation (C:1).

\[ H_x\big|_{z=0} = -\frac{jk_n^2}{\omega \mu} \sum_{i=0}^{\infty} A_i \cos(k_n t) \cos\left(\frac{iny}{w}\right) = \]

\[ = -\frac{jk_n^2}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{w k_n \tan(k_n t)} \int_0^w M^A \cos\left(\frac{iny}{w}\right)dy \]

and

\[ H_x\big|_{z=-t} = -\frac{jk_n^2}{\omega \mu} \sum_{i=0}^{\infty} A_i \cos\left(\frac{iny}{w}\right) = \]

\[ = -\frac{jk_n^2}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos\left(\frac{iny}{w}\right)}{w k_n \sin(k_n t)} \int_0^w M^A \cos\left(\frac{iny}{w}\right)dy \]
Appendix D  \( H_n(-M^B) \big|_{\text{tan}} \) TE-case

Derivation of field expression for the tangential magnetic field in region B produced by the magnetic current \(-M^B\), i.e. \( H_n(-M^B) \big|_{\text{tan}} \).

For the TE-case:

\[-M^B = iM^B\]

The geometry for the problem is shown in Fig. D:1.

![Figure D:1. Geometry for determination of the field in region B.](image)

Following the same procedure as in appendix C but now with the following boundary conditions for the vector potential, \( F \):

\[
\begin{align*}
\frac{dF}{dz} &= 0 \quad \text{at} : z = 0 \\
\frac{dF}{dy} &= 0 \quad \text{at} : y = 0, y = w \\
\frac{dF}{dz} &= M^B \quad \text{at} : z = -t 
\end{align*}
\]

leads to the following expression for the electric vector potential:
\[ F(y, z) = \sum_{i=0}^{\infty} A_i \cos(k_{zt} z) \cos \left( \frac{in\gamma}{w} \right) \]

\[ A_i = \frac{\alpha_i}{i \omega k_{zt} \sin(k_{zt} t)} \int_0^\infty M^B \cos \left( \frac{i\pi y'}{w} \right) dy' \]

\[ \alpha_0 = 1, \quad \alpha_i = 2 ; i \geq 1 \]

\[ k_{zt}^2 = k_{at}^2 - \left( \frac{i \pi}{w} \right)^2 \]

And the tangential magnetic field, \( H_x \), at \( z=0 \) and \( z=-t \) can be written as:

\[ H_x \big|_{z=0} = -\frac{-jk^2}{\omega \mu} \sum_{i=0}^{\infty} A_i \cos \left( \frac{i\pi y}{w} \right) = \]

\[ = -\frac{-jk^2}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w} \right)}{i \omega k_{zt} \tan(k_{zt} t)} \int_0^\infty M^B \cos \left( \frac{i\pi y'}{w} \right) dy' \]

and

\[ H_x \big|_{z=-t} = -\frac{-jk^2}{\omega \mu} \sum_{i=0}^{\infty} A_i \cos(k_{zt} t) \cos \left( \frac{i\pi y}{w} \right) = \]

\[ = -\frac{-jk^2}{\omega \mu} \sum_{i=0}^{\infty} \frac{\alpha_i \cos \left( \frac{i\pi y}{w} \right)}{i \omega k_{zt} \tan(k_{zt} t)} \int_0^\infty M^B \cos \left( \frac{i\pi y'}{w} \right) dy' \]