Anders Bergman

DIFFERENCE AMPLIFIER FOR USE IN ACCURATE VOLTAGE TRANSFORMER CALIBRATIONS

SP REPORT 1989:27
Electrical Metrology
Borås 1989

ABSTRACTS

The operating conditions are investigated for a difference amplifier used for calibration of voltage transformers and which is connected between the outputs of two voltage transformers. The criteria on the performance of the difference amplifier are developed for several applications and set in such a form as to be readily verifiable.

KEYWORDS: Voltage transformer, transformer calibration, differential amplifier, isolation amplifier

STATENS PROVNINGSANSTALT
RAPPORT 1989:27
ISBN 91-7848-177-5
ISSN 0284-5172
Borås 1989

SWEDISH NATIONAL TESTING INSTITUTE
REPORT 1989:27

Postal adress
P.O. Box 857, S-501 15 BORÅS
SWEDEN
Telephone int +46 33 165000
Telefax int +46 33 135502
Telex 36252 , Testing S

CONTENTS

SUMMARY

Calibration of a voltage transformer is commonly performed by comparing the voltage transformer against a known standard by means of one of several available bridge techniques.

The difference between the outputs of the standard and the voltage transformer must either be compensated to zero or directly measured. It is in both cases advantageous to utilize an accurate difference and isolation amplifier to both amplify the weak difference signal and to transfer the signal to earth potential.

This paper investigates the influence of the inevitable errors of the difference amplifier on the total measurement. The main parameters investigated are the amplifier amplitude error (ratio) and its phase displacement, the isolation mode rejection and the noise contributions. A set of specifications is developed for the difference amplifier based on a set of reasonable and commonly used assumptions regarding the required accuracy for calibrations of different voltage transformers.

The results obtained prove that the requirements on the difference amplifier are high, but well within the range possible to realize with commercially available building blocks.

1 GENERAL

The use of a difference amplifier in voltage transformer calibrations is shown in figure 1. The voltage standard could be a standard transformer or a voltage divider. The difference in the output voltage of the transformer under test and the voltage standard is fed to the differential input of the difference amplifier. The input signal to the difference amplifier thus has an isolation mode voltage equal to the output voltage of the voltage transformer, normally in the range of 50-100 V. For simplicity 100 V has been assumed in the following. The difference voltage may range from some 100 μV to 10 V depending on the accuracy of the transformer. At test voltages below nominal voltage, the difference voltage will be correspondingly lower.

In the following the ratio error and phase displacement incurred by the amplifier will be investigated and the permissible limits will be identified.

The contribution to the output signal caused by the isolation mode voltage will be characterized and permissible limits identified.

The noise contribution of the amplifier will be discussed and the permissible limits identified.

The above three parameters are treated separately and any effect of superpositioning of the parameters is considered negligible. It is also noted that ratio error and phase displacement may be considered independant of the test voltage, whereas the evaluation of isolation mode rejection and noise contribution must take the test voltage into account.

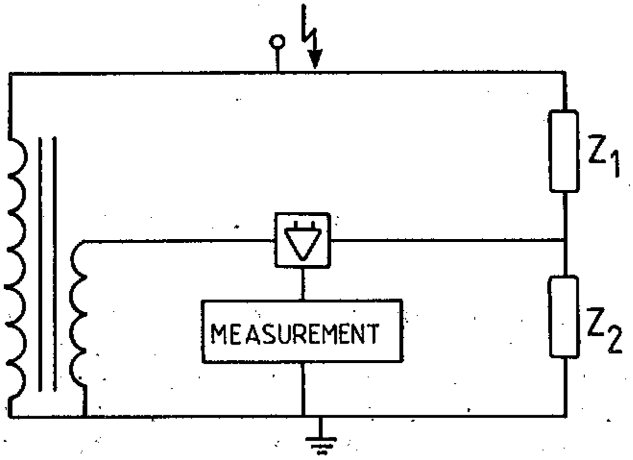

Figure 1. Voltage transformer calibration circuit

2 RATIO AND PHASE DISPLACEMENT

2.1 Difference signal

The output voltage of the voltage transformer and of the voltage standard can be written as:

Voltage standard	$U \sin \omega t$
Transformer under test	$U(1-k) \sin(\omega t+\delta)$

where k is the ratio error and δ is the phase displacement.

The difference voltage is thus:

$$\Delta U = U \sin \omega t - U(1-k)\sin(\omega t+\delta) =$$

$$= U(1-(1-k) \cos \delta) \sin \omega t - U(1-k)\sin \delta \cos \omega t =$$

$$= U_r \sin \omega t + U_i \cos \omega t$$

where:

$$U_r = U(1-(1-k)\cos \delta)$$

$$U_i = - U(1-k)\sin \delta$$

The quantities U_r and U_i can be measured directly at the output of the difference amplifier if the amplifier is ideal. The quantity U_r is the instantaneous voltage at $\sin\omega t=0$ and U_i is the instantaneous voltage at $\sin\omega t=1$. U_r and U_i may also be determined by applying a compensating voltage of known amplitude and phase until null difference is obtained.

The unknown quantities k and δ may now be written:

$$k = \frac{U_r}{U \cos \delta} + 1 - \frac{1}{\cos \delta} \qquad \simeq U_r/U$$

$$\delta = \arcsin\left(\frac{-U_i}{U\,(1-k)}\right) \qquad \simeq \arcsin(U_i/U) \simeq U_i/U$$

For use in accurate evaluation of the errors of voltage transformers and for evaluation of the measuring uncertainty of the measuring equipment (the error of the error), the above full formulas for k and δ must be used. The approximate formulas will be sufficiently accurate for determinations of the relative uncertainty of the measuring uncertainty of k and δ.

2.2 Requirements

The requirement on the accuracy of voltage transformers varies with the intended application. The maximum permissible measuring uncertainty will be different depending on which type of voltage transformer is being tested. The following table is a summarizes a set of reasonable and commonly used requirements. Note that phase displacements are given in radians.

Voltage transformer application	Transformer max permitted error ratio $\lvert k \rvert$	phase $\lvert \delta \rvert$	Measuring uncertainty ratio $\lvert k-k' \rvert$	phase $\lvert \delta-\delta' \rvert$
Protections	10^{-1}	10^{-1}	10^{-3}	10^{-3}
Control	10^{-2}	10^{-2}	10^{-4}	10^{-4}
Metering	$2*10^{-3}$	$3*10^{-3}$	10^{-4}	10^{-4}
Standard	$3*10^{-4}$	$5*10^{-4}$	10^{-5}	10^{-5}
Research	$3*10^{-4}$	$5*10^{-4}$	10^{-7}	10^{-7}

The relative uncertainty of the measuring uncertainty should be less than 10^{-1}.

2.3 Amplifier errors

Assume now that the difference amplifier has an amplitude error of ε and a phase displacement of ϕ. If the output of the difference amplifier is measured by instruments with negligible errors, then the measuring uncertainty is equal to the amplifier error.

The output voltage of the difference amplifier is now:

$$\begin{aligned} U_{fel} &= U_r\ (1-\varepsilon)\sin(\omega t+\phi) + U_i\ (1-\varepsilon)\cos(\omega t+\phi) = \\ &= (1-\varepsilon)((U_r\ \cos\phi - U_i\ \sin\phi)\ \sin\omega t + \\ &\qquad (U_r\ \sin\phi + U_i\ \cos\phi)\ \cos\omega t) = \\ &= M_r\ \sin\omega t + M_i\ \cos\omega t \end{aligned}$$

M_r and M_i are the quantities actually measured at the output of a difference amplifier with errors. Calculation of the voltage transformer errors k and δ from M_r and M_i will provide the values k' and δ' which are different from the correct values k and δ by an amount which is dependant on the errors of the difference amplifier.

$$k' = \frac{M_r}{U \cos \delta} + 1 - \frac{1}{\cos \delta}$$

$$\delta' = \arcsin(\frac{-M_i}{U (1-k)})$$

2.4 Ratio

The measuring uncertainty in the determination of the ratio with regard to the errors of the difference amplifier is:

$$k-k' = \frac{U_r}{U \cos \delta} + 1 - \frac{1}{\cos \delta} - (\frac{M_r}{U \cos \delta} + 1 - \frac{1}{\cos \delta})=$$

$$= (U_r - M_r)/ U \cos \delta =$$

$$= (U_r -(1-\varepsilon)(U_r \cos \phi - U_i \sin \phi)/U \cos \delta =$$

$$= (\frac{U_r}{U \cos \delta} - (1-\varepsilon)(\frac{U_r}{U \cos \delta} \cos \phi - \frac{U_i}{U \cos \delta} \sin \phi)$$

For the evaluation of k-k', where a relative uncertainty of 10% is acceptable, the approximations

$U_r/U \cos \delta = k$
$U_i/U \cos \delta = \delta$

are valid. Thus:

$$k-k' \approx k - (1-\varepsilon)(k \cos \phi - \delta \sin \phi) =$$

$$= k(1 - \cos \phi + \varepsilon \cos \phi) + (1-\varepsilon)\delta \sin \phi)$$

Assuming that $\varepsilon<10^{-2}$, $\phi<10^{-2}$ and letting
$(1-\varepsilon) = 1$
$\cos \phi = 1$
$\varepsilon \cos \phi = \varepsilon$
$\sin \phi = \phi$

we obtain:

$$k-k' \approx k(1 - 1 + \varepsilon) + \delta\phi = $$

$$= k\varepsilon + \delta\phi$$

2.5 Phase displacement

The measuring uncertainty in the determination of the phase displacement with regard to the errors of the difference amplifier is:

$$\delta-\delta' = \arcsin \frac{-U_i}{U(1-k)} - \arcsin \frac{-M_i}{U(1-k)} =$$

$$= \arcsin \alpha - \arcsin \beta$$

The phase displacement of actual transformers ranges from 0.1 rad to almost zero.

The first terms in the series expansion of arcsin x are

$$\arcsin x = x + x^3/6 + 3x^5/40$$

The truncation error of a series $a_n x^n$ which has a radius of convergence less than 1, is less than twice the value of the first neglected term.

Evaluating arcsin x with the two first terms of the expansion thus has an error of less than 1.5 µrad for the worst case of $\delta=0.1$.

The error term $\delta-\delta'$ may therefore be expressed by the two first terms in the series expansion of arcsin.

$$\delta-\delta' = \arcsin\alpha - \arcsin\beta \approx \alpha - \beta + (\alpha^3 - \beta^3)/6 =$$

$$= (\alpha - \beta)(1 - (\alpha^2 + \alpha\beta + \beta^2)/6)$$

Now α is on the order of δ. See **2.1 Difference signal.** Furthermore α and β are approximately equal. Since the worst case is $\delta=0.1$, the term $(\alpha^2 + \alpha\beta + \beta^2)/6$ is of the order of $\delta^2/2 \approx 0.01$. Since a relative error of 10% is permissible in the evaluation of $\delta-\delta'$, the approximation:

$$(\alpha^2 + \alpha\beta + \beta^2)/6) \approx 0$$

is valid.

Therefore:

$$\delta-\delta' \approx \alpha - \beta =$$

$$= \frac{-U_i}{U(1-k)} - \frac{-M_i}{U(1-k)} =$$

$$= (M_i - U_i)/U(1-k) =$$

$$= ((1-\varepsilon)(U_r \sin\phi + U_i \cos\phi) - U_i)/U(1-k) =$$

$$= (U_r(1-\varepsilon)\sin\phi + U_i(\cos\phi - \varepsilon\cos\phi - 1))/U(1-k)$$

For the evaluation of $\delta-\delta'$, where a relative error of 10^{-1} is acceptable, the approximations

$U_r/U \approx k$
$Ui/U \approx \delta$

are valid.

$$\delta-\delta' \approx (k(1-\varepsilon)\sin\phi + \delta(\cos\phi - \varepsilon\cos\phi - 1))/(1-k)$$

Assuming that $\varepsilon<10^{-2}$, $\phi<10^{-2}$ and letting
$(1-\varepsilon) = 1$
$\cos\phi = 1$
$\varepsilon \cos\phi = \varepsilon$
$\sin\phi = \phi$
we obtain

$$\delta-\delta' \approx (k\phi + \delta(1 - \varepsilon - 1))/(1-k) =$$

$$= (k\phi - \delta\varepsilon)/(1-k)$$

2.6 Evaluation

The measurement uncertainty due to errors in the difference amplifier has been found to be:

$$k-k' \approx k\varepsilon + \delta\phi$$

$$\delta-\delta' \approx (k\phi - \delta\varepsilon)/(1-k)$$

A sufficient condition for the fulfillment of the requirement on the measurement is found by assuming $\varepsilon=\phi$. This assumption is also reasonable in view of results that are obtainable with operational amplifier designs.
Thus:

$k-k' \approx \varepsilon(k+\delta)$
$\delta-\delta' \approx \varepsilon(k-\delta)/(1-k)$

Since both k and δ may be negative, the most stringent requirement on the difference amplifier is obtained for:

$$|k-k'| \approx |\varepsilon|(|k| + |\delta|)$$

$$|\delta-\delta'| \approx |\varepsilon|(|k| + |\delta|)/(1 - |k|)$$

For cases where $k \leq 10^{-2}$, set $1-k=1$. We obtain:

$$|k-k'| \approx |\delta-\delta'| \approx |\varepsilon|(|k| + |\delta|)$$

Using the values tabulated for k, δ, k-k' and δ-δ' in section Requirements we obtain the following maximum permissible errors for the difference amplifier.

Voltage transformer application	Transformer max permitted error ratio	phase	Amplifier error ratio=phase
Protections	10^{-1}	10^{-1}	$4.5*10^{-3}$
Control	10^{-2}	10^{-2}	$5*10^{-3}$
Metering	$2*10^{-3}$	$3*10^{-3}$	$2*10^{-2}$
Standard	$3*10^{-4}$	$5*10^{-4}$	10^{-2}
Research	$3*10^{-4}$	$5*10^{-4}$	10^{-4}

3 ISOLATION MODE REJECTION

Due to leakage across the high voltage barrier in the difference amplifier, an error voltage will appear across the output. The isolation mode rejection (IMR) is the ratio between the driving voltage V_{Iso} and the voltage E which would have to be applied across the input of the difference amplifier in order to drive the output voltage to zero. The isolation mode rejection should not be confused with the common mode rejection which applies to the voltage betweeen the input terminals and the input common.

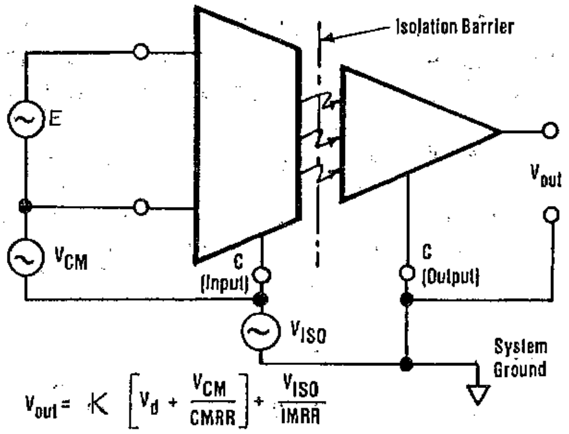

Figure 2. Definitions of isolation mode voltage

The definition of IMR is thus:

$IMR = V_{Iso}/E$

Instead of the customary IMR figure, the quantity IMR/K may be defined as a figure of merit for the amplifier.

$IMR/K = V_{Iso}/E*K$

In the below table the quantity ΔU is the difference voltage at the input of the amplifier. E is the maximum permissible error voltage due to isolation mode effects. Both quantities are defined from the requirements of section 2.2, **Requirements** by assuming that the maximum permissible error voltage is equal to the maximum permissible measuring uncertainty multiplied by the output voltage (100 V) from the voltage transformer.

The gain figure K included in the table has been selected as the lowest gain which would be used in each case. The lowest gain gives the highest requirement on the figure of merit IMR/K.

Voltage transformer application	ΔU V	E mV	K	IMR dB	IMR/K dB
Protections	10	100	1	60	60
Control	1	10	3	80	70
Metering	0.2	10	10	80	60
Standard	0.03	1	100	100	60
Research	0.03	0.01	350	140	89

Full performance at 10% of rated voltage can be reatined either by increasing the gain of the amplifier by a factor of 10 or by increasing the requirement on the IMR and IMR/K figures by 20 dB.

4 NOISE

The noise contribution is most severe when the difference voltage is lowest. An evaluation of the noise of the amplifier for its highest gain is therefore sufficient to cover all cases.

The permissible voltage noise is listed in the table below as an equivalent voltage noise referred to the input of the difference amplifier for a voltage transformer output voltage of 100V. Evaluating the noise for a frequency range of 40 Hz to 70 Hz is more than conservative for applications at power frequency. If full performance is required at 10 % of rated voltage the maximum permitted voltage noise would be 1/10 of the values listed below.

Voltage transformer application	ΔU V	Noise mV_{rms}
Protections	10	100
Control	1	10
Metering	0.2	10
Standard	0.03	1
Research	0.03	0.01

5 SUMMARY

The requirements on a difference amplifier for use as primary sensing element in a voltage transformer calibration circuit have been investigated as regards accuracy, isolation mode rejection and noise.

The results show that for the most stringent application identified, namely research work on national standard transformers, the requirements on the amplifier are:

- error $< 10^{-4}$
- IMR/K > 89 dB
- noise $< 10\ \mu V_{rms}$ (40-70 Hz)

and for full performance at 10 % of rated voltage:

- error $< 10^{-4}$
- IMR/K > 109 dB or 89 dB and higher gain
- noise $< 1\ \mu V_{rms}$ (40-70 Hz)

These requirements are high but by no means impossible to realize with commercially available operational amplifiers.

It is also to be noted that the requirements are set in such a form as to be readily measured on a finished design.

6 REFERENCES

RICHTER, Hans. Über die Fehlerbestimmung von Normalspannungswandlern mit Spannungsteilern. Braunschweig: Technischen Universität Carolo-Wilhelmina zu Braunschweig, 1983, 157 pages

Integrated Circuits Data Book. Burr-Brown Corporation, 1986, 1244 pages

VASASTADENS BOKBINDERI AB
1989