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Density of Moist Air Monitored by Laser Refractometry

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Abstract

Air-buoyancy corrections are often a dominant source of error in the transfer of the unit of mass through weighing of nominally equal weights of different densities. The possibilities of monitoring air density through measurement of the refractive index of air are considered. A critical discussion is given of the present knowledge of the refractivities of air and its constituent gases. A simple laser refractometer capable of resolving air-density variations of $0.15 \times 10^{-3} \text{ kg m}^{-3}$, corresponding to a change in air buoyancy of $9 \mu\text{g}$, is demonstrated in weighings of the Swedish prototype kilogram with stainless-steel standards.

1. Introduction

In the transfer of the unit of mass through weighing of Pt-Ir prototype kilograms with secondary standards in stainless steel, a major source of uncertainty arises in the determination of air-buoyancy effects. To compensate for an apparent mass difference, $q_w(V_1 - V_2)$, in the weighing of two kilograms (of volume V_1 and V_2) in air of density ρ_w , which is typically of the order of 100 mg, to an accuracy comparable with present-day balance resolution of several μg , requires a determination of air density to several parts in 10^4 [1]. An established procedure, recommended internationally, is to calculate the air density, ρ_w , with an equation based on the non-ideal gas equation and a model for air composition derived from empirical observations [2]. In generalized form:

$$\rho_w = \frac{p}{ZRT} \cdot \sum_i x_i M_i \quad (1)$$

an equation which requires precise, absolute and simultaneous measurements of atmospheric pressure, p ; of temperature, T ; and, where practically possible, of constituent gas concentration, x_i . The molecular

masses, M_i , and the universal gas constant, R , are well known, while the air compressibility, Z , is itself calculated from an empirical function of p and T .

In replies to a recent international questionnaire on air-density determination in precision weighing [3], a number of alternative ways of determining air-buoyancy effects were mentioned. This article considers further the proposal of monitoring changes in air density through measurements of the optical refractivity of air. The Lorenz-Lorentz equation is the basis for this, since it gives the refractivity, A , of a macroscopic gas as the product of the molecular polarizability and the number density of molecules in the gas [4]. In a form analogous to (1), one may write:

$$\frac{n^2 - 1}{n^2 + 2} \approx \frac{2}{3} (n - 1) = \frac{p}{ZRT} \cdot \sum_i x_i A_i \quad (2)$$

where $n - 1 \approx 2 \times 10^{-4}$, n being the refractive index, and $A = (4/3) \pi N_A (4 \pi \epsilon_0)^{-1} \alpha$, where α is the molecular polarizability, N_A is Avogadro's number and ϵ_0 is the permittivity of free space.

In this paper we consider the present knowledge of the refractivity of air and its constituent gases, required to interpret changes in air composition during a weighing at atmospheric pressure. Results are also presented of measurements performed with a simple laser refractometer with sufficient resolution to monitor changes in air density corresponding to $9 \mu\text{g}$ in air buoyancy in recent weighings of the Swedish prototype kilogram with secondary standards.

2. Refractivity of Air and its Constituent Gases

2.1. Refractivity of Air

The absolute refractivity of air, its dispersion with optical wavelength and its dependence on pressure and temperature are well investigated, so that it may be calculated, from empirical formulae given by Edlén [5],

to an accuracy quoted to be about 5×10^{-8} in n . Later investigations [6] of the deviations from ideal-gas behaviour concerning the thermodynamic properties of air, used for example in the air-density equation referred to above, do not significantly alter Edlén's formulation of the dependence of air refractivity on pressure and temperature [7]. Recent measurements of the refractive index of air with a variety of refractometers [8, 9] essentially confirm Edlén's values, while progress towards higher accuracy in refractivity measurements is slow.

2.2. Air Composition

The formulation of the density of air [2, 7], employs a model for standard air in which it is assumed, on the basis of observation, that the relative concentrations of N_2 , Ar and the sum of O_2 and CO_2 are constant, and that the concentration of water vapour, x_v , displaces an equal concentration of dry air. The molar concentrations, weights and refractivities for the most abundant gases in standard air are listed in Table 1. Other gases, such as He, CH_4 , etc. make up the remaining 0.003% of air composition.

2.3. Refractivity of Atmospheric Gases

Values of the refractivity and dispersion for the constituent gases N_2 , O_2 and Ar have been determined from optical refractometry with visible light by Cuthbertson and Cuthbertson [10], as tabulated in the classical reference work of Landolt and Börnstein [11]. Dispersion formulae for H_2O and CO_2 are given in reference [12]. An example of more recent optical measurements of the refractivity of some of the atmospheric gases can be found in the work of Kerl [13]. The values of A_i given in Table 1 are those estimated from an extrapolation of the optical-dispersion formulae to the limit of infinitely long wavelength (i.e. $\sigma \rightarrow 0$), as well as those evaluated at the laser wavelength of interest.

The consistency of the absolute values of the refractivity and dispersion of air, given by Edlén [5], with those of the constituent gases of air, can be tested by simply adding the refractivities of each gas as determined separately, assuming that each contribution to the refractivity is proportional to the molar fraction of each constituent gas. In Fig. 1 is shown the dispersion of standard, dry air over a wavenumber interval from $\sigma = 0$ to $\sigma = 20\,000\text{ cm}^{-1}$. Curves I and II show estimates of the dispersion obtained by summing values from references [13] and [11], respectively. Curve IV is that of Edlén [5]. The values are calculated for STP, i.e., $p = 101\,325\text{ Pa}$ and $T = 288.15\text{ K}$. It is apparent that these estimates differ from each other by more than the quoted uncertainties.

One may also compare the various estimates of the refractivity of air at $\sigma = 0$. Table 1 gives the value extrapolated from Edlén's formula: a sum of values from Ref. 13 gives $4.312\text{ cm}^3\text{ mol}^{-1}$.

Fig. 1. The dispersion of dry, standard air from published optical measurements, and its extrapolation (broken lines) to long wavelengths. Explanation of the different curves is given in the text (Sect. 2.3). The error bars shown are derived from the uncertainties quoted in the various optical measurements referred to in the text

Table 1. Air and its principal constituent gases

	Dry Air	Principal constituent gases of dry air ($x_v = 0$)				Water vapour
	($x_v = 0$)	N_2	O_2	Ar	CO_2	H_2O
x_i	1	0.78101	0.20939	0.00917	0.00040	x_v
$M_i (10^{-3}\text{ kg mol}^{-1})$	28.9635	28.0134	31.9988	39.948	44.010	18.015
$A_i @ \sigma = 0 (10^{-6}\text{ m}^3\text{ mol}^{-1})$	4.295	4.400 ^a	3.964	3.971	6.580	3.599
$A_i @ \sigma = (1/0.633)\mu\text{m}^{-1}$ ($10^{-6}\text{ m}^3\text{ mol}^{-1}$)	4.351	4.448 ^b	4.035	4.024	6.688	3.678

^a $A_{N_2} = 4.389(2) \times 10^{-6}\text{ m}^3\text{ mol}^{-1}$ from long-wavelength measurement (see text, Sect. 2.4)

^b Corrected for long-wavelength discrepancy

2.4. Long-wavelength Normalization

The refractive index of a medium approaches the square root of the dielectric constant in the limit of infinitely long wavelengths ($\sigma \rightarrow 0$). There exist measurements of accuracy comparable with the above optical refractometry, both on the refractivity of N_2 and Ar at microwave frequencies [14], and on the dielectric constants of these gases as determined by the cyclic-expansion method [15]. These long-wavelength measurements are in good agreement, but differ particularly for nitrogen from the extrapolated values of the optical data: Refs. 14 and 15 give $A_{N_2}(0) = 4.389(1)$ and $4.390(2) \text{ cm}^3 \text{ mol}^{-1}$, respectively. The magnitude of this discrepancy is of the same order as the differences between the extrapolated-optical-data estimates for air. Curve III shows the refractivity of air as estimated from a sum of data in Ref. 11, except that the value of the dominating contribution from nitrogen is corrected so as to coincide with the dielectric and microwave measurements at long wavelengths. This curve lends support to Edlén's values as well as to the model for standard air.

2.5. Deviations from Ideal-gas Behaviour

Interactions between the molecules in a gas lead to deviations from the Lorenz-Lorentz equation for the refractivity of an ideal gas. By analogy with the compressibility, one may express the refractivity as a virial expansion [16]:

$$\frac{n^2 - 1}{n^2 + 2} = \rho(A + B\rho + \dots)$$

Direct measurements for the second virial coefficients, B , of the refractivity on N_2 , Ar and CO_2 have been made recently [17], while no such measurements appear to have been performed for air. An order-of-magnitude calculation, in which the non-ideal-gas contributions from these constituent gases are added, shows that deviations from ideal-gas behaviour for the refractive index of air at atmospheric pressure should be negligibly small, compared with the uncertainties in the optical measurements discussed above.

3. Comparing Air Refractivity with Air Density

The basis for a comparison of the refractivity and weight density of air is furnished by the analogous forms of (1) and (2). Of course, the molecular weights, M_i , and the refractivities, A_i , of the constituent gases do not necessarily scale together. So, while $p/(ZRT)$ will give equivalent values for the number density in

both expressions (1) and (2) under all conditions, variations in the relative concentrations of the constituent gases in air may lead to changes in the proportionality between ρ_w and $n - 1$.

The absolute air density may in principle be derived from a measurement of the absolute air refractivity, together with some estimates of the constituent gas concentrations (see Sect. 2.2), from the equation

$$\rho_w = \frac{2}{3} \frac{M}{A} (n - 1) \quad (3)$$

where $M = \sum_i x_i M_i$, and $A = \sum_i x_i A_i$.

A simple differentiation of (1) and (2) gives a relation between a change in weight density, $\Delta\rho_w$, and a change in refractive index, Δn :

$$\Delta\rho_w = \frac{2}{3} \frac{M}{A} \Delta(n - 1) + \frac{p}{ZRT} \left[\sum_i M_i \Delta x_i - \frac{M}{A} \sum_i A_i \Delta x_i \right] \quad (4)$$

In the present work, a laser Michelson interferometer of fixed mechanical length l is employed to monitor changes in optical path length in air, given by the product nl .

4. A Simple Refractometer for Monitoring Changes in Air Density

4.1. Experimental Set-up

This work forms part of the tenth comparative weighing of the Swedish prototype kilogram with principal kilogram standards in stainless steel and in gilded brass; the first comparison was performed by Ekstrand and Ångström in 1894 [18]. The Pt-Ir kilogram K 40 has been weighed against the international kilogram prototype three times, most recently in 1984 [19].

A Michelson laser interferometer (model HP 5526 A), usually used in our laboratory for length metrology, was slightly modified so that the mechanical length, l , of the arm of the interferometer open to air was fixed (at about 54 cm) by mounting the optics on two plates separated by three fused-silica rods, of low thermal expansion. Changes in the optical path length of the open arm were measured to a resolution of $1/40$ th of the He-Ne laser wavelength ($0.633 \mu\text{m}$).

The refractometer was placed close to the kilogram prototype balance (Mass-comp 2, Volland Co), together with sensors for measurement of air pressure (Texas Inst. 2908), air temperature (Pt 100 Systemtechnik AB), air humidity (EG & G mirror hygrometer 660) and CO_2 concentration (infrared absorption spectrometer MIRAN 1 A). The complete measurement set-up was installed in the fully air-conditioned and vibration-

isolated environment of the National Measurement Laboratory's comparison room. The room temperature was held at $(20.00 \pm 0.02)^\circ\text{C}$. A HP-85 microcomputer collected data from each of these instruments, except the CO_2 determination.

4.2. Weighing Procedure

The complete set-up was left undisturbed for five days prior to measurement. Weighings of the Swedish prototype kilogram, K 40, and secondary kilogram standards of stainless steel, G1 and MJV2, were then carried out in the prototype balance. Details of the weighing procedure will be published elsewhere. It is sufficient here to note that exchange weighing was performed, where each weight was weighed in turn on a single pan against a fixed counterweight of density close to that of stainless steel. Each weighing took approximately 11 minutes, including time for the placing of the weight on the scale pan in the arrested position; a period of free oscillation; measurements of the scale reading for six turning-points in the oscillation; arrest of the scale pan; removal of the weight from the scale pan; and exchange of the weight. Eight such weighings were performed in each protocol of the comparative weighing of two kilograms, and 30 such protocols made up the complete weight comparison.

4.3. Aerostatic Weighing

The balance was largely open to the laboratory air: it was thus necessary to follow changes in air density in the vicinity of the balance at every measurement. The various air parameters – pressure, temperature, humidity and optical path length – were thus registered within several seconds of each reading of a turning-point in the oscillation of the balance. The compensation of the balance reading for air buoyancy required knowledge of the density of the counterweight and the sensitivity of the balance at each measurement. (The densities of the kilograms had been previously determined by hydrostatic weighing.)

The density of the counterweight was determined by aerostatic weighing against the Pt-Ir kilogram: Fig. 2 is a plot of the equilibrium scale reading (the average of 5 turning points) for the balance as a function of the air density, as calculated using (1). The slope of the straight line fitted, by regression, to the data gave the difference in effective volume of $78 (2) \text{ cm}^3$. 'Effective volume' means the volume that the counterweight would have in an equal-armed balance: the counterweight arm is much shorter than the pan arm. The balance is based on a design described in Ref. 20.

The sensitivity of the balance was determined by recording the difference in scale reading at each kilo-

Fig. 2. Plot of the equilibrium scale reading for the balance as a function of air density with the prototype kilogram K40 on the weighing pan, and a stainless-steel counterweight

gram weighing where a 1 mg weight was placed on the balance beam from the scale reading with only the kilogram on the scale pan. It was necessary, however, to correct the scale reading for the effects of air buoyancy, which itself required an estimate of the balance sensitivity. An iterative calculation was thus performed and convergence to a self-consistent solution was obtained after three iterations. Convergence was deemed to be satisfactory when the calculated differences in weight for a weighing of a pair of kilograms for two consecutive iterations agreed within $4 \mu\text{g}$.

4.4. Calculation of Air Density

The scale reading of the balance at each turning-point of free oscillation was corrected for the momentary air buoyancy. The air density, required to estimate the latter, was calculated by two different methods: firstly, using (1) with the measured values of pressure, temperature, humidity and CO_2 concentration; and secondly, from the refractive index measurements. The latter method is now discussed in detail:

In principle, the present apparatus could be used to measure the absolute air refractivity by measuring the change in optical path length during evacuation of the interferometer. As discussed in Sect. 3, this measurement would allow an estimate of the absolute air density. However, the only values for A_i available at present are those derived from refractivity measurements with a gas density for each constituent calculated from the non-ideal gas equation (Sect. 2.3), and it is arguable whether these are better than the air-density calculations using (1) [2]. True *ab initio* values of A_i are available only in rare cases such as from configuration interaction (CI) calculations on He [21].

So, in the present work, it was decided to determine the absolute air density from (1) at one arbitrary mo-

ment. Alternative methods of determining the absolute air density for example, through the weighing of bodies of equal weight and surface area but different volumes have been reported [22, 23]. The refractive-index measurements do, however, enable a reasonably accurate monitoring of *changes* in the air density with only one parameter [compared with 4 parameters in (1)], provided that the air composition is relatively stable.

Figure 3 shows the air density calculated (a) by use of (1) and (b) from the refractivity measurements, as a function of the approximately 1400 observations over a kilogram comparison of 30 protocols, and about 3 days. The air density, ρ_w , estimated from the refractivity measurements was calculated from the expression:

$$\rho_w = (\rho_w)_0 - \frac{2}{3} \frac{M}{A} \frac{\lambda}{40} \frac{1}{2l} [\Delta(nl) - \Delta(nl)_0] \quad (5)$$

where $(\rho_w)_0$ is the average air density during the very first weighing of the comparison [calculated from (1)];

$M/A = 6.6315 \times 10^{-3} \text{ kg m}^{-3}$, is the ratio of the mass and refractivity of air calculated from the values given in Table 1 for the measured composition of air at the very first weighing, averaged over six turning-points;

$\lambda = 0.6329914(3) \text{ } \mu\text{m}$, is the (*in vacuo*) wavelength of the He-Ne laser, as specified by the manufacturers [24];

$\Delta(nl)$ is the change in optical path length in the Michelson interferometer, measured in units of $\lambda/40$;

$\Delta(nl)_0$ is the value of $\Delta(nl)$ at the very first weighing; and

$l = 0.53919(2) \text{ m}$, is the mechanical length of the Michelson interferometer.

The conditions under which the very first weighing was performed were as follows:

$p = 98\,802 \text{ Pa}$, $t = 20.011^\circ\text{C}$, water-vapour concentration = 1.143%, CO_2 concentration = 0.046%, with an air density of $\rho_w = 1.16944 \text{ kg m}^{-3}$, calculated from (1) [2], and an air refractive index of $n = 1 + 2.6453 \times 10^{-4}$, calculated from (2) [5].

Equation (5) may be easily derived from (4) on the assumption that no appreciable change in air composition occurs (i.e. $\Delta x_i = 0$). As can be seen from Fig. 3, the laser refractometer produces a signal which follows changes in air density reasonably well. In the next section, a discussion is given of possible sources of discrepancy in the calculations of air density with the two methods.

The finite resolution of the interferometer ($\lambda/40$) can be seen in the step-like changes in the air density calculated from the refractive-index measurements. The corresponding change in air buoyancy in the comparative weighing of a Pt-Ir kilogram against a stainless-steel kilogram for the air-density change is approximately 9 μg .

Fig. 3. Plots of air density calculated (a) by the BIPM-recommended formulae [2], and (b) from refractivity measurements, versus observation number over a complete (3 day) kilogram comparison

5. Discussion of Sources of Discrepancy in Air Density Calculations

5.1. Measurement of Pressure and Temperature. Calculation of Gas Number Density

The instruments (specified in Sect. 4.1) employed to measure the absolute air pressure and temperature had the following associated uncertainties: $\pm 6 \text{ Pa}$ and $\pm 0.005^\circ\text{C}$, respectively. Both of these instruments were calibrated in association with the present kilogram comparison. Calibration certificates are referred to in the reference list [25, 26]. The uncertainties in air number density, calculated from p/ZRT , due to uncertainties in these air parameters are: $\pm 0.07 \times 10^{-3} \text{ kg m}^{-3}$ and $\pm 0.02 \times 10^{-3} \text{ kg m}^{-3}$, respectively. The number density enters in the same form in both ways [equations (1) and (5)] of calculating the air weight density.

5.2. Air Composition

The instruments (specified in Sect. 4.1) employed to measure the absolute air humidity and the CO_2 concentration had the following associated uncertainties: $\pm 0.010\%$ relative concentration H_2O ($\pm 0.5\% \text{ RH}$) and $\pm 0.005\%$ relative concentration CO_2 , respectively. Both of these instruments were calibrated in association with the kilogram comparison. Calibration certificates are referred to in the reference list [27, 28]. The uncertainties in air density, calculated with either

Table 2. Change in constituent gas concentration to cause a significant difference ($+0.15 \times 10^{-3} \text{ kg m}^{-3}$) in air density derived from refractivity measurements

Gas	Δx_i	x_i
N ₂ :	-0.246%	77.219%
O ₂ :	+0.069%	20.698%
Ar:	+0.027%	0.907%
CO ₂ :	-1.060%	0.045%
H ₂ O:	-0.057%	1.130%

method, due to uncertainties in these air parameters are: $\pm 0.05 \times 10^{-3} \text{ kg m}^{-3}$ and $\pm 0.03 \times 10^{-3} \text{ kg m}^{-3}$, respectively.

The air-density values in Fig. 3, derived from (5) using the refractivity measurements, are obtained under the assumption that the composition of the air did not change appreciably during the three days of weighing. In fact, the following variations in water vapour content, x_v , and CO₂ concentration were recorded:

water vapour: min. $x_v = 1.103\%$ (ca. 46% RH), max. $x_v = 1.190\%$ (ca. 50% RH),

CO₂: min. 0.045%, max. 0.048%.

The importance of these variations in the constituent-gas concentration in the estimation of air-density changes from the optical measurements may be judged from an evaluation of the sum of terms on the right hand side of (4). Substituting the values for the molecular masses, M_i , and refractivities, A_i , from Table 1 in (4), we list in Table 2 the concentration changes that would correspond to the resolution of the laser interferometer, i.e., $+\lambda/40$ in optical path, or ca. $0.15 \times 10^{-3} \text{ kg m}^{-3}$ in air density.

Thus, because the molecular masses and refractivities scale in different ways for the different constituent gases, the sensitivity of the derived air density to concentration changes varies widely. The gases which could contribute most to discrepancies in the air density derived from the refractivity are H₂O, O₂ and Ar. The dominant, observed variation in air composition is of the water-vapour content, by about $+0.087\%$ during the weighings. According to the model of standard air employed in current formulations of the air density (see Sect. 2.2), an equal concentration of dry air displaced by the water vapour would result in relative changes in the gases O₂ and Ar of -0.018% and -0.0008% , respectively. The variation in water-vapour content by $+0.087\%$ would thus lead to changes in the contributions of these gases to the derived air density plotted in Fig. 3 of $-0.23 \times 10^{-3} \text{ kg m}^{-3}$ for H₂O; $-0.04 \times 10^{-3} \text{ kg m}^{-3}$ for O₂, and $-0.004 \times 10^{-3} \text{ kg m}^{-3}$ for Ar.

Possible correlations in differences in air density estimated from (1) and (5) with changes in air humidity are examined in Fig. 4. The situation is unfortunately not so clear, either because of other sources of 'noise'

Fig. 4. Plots of (a) air humidity and (b) difference in air-density estimates, versus observation number

(as considered in Sects. 5.1 and 5.3) or due to atmospheric changes more complex than those of standard air. One may speculate that the observed correlation of discrepancies in the derived air density with periods of time where the air density is nearly stationary for a longer time (typically 4 or 5 hours), apparent from Fig. 3, is an indication of the latter.

The recorded change in CO₂ concentration and the expected reciprocal change in O₂ concentration for standard air would have negligible influence on the air-density values derived from the refractivity measurements, within the current resolution of the refractometer. Normal variations in the concentrations of the other constituent gases are unlikely to be resolvable.

The discussion in this section may be summarized by noting that changes in air density may be monitored with single-parameter refractive-index measurements with the present laser interferometer, at a resolution of $0.15 \times 10^{-3} \text{ kg m}^{-3}$ (ca. $9 \mu\text{g}$ in air buoyancy for Pt-Ir/steel kilogram comparisons). However, supplementary measurements of the air humidity will be necessary if the water-vapour content is expected to vary by more than 0.06% (humidity, by more than 2%).

5.3. Laser Interferometric Measurements

Factors which may influence the observed optical path length of the interferometer described in Sect. 4.1 were:

(a) *Changes in the Mechanical Length of the Interferometer.* During the three days of weighing, the temperature in the laboratory varied between 20.010°C and 20.020°C. Taking into account the thermal ex-

pansion coefficient (1×10^{-6}) of the 54 cm long fused-silica rods of the interferometer, the associated change in length would be about 6 nm. This change could be misinterpreted as a change in air density of $0.08 \times 10^{-3} \text{ kg m}^{-3}$.

(b) *Changes in the Frequency of the Laser.* The frequency-stability of the He-Ne laser is estimated by the manufacturer [24] to be $\pm 5 \times 10^{-7}$, i.e., ca. $\pm 240 \text{ MHz}$. The corresponding error in fringe-counting over 54 cm would be ± 0.5 fringe, which could be misinterpreted as a change in air density of $\pm 2 \times 10^{-3} \text{ kg m}^{-3}$. In the middle of the 18th weighing protocol, the observed optical path length changed in a discrete jump by $+44 \lambda/40$. Subsequent values remained at the higher level. It was felt justified to correct the post-jump optical-path values used in the plot of Fig. 3 so as to provide continuity with the values prior to the 18th protocol. The jump seems most likely to be due to a change in longitudinal mode of the laser: the free spectral range of the laser was estimated to be about 500 MHz (the laser is about 0.3 m long). After this correction, the observed agreement between estimates of the air density from laser and pressure measurements (see Fig. 3) implies that the frequency stability, apart from a hop in mode, was better than the manufacturer's specifications.

6. Summary

A simple laser refractometer of fixed length, and constructed from "off-the-shelf" components, has been used to monitor changes in air density, making possible the compensation for variation in air-buoyancy effects with a single parameter in precision weighing of equal weights of different densities. The resolution of the device is about $0.15 \times 10^{-3} \text{ kg m}^{-3}$ in air density, or ca. $9 \mu\text{g/kg}$ when comparing Pt-Ir and stainless-steel kilograms. The absolute air density needs to be determined at one point. Supplementary measurements of air humidity are necessary in cases where the humidity is expected to vary by more than 2% during a comparison. The present technique compares favourably with the established method of calculating air density from various atmospheric parameters [2], or by other methods of determining air density [22, 23].

In weighings comparing the Swedish prototype Pt-Ir kilogram, K40, with a stainless-steel kilogram MJV2, in May 1987, the observed weight difference was $-0.526(6) \text{ mg}$ after compensation for air buoyancy using the BIPM-recommended formulae for air density [2], and $-0.525(6) \text{ mg}$ after compensation for air buoyancy with air-density values derived from la-

ser refractometry (the given uncertainties are the statistical standard deviations in the mean of the weight differences over 9 weighing protocols). These weight differences are preliminary results.

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