# CONSIDERING TEMPERATURES IN OPERATIONAL PLANNING OF DISTRICT HEATING SYSTEMS

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# Abstract


**Considering temperatures in operational planning of district heating systems**

The project SeCoHeat aims at assessing the additional profits that district heating systems can make by participating in new electricity markets, such as ancillary service markets. A model has been developed to schedule district heating units on an hourly basis to minimize heat and electricity production costs while maximizing revenues from electricity markets. This model works with hourly energy flows. In this report, the importance of considering temperature quality in district heating networks when scheduling district heating units is investigated.

Temperature quality refers to controlling mass flows and supply temperatures to ensure acceptable confort at the end-consumers. Traditional scheduling models use an energy-only formulation where energy is related to the product of temperaturate and mass flows. They do not consider these two quantities separately and, therefore, are unable to capture temperature quality aspects. On the other hand, these traditional scheduling models are less complex than a full representation of both temperatures and mass flows. Traditional scheduling models based on an energy-only formulation can be expressed as MILP optimization problems, whereas considering both temperatures and mass flows lead to MINLP optimization problems which are must harder to solve.

Several simplications and reformulations have been proposed in this report to make the full MINLP problem less complex. These simplifications reduce the number of non-linear equations and, for one of them, even leads to a MILP formulation. In addition, a literature review about exisiting linear reformulations of the full MINLP problem and the importance of considering temperature quality is performed.

This report gives a ground to further develop scheduling models that make tradeoffs between model complexity and accurate representation of temperature quality.



Keywords: disctrict heating systems, temperature quality, MILP, MINLP, scheduling model.

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# 1 Introduction

The project SeCoHeat aims at assessing the additional profits that district heating systems can make by participating in new electricity markets, such as ancillary service markets. A model has been developed to schedule district heating units on an hourly basis to minimize heat and electricity production costs while maximizing revenues from electricity markets, see [6]. This model works with hourly energy flows. Previous works, however, have shown that disregarding district heating temperatures may lead to inaccurate scheduling [15]. In the present report, the importance of considering district heating temperatures in scheduling models is investigated.

Section 2 introduces quantities related to district heating temperatures. Section 3 proposes mathematical formulations to consider district heating temperatures in scheduling models. In Section 4, a literature review of previous works related to the importance and modelling of district heating temperatures is presented.

# 2 Temperatures and mass flows

Temperatures in a district heating system are illustrated in Figure 1, which depicts a district heating system consisting of generating units and storage (left-hand side of the figure) connected to a district heating network (right-hand side of the figure).

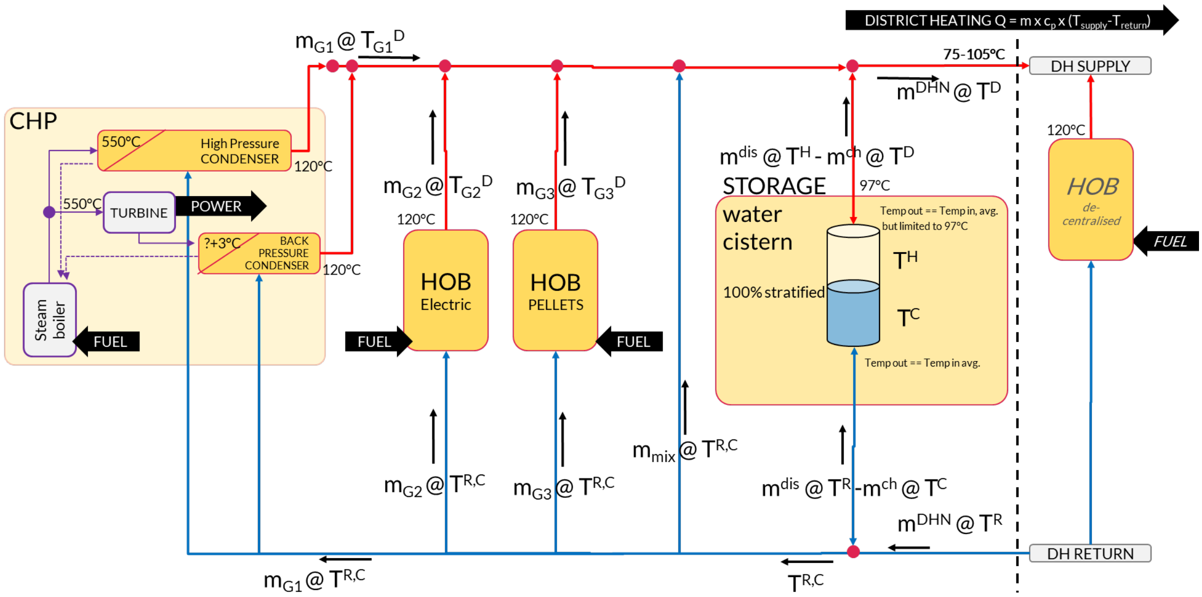


**Figure 1:** Illustration of temperatures in a district heating system

The generating units $G$ generate hot water at a certain temperature $T_G^D$ (all 120°C in the figure but they could be different) which can be either fed to the district heating network or stored in a thermal storage (water cistern in the figure). The hot water is circulated out of the generation units at a mass flow of $m_G$.

The water heated by the generating units can be mixed with cold return water (simplification) at the adjusted return temperature $T^{R,C}$ (see below for what "adjusted" means) with mass flow rate $m^{\text{mix}}$ and water discharing from the storage from the high temperature part of the storage (temperature $T^H$). The temperature out of the water cistern is however limited to a maximum temperature $T^{\max}$ (97°C in the figure). The water cistern is either charged with a flow rate of $m^{\text{ch}}$ or discharged with a flow rate $m^{\text{dis}}$. The temperature resulting from this mixing is the delivery temperature $T^D$ that is fed into the district heating network. The district heating system owner controls the delivery temperature to ensure proper temperature quality in the district heating network. In the simplified picture, the delivery temperature can be controlled by

1. Starting up new generating units.
2. Changing the mass flow rates of the heated water from the generation units (adjust $m_G$).
3. Changing how much return water is mixed in with the hot water (adjust the flow rate $m^{\text{mix}}$).

The water is delivered to the district heating network with a certain mass flow $m^{\text{DHN}}$ to supply the appropriate amount of heat. The water returns from the district heating network at the same mass flow $m^{\text{DHN}}$ but with a return temperature $T^R$ lower than $T^D$. The return water is fed into the storage at the return temperature $T^R$ when the storage discharges hot water into the district heating network. When hot water is charged in the storage, water from the cold part of the storage (temperature $T^C$) is mixed with the return water. The resulting temperature of the return water ("adjusted" return temperature) flowing to the generating units and being mixed with the hot water is $T^{R,C}$ .

# 3 Mathematical formulation

In the formulation below, decision variables are written out in bold. Parameters are written out in normal font. The mathematical formulation is non linear in the decision variables because of the product of mass (or mass flows) and temperatures.

In addition to the full formulation, we will consider several assumptions below. In Tables 10 and 11, it can be seen how certain equations become linear and some decision variables become parameters.

**Assumption 1 (A1): Fixed mass flow formulation**

With this assumption, we assumed that the mass flows are set to their nominal value. The following decision variables become parameters:

- $\boldsymbol{m_t}^{\text{DHN}} \Rightarrow m_t^{\text{DHN}}$
- $\boldsymbol{m_{g,t}} \Rightarrow m_{g,t}$
- $\boldsymbol{T_t^D} \Rightarrow T_t^D$ (through equation (1))

The charging and discharging mass flows are still decision variables, as is the mixing mass flow.

**Assumption 2 (A2): Storage's cold water at return temperature**

With this assumption, $\boldsymbol{T_{s,t}^C} = T_t^R$, which also entails that $\boldsymbol{T_t^{R,C}} = T_t^R$ and that there is no need to keep track of the cold mass of the storage.

**Assumption 3 (A3): Storage's hot water temperature fixed**

This assumption assumes that the storage has a maximum temperature (for example less than 100°C due to being open storage) and that it is always loaded with water at temperature higher than this maximum temperature. Then, it can be assumed that the temperature of the water in the storage is always at that maximum temperature. With this assumption, $\boldsymbol{T_{s,t}^H} = T_s^{\max}$

**Assumption 4 (A4): All hot water temperatures known**

With this assumption, in addition to the hot water temperature in the storage, the temperatures of all production units and the supply temperature are also assumed to be set to pre-determined values (not necessarily time-independent). Therefore, they become parameters: $\boldsymbol{T_{g,t}^D} \Rightarrow T_{g,t}^D$ and $\boldsymbol{T_t^D} \Rightarrow T_t^D$ . Through Equation (1), the supply mass flow also becomes a parameter: $\boldsymbol{m_t}^{\text{DHN}} \Rightarrow m_t^{\text{DHN}}$

## 3.1 Energy balance

The energy consumption of the district heating network and customers is

$$Q_t^D = \boldsymbol{m}_{\boldsymbol{t}}^{\text{DHN}} C_p \left( \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{D}} - T_t^R \right) \tag{1}$$

Comments: The return temperature is not controllable but depends on what happens on the customer side and is considerded to be an exogenous parameter. The heat demand $Q_t^D$ is an exogenous parameter.

Table 1 shows the impact of some of the assumptions on the linearity of Equation (1). A1 and A4 make the equation linear. Assumptions not shown in the table do not impact the equation.

**Table 1:** Impact of assumptions on Equation (1)

| |
|---|
| A1: Fixed mass flow<br>$Q_t^D = m_t^{\text{DHN}} C_p \left( T_t^D - T_t^R \right)$ |
| A4: All hot water temperatures known:<br>$Q_t^D = m_t^{\text{DHN}} C_p \left( T_t^D - T_t^R \right)$ |

The energy supplied by generating units is

$$\boldsymbol{Q}_{\boldsymbol{g,t}} = \boldsymbol{m}_{\boldsymbol{g,t}} C_p \left( \boldsymbol{T}_{\boldsymbol{g,t}}^{\boldsymbol{D}} - \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{R,C}} \right) \tag{2}$$

Comment: $\boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{R,C}}$ depends on discharging activities from the storage and is therefore a decision variable (although $T_t^R$ is not).

Table 2 shows the impact of the assumptions on Equation (2). A1 makes the equation linear. A4 is not enough by iteself to make the equation linear but in combination with A2, the equation becomes linear. A3 does not have any impact on the equation.

**Table 2:** Impact of assumptions on Equation (2)

| |
|---|
| A1: Fixed mass flow<br>$\boldsymbol{Q}_{\boldsymbol{g,t}} = m_{g,t} C_p \left( \boldsymbol{T}_{\boldsymbol{g,t}}^{\boldsymbol{D}} - \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{R,C}} \right)$ |
| A2: Storage's cold water at return temperature<br>$\boldsymbol{Q}_{\boldsymbol{g,t}} = \boldsymbol{m}_{\boldsymbol{g,t}} C_p \left( \boldsymbol{T}_{\boldsymbol{g,t}}^{\boldsymbol{D}} - T_t^R \right)$ |
| A1 & A2:<br>$\boldsymbol{Q}_{\boldsymbol{g,t}} = m_{g,t} C_p \left( \boldsymbol{T}_{\boldsymbol{g,t}}^{\boldsymbol{D}} - T_t^R \right)$ |
| A4: All hot water temperatures known<br>$\boldsymbol{Q}_{\boldsymbol{g,t}} = \boldsymbol{m}_{\boldsymbol{g,t}} C_p \left( T_{g,t}^D - \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{R,C}} \right)$ |
| A2 & A4<br>$\boldsymbol{Q}_{\boldsymbol{g,t}} = \boldsymbol{m}_{\boldsymbol{g,t}} C_p \left( T_{g,t}^D - T_t^R \right)$ |

The energy discharged from and charged in storage units is

$$\boldsymbol{Q}_{\boldsymbol{s,t}}^{\text{dis}} = \boldsymbol{m}_{\boldsymbol{s,t}}^{\text{dis}} C_p \left( \min \left( \boldsymbol{T}_{\boldsymbol{s,t}}^{\boldsymbol{H}}, T_s^{\max} \right) - T_t^R \right) \tag{3}$$

$$\boldsymbol{Q}_{\boldsymbol{s,t}}^{\text{ch}} = \boldsymbol{m}_{\boldsymbol{s,t}}^{\text{ch}} C_p \left( \boldsymbol{T}_{\boldsymbol{t}}^{\boldsymbol{D}} - \boldsymbol{T}_{\boldsymbol{s,t}}^{\boldsymbol{C}} \right) \tag{4}$$

**Table 3:** Impact of assumptions on Equation (3)

| |
|---|
| A3: Storage's hot water temperature fixed<br>$\boldsymbol{Q}^{\text{dis}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}} C_p \left(T^{\max}_s - T^R_t\right)$ |
| A4: All hot water temperatures known<br>$\boldsymbol{Q}^{\text{dis}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}} C_p \left(T^{\max}_s - T^R_t\right)$ |

Table 3 shows the impact of the assumptions on Equation (3). A3 (and therefore A4) has an impact on this equation and makes it linear.

Table 4 shows the impact of the assumptions on Equation (4). A3 does not have any impact on the charging equation. A1 and A4 are not enough by themselves to make the equation linear but in combination with A2, the equation becomes linear.

**Table 4:** Impact of assumptions on Equation (4)

| |
|---|
| A1: Fixed mass flows<br>$\boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}} C_p \left(T^D_t - \boldsymbol{T}^{\boldsymbol{C}}_{\boldsymbol{s,t}}\right)$ |
| A2: Storage's cold water at return temperature<br>$\boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}} C_p \left(\boldsymbol{T}^{\boldsymbol{D}}_{\boldsymbol{t}} - T^R_t\right)$ |
| A1 & A2<br>$\boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}} C_p \left(T^D_t - T^R_t\right)$ |
| A4: All hot water temperatures known<br>$\boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}} C_p \left(T^D_t - \boldsymbol{T}^{\boldsymbol{C}}_{\boldsymbol{s,t}}\right)$ |
| A2 & A4<br>$\boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}} = \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}} C_p \left(T^D_t - T^R_t\right)$ |

The total energy balance is ensured by

$$\sum_{g=1}^{N_G} \boldsymbol{Q}_{\boldsymbol{g,t}} + \sum_{s=1}^{N_S} \left(\boldsymbol{Q}^{\text{dis}}_{\boldsymbol{s,t}} - \boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}}\right) = Q^D_t \tag{5}$$

## 3.2 Mass flow constraints

At the supply node:

$$\sum_{g=1}^{N_G} \boldsymbol{m}_{\boldsymbol{g,t}} + \boldsymbol{m}_{\text{mix},\boldsymbol{t}} + \sum_{s=1}^{N_s} \left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}} \tag{6}$$

At the return node (same as in the supply node):

$$\sum_{g=1}^{N_G} \boldsymbol{m}_{\boldsymbol{g,t}} + \boldsymbol{m}_{\text{mix},\boldsymbol{t}} + \sum_{s=1}^{N_s} \left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}} \tag{7}$$

## 3.3 Water mixing constraints

See for example [15]. Water mixing at the supply node implies that

$$\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}T^{D}_{g,t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T^{R,C}_{t}} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}\min\left(\boldsymbol{T^{H}_{s,t}}, T^{\max}_{s}\right) - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\boldsymbol{T^{D}_{t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}}\boldsymbol{T^{D}_{t}} \tag{8}$$

Note that the return water that is mixed has a temperature $T^{R,C}_t$ that can be different from the water returning from the DHN in case the storage is charged (in which case water from the cold part of the storage is mixed with the return water from DHN).

Table 5 shows the impact of the assumptions on Equation (8). Only combinations A1, A2 and A3 or A4, A2 and A3 make the equation linear.

**Table 5:** Impact of assumptions on Equation (8)

| | |
|---|---|
| A1: Fixed mass flows: | $\sum_{g=1}^{N_G}\left(m_{g,t}\boldsymbol{T^{D}_{g,t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T^{R,C}_{t}} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}\min\left(\boldsymbol{T^{H}_{s,t}}, T^{\max}_{s}\right) - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}T^{D}_{t}\right) = m^{\text{DHN}}_{t}T^{D}_{t}$ |
| A2: Storage's cold water at return temperature: | $\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}T^{D}_{g,t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}T^{R}_{t} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}\min\left(\boldsymbol{T^{H}_{s,t}}, T^{\max}_{s}\right) - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\boldsymbol{T^{D}_{t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}}\boldsymbol{T^{D}_{t}}$ |
| A3: Storage's hot water temperature fixed | $\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}T^{D}_{g,t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T^{R,C}_{t}} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}T^{\max}_{s} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\boldsymbol{T^{D}_{t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}}\boldsymbol{T^{D}_{t}}$ |
| A1 & A2 & A3: | $\sum_{g=1}^{N_G}\left(m_{g,t}\boldsymbol{T^{D}_{g,t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}T^{R}_{t} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}T^{\max}_{s} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}T^{D}_{t}\right) = m^{\text{DHN}}_{t}T^{D}_{t}$ |
| A4: : All hot water temperatures known | $\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}}T^{D}_{g,t}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T^{R,C}_{t}} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}T^{\max}_{s} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}T^{D}_{t}\right) = m^{\text{DHN}}_{t}T^{D}_{t}$ |
| A2 & A4 | $\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}}T^{D}_{g,t}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}T^{R}_{t} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}T^{\max}_{s} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}T^{D}_{t}\right) = m^{\text{DHN}}_{t}T^{D}_{t}$ |

Similarly, at the return node:

$$\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}T^{R,C}_{t}}\right) + \boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T^{R,C}_{t}} + \sum_{s=1}^{N_s}\left(\boldsymbol{m}^{\text{dis}}_{\boldsymbol{s,t}}T^{R}_{t} - \boldsymbol{m}^{\text{ch}}_{\boldsymbol{s,t}}\boldsymbol{T^{C}_{s,t}}\right) = \boldsymbol{m}^{\text{DHN}}_{\boldsymbol{t}}T^{R}_{t} \tag{9}$$

Note that the return temperatures to all generating units are equal to $\boldsymbol{T^{R,C}_{t}}$ (that is also the temperature of the water mixed with the hot water) and depend on the return temperature from the DHN and whether the storage is being charged (in which case water from the cold part of the storage is mixed with the return water from DHN).

Table 6 shows the impact of the assumptions on Equation (9). A2 is enough to make the equation linear since it then reduces to a mass conservation equation. Combining A2 with A1, a few terms become parameters instead of decision variables.

## 3.4 Stratified thermal storage constraints

### 3.4.1 Energy-oriented model

Energy balance for all STS $s$:

$$\boldsymbol{V_{s,t}} = (1-\rho_s)\cdot\boldsymbol{V_{s,t-1}} + \boldsymbol{Q}^{\text{ch}}_{\boldsymbol{s,t}}\cdot\eta^{\text{ch}}_{s} - \frac{\boldsymbol{Q}^{\text{dis}}_{\boldsymbol{s,t}}}{\eta^{\text{dis}}_{s}} \tag{10}$$

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**Table 6:** Impact of assumptions on Equation (9)

| |
|---|
| A1: Fixed mass flows: |
| $\sum_{g=1}^{N_G}\left(m_{g,t}\boldsymbol{T_t^{R,C}}\right)+\boldsymbol{m}_{\text{mix},\boldsymbol{t}}\boldsymbol{T_t^{R,C}}+\sum_{s=1}^{N_s}\left(\boldsymbol{m_{s,t}^{\text{dis}}}T_t^R-\boldsymbol{m_{s,t}^{\text{ch}}}\boldsymbol{T_{s,t}^C}\right)=m_t^{\text{DHN}}T_t^R$ |
| A2: Storage's cold water at return temperature: |
| $\sum_{g=1}^{N_G}\left(\boldsymbol{m_{g,t}}T_t^R\right)+\boldsymbol{m}_{\text{mix},\boldsymbol{t}}T_t^R+\sum_{s=1}^{N_s}\left(\boldsymbol{m_{s,t}^{\text{dis}}}T_t^R-\boldsymbol{m_{s,t}^{\text{ch}}}T_t^R\right)=\boldsymbol{m_t^{\text{DHN}}}T_t^R$ |
| A1 & A2 |
| $\sum_{g=1}^{N_G}\left(m_{g,t}T_t^R\right)+\boldsymbol{m}_{\text{mix},\boldsymbol{t}}T_t^R+\sum_{s=1}^{N_s}\left(\boldsymbol{m_{s,t}^{\text{dis}}}T_t^R-\boldsymbol{m_{s,t}^{\text{ch}}}T_t^R\right)=m_t^{\text{DHN}}T_t^R$ |

Min / max energy content in storage:

$$V^{\min}\leq\boldsymbol{V_t}\leq V_{\max} \tag{11}$$

Max charge and discharge rates:

$$V_s^{\text{max-dis}}\leq\boldsymbol{V_{s,t}}-\boldsymbol{V_{s,t-1}}\leq V_s^{\text{max-ch}} \tag{12}$$

To fix value, there is two options. Either set an end value:

$$\boldsymbol{V_{s,N_T}}=V_s^{\text{end}} \tag{13}$$

or set an end range:

$$\boldsymbol{V_{s,N_T}}\geq V_s^{\text{end-min}} \tag{14}$$
$$\boldsymbol{V_{s,N_T}}\leq V_s^{\text{end-max}} \tag{15}$$

#### 3.4.2 Mass and temperature-oriented model

With the assumption of a fully stratified storage, the masses and temperatures of hot and cold water in the storage must fulfill the following equations, see for example [15].

$$\boldsymbol{M_{s,t}^H}\boldsymbol{T_{s,t}^H}=\boldsymbol{M_{s,t-1}^H}\boldsymbol{T_{s,t-1}^H}+\boldsymbol{m_{s,t}^{\text{ch}}}\boldsymbol{T_t^D}-\boldsymbol{m_{s,t}^{\text{dis}}}\boldsymbol{T_{s,t}^H} \tag{16}$$
$$\boldsymbol{M_{s,t}^C}\boldsymbol{T_{s,t}^C}=\boldsymbol{M_{s,t-1}^C}\boldsymbol{T_{s,t-1}^C}-\boldsymbol{m_{s,t}^{\text{ch}}}\boldsymbol{T_{s,t}^C}+\boldsymbol{m_{s,t}^{\text{dis}}}T_t^R \tag{17}$$
$$\boldsymbol{M_{s,t}^H}=\boldsymbol{M_{s,t-1}^H}+\left(\boldsymbol{m_{s,t}^{\text{ch}}}-\boldsymbol{m_{s,t}^{\text{dis}}}\right) \tag{18}$$
$$\boldsymbol{M_{s,t}^C}=\boldsymbol{M_{s,t}^C}-\left(\boldsymbol{m_{s,t}^{\text{ch}}}-\boldsymbol{m_{s,t}^{\text{dis}}}\right) \tag{19}$$

Table 7 shows the impact of the assumptions on Equation (16). A1 and A3 together, or A4 by itself (A4 contains A3), make the equation linear.

Table 8 shows the impact of the assumptions on Equation (17). Only A2 has an impact on the equation and it makes it linear.

The mass flowrates for charging and discharging are limited (assumption: same limits for charging and discharging)

$$m_s^{\min}\leq\boldsymbol{m_{s,t}^{\text{dis}}},\boldsymbol{m_{s,t}^{\text{ch}}}\leq m_s^{\max} \tag{20}$$

The masses of water have limits (note that it's enough to put limits on either the cold or the hot part, thanks to Equations (18) and (19))

$$M_s^{\min}\leq\boldsymbol{M_{s,t}^H}\leq M_s^{\max} \tag{21}$$

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**Table 7:** Impact of assumptions on Equation (16)

| |
|---|
| A1: Fixed mass flows: $\boldsymbol{M^H_{s,t}T^H_{s,t}} = \boldsymbol{M^H_{s,t-1}T^H_{s,t-1}} + \boldsymbol{m^{\text{ch}}_{s,t}}T^D_t - \boldsymbol{m^{\text{dis}}_{s,t}T^H_{s,t}}$ |
| A3: Storage's hot water temperature fixed $\boldsymbol{M^H_{s,t}}T^{\max}_s = \boldsymbol{M^H_{s,t-1}}T^{\max}_s + \boldsymbol{m^{\text{ch}}_{s,t}T^D_t} - \boldsymbol{m^{\text{dis}}_{s,t}}T^{\max}_s$ |
| A1 & A3 $\boldsymbol{M^H_{s,t}}T^{\max}_s = \boldsymbol{M^H_{s,t-1}}T^{\max}_s + \boldsymbol{m^{\text{ch}}_{s,t}}T^D_t - \boldsymbol{m^{\text{dis}}_{s,t}}T^{\max}_s$ |
| A4: All hot water temperatures known $\boldsymbol{M^H_{s,t}}T^{\max}_s = \boldsymbol{M^H_{s,t-1}}T^{\max}_s + \boldsymbol{m^{\text{ch}}_{s,t}}T^D_t - \boldsymbol{m^{\text{dis}}_{s,t}}T^{\max}_s$ |

**Table 8:** Impact of assumptions on Equation (17)

| |
|---|
| A2: Storage's cold water at return temperature: $\boldsymbol{M^C_{s,t}}T^R_t = \boldsymbol{M^C_{s,t-1}}T^R_t - \boldsymbol{m^{\text{ch}}_{s,t}}T^R_t + \boldsymbol{m^{\text{dis}}_{s,t}}T^R_t$ |

## 3.5 Summary

Table 9 gives an overview of the parameters. Table 10 gives an overview of the decision variables in the full formulation from Section 3. Some of these decision variables become parameters in some of the simplified formulations.

**Table 9:** List of parameters

| Symbol | Description |
|---|---|
| $C_p$ | Specific heat capacity of water |
| $\eta^{\text{ch}}_s$ | Charging efficiency for storage |
| $\eta^{\text{dis}}_s$ | Discharging efficiency for storage |
| $\rho_s$ | Hourly loss factor for storage |
| $m^{\min}_s$ | minimum charging / discharging flow |
| $m^{\max}_s$ | maximum charging / discharging flow |
| $M^{\min}_s$ | Minimum mass of hot and cold water in storage |
| $M^{\max}_s$ | Maximum mass of hot and cold water in storage |
| $T^{\max}_s$ | Maximum temperature out of storage |
| $V^{\text{end}}_s$ | Fixed end-value for storage |
| $V^{\text{end-min}}_s$ | Minimum end-value for storage |
| $V^{\text{end-max}}_s$ | Maximum end-value for storage |
| $V^{\min}$ | Minimum heat capacity in storage |
| $V^{\max}$ | Maximum heat capacity in storage |
| $V^{\text{max-ch}}$ | Maximum charging rate in storage |
| $V^{\text{max-dis}}$ | Maximum discharging rate in storage |

Table 11 gives an overview on the equations in the mathematical formulation and whether or not they are linear. It can be seen that there are only two combinations of assumptions that make the model linear:

**A1 & A2 & A3**

Constant mass flows, storage's cold water at return temperature and storage's hot water fixed.

**Table 10:** List of variables (V in red) and parameters (P in green) in the formulation and under the different assumptions

| Symbol | Full formulation | A1 | A2 | A3 | A4 |
|---|---|---|---|---|---|
| $m_{s,t}^{\text{ch}}$ | V | V | V | V | V |
| $m_t^{\text{DHN}}$ | V | P | V | V | P |
| $m_{s,t}^{\text{dis}}$ | V | V | V | V | V |
| $m_{g,t}$ | V | P | V | V | V |
| $m_{\text{mix},t}$ | V | V | V | V | V |
| $M_{s,t}^{C}$ | V | V | V | V | V |
| $M_{s,t}^{H}$ | V | V | V | V | V |
| $Q_{s,t}^{\text{ch}}$ | V | V | V | V | V |
| $Q_t^{D}$ | P | P | P | P | P |
| $Q_{s,t}^{\text{dis}}$ | V | V | V | V | V |
| $Q_{g,t}$ | V | V | V | V | V |
| $T_{s,t}^{C}$ | V | V | P | V | V |
| $T_t^{D}$ | V | P | V | V | P |
| $T_{g,t}^{D}$ | V | V | V | V | V |
| $T_{s,t}^{H}$ | V | V | V | P | P |
| $T_t^{R,C}$ | V | V | P | V | V |
| $T_t^{R}$ | P | P | P | P | P |
| $V_{s,t}$ | V | V | P | V | V |

**A2 & A4**

Storage's cold water at return temperature and all hot temperatures fixed (note that A4 includes A3).

These two sets of assumptions correspond to either setting the mass flows or the temperatures constant in the supply and return pipes, and to assume that the temperature at which the storage is charged and discharged is known and time-independent. Assuming that the temperature at which the storage is known is not enough. Indeed, if the temperature is known and varies with time, then the cold and hot temperatures in the storage will be decision variables and, for example, Equation (16) would remain nonlinear.

**Table 11:** List of linear and nonlinear equations

| Equation number | Full formulation | A1 | A2 | A3 | A4 | A2 & A4 | A1 & A2 | A1 & A3 | A1 & A2 & A3 |
|---|---|---|---|---|---|---|---|---|---|
| (1) | NL | L | NL | NL | L | L | L | L | L |
| (2) | NL | L | NL | NL | NL | L | L | L | L |
| (3) | NL | NL | NL | L | L | L | NL | L | L |
| (4) | NL | NL | NL | NL | NL | L | L | NL | L |
| (5) | L | L | L | L | L | L | L | L | L |
| (6) and (7) | L | L | L | L | L | L | L | L | L |
| (8) | NL | NL | NL | NL | NL | L | NL | NL | L |
| (9) | NL | NL | L | NL | NL | L | L | NL | L |
| (10), (11), (12), (13), (14) and (15) | L | L | L | L | L | L | L | L | L |
| (16) | NL | NL | NL | NL | L | L | NL | L | L |
| (17) | NL | NL | L | NL | NL | L | L | NL | L |
| (18) and (19) | L | L | L | L | L | L | L | L | L |
| (20) | L | L | L | L | L | L | L | L | L |
| (21) | L | L | L | L | L | L | L | L | L |

# 4 Literature study

In this section, existing work on three topics related to the consideration of temperature quality are reviewed:

1. Modelling of the temperature dynamics in water cisterns.
2. Impact of supply or return temperatures on efficiencies
3. Modelling of temperature quality in MILP formulations

## 4.1 Temperature dynamics in water cistern

Two models for the storage are presented:

**Fully-mixed model**

The storage is modelled as one body of water. Appropriate to estimate losses but not appropriate to consider the impact of supply and return temperatures on the storage content.

**Fully-stratified model**

The storage is modelled as two bodies of water, one cold and one hot. Approximates in a better way a real storage in order to consider the impact of supply and return temperatures.

### 4.1.1 Fully-mixed model

Temperature dynamics based on initial value in the storage and subsequent charging and discharging are considered by the following equation (using a fully mixed model for the storage), from [18]:

$$T_{HWT(t)} = T_{HWT(t-1)} + \left( \frac{TES^{HWT}_{disch.(t)} / \eta_{disch.}}{Vol \cdot Cp_{water} \cdot \rho_{water}} \right) - \left( \frac{TES^{HWT}_{ch.(t)} \cdot \eta_{ch.}}{Vol \cdot Cp_{water} \cdot \rho_{water}} \right) \tag{22}$$

$T_{HWT(t)}$ is the temperature in the storage in degrees Celsius. $TES^{HWT}_{disch.(t)}$ is the energy discharged into the district heating network from the storage in MWh. $\eta_{disch.}$ is the discharging efficiency. Vol is the volume of the storage (m3). $Cp_{water}$ is the specific heat capacity of water. $\rho_{water}$ is the density of water. The discharging and charging efficiencies are very often assumed to be constant for accumulator tanks and equal to 98% [18], [5]. See also [1] for another model that considers the mixing effect in the charge and discharge losses. The energy stored $TES^{HWT}_{stored(t)}$ in the storage is modelled by (from [18]):

$$TES^{HWT}_{stored(t)} = TES^{HWT}_{stored(t-1)} + TES^{HWT}_{ch.(t)} \cdot \eta_{ch.} - \frac{TES^{HWT}_{disch.(t)}}{\eta_{disch.}} - TES^{HWT}_{loss(t)} \tag{23}$$

Where $TES^{HWT}_{loss(t)}$ are the energy losses, computed as (from [18]):

$$TES^{HWT}_{loss(t)} = \left(T_{HWT(t)} - T_{out(t)}\right) \cdot U \cdot A_{HWT} \tag{24}$$

Where $T_{out(t)}$ is the ambient temperature, $U$ is the heat transfer coefficient of the storage and $A_{HWT}$ is the surface area of the storage. Other models for energy losses in fully mixed models can be found in [24]. If the fully mixed model is used in an optimization framework, the optimisation variables will be $T_{HWT(t)}$, $TES^{HWT}_{stored(t)}$, $TES^{HWT}_{disch.(t)}$ and $TES^{HWT}_{ch.(t)}$. The fully mixed model is linear. For SeCoHeat, we also want to consider the temperature of the water that is charged / discharged from the storage and its effect on the supply temperature. The fully mixed model does not allow us to do this since it only models an average temperature in the storage. The fully stratified model presented in the next section may be more appropriate.

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### 4.1.2 Fully stratified

In the fully stratified model, the TES is modelled by two masses of water, one hot and one cold. When charging the storage, hot water is added to the storage from the supply pipe and cold water is released into the return pipe. When discharging the storage, cold water is added to the storage from the return pipe and hot water is released into the supply pipe. See also the TES in 1. To model the temperature in the hot and cold parts, the following temperature mixing equations apply (see [15], [11], [10], [27] and [17]). They relate masses, temperatures and mass flows in and out of the storage.

$$M^H_{s,t}T^H_{s,t} = M^H_{s,t-1}T^H_{s,t-1} + m^{\text{ch}}_{s,t-1}T^D_{T-1}\Delta t - m^{\text{dis}}_{s,t-1}T^H_{s,t-1}\delta t \tag{25}$$

$$M^C_{s,t}T^C_{s,t} = M^C_{s,t-1}T^C_{s,t-1} - m^{\text{ch}}_{s,t-1}T^C_{T-1}\delta t + m^{\text{dis}}_{s,t-1}T^R_{s,t-1}\delta t \tag{26}$$

$M^H_{s,t}$ and $M^C_{s,t-1}$ are the masses of the hot and cold water. $T^H_{s,t}$ and $T^C_{s,t}$ are the temperatures of the hot and cold water. $m^{\text{ch}}_{s,t-1}$ and $m^{\text{dis}}_{s,t-1}$ are the mass flows corresponding to charging and discharging the storage (hot water in the storage from the supply pipe and hot water from the storage into the supply pipe, respectively). The masses of the hot and cold water parts depend on the charge and discharge flows, and of the length of the time step $\Delta t$, see also Figure 1:

$$M^H_{s,t} = M^H_{s,t-1} + \left(m^{\text{ch}}_{s,t-1} - m^{\text{dis}}_{s,t-1}\right)\Delta t \tag{27}$$

$$M^C_{s,t} = M^C_{s,t-1} - \left(m^{\text{ch}}_{s,t-1} - m^{\text{dis}}_{s,t-1}\right)\Delta t \tag{28}$$

As mentioned above, for SeCoHeat, we also want to consider the temperature of the water that is charged / discharged from the storage and its effect on the supply temperature. The fully stratified model allows us to do this. In an optimization framework, the optimization variables will be $M^H_{s,t}$ and $M^C_{s,t}$, $T^H_{s,t}$and $T^C_{s,t}$, $m^{\text{ch}}_{s,t}$ and $m^{\text{dis}}_{s,t}$. The formulation is non linear because of the product of masses and temperatures on the one hand and of the mass flows and temperatures, on the other hand. The challenge with the fully stratified model is the non-linearity.

## 4.2 Consideration of supply or return temperature on effiencies

Efficiencies of heat pumps (coefficient of performance - COP) and of combined heat and power plants (alpha values) depend on temperatures.

The benefits of a control strategy that aims at achieving lower supply temperatures are investigated for one single plant in [19], focusing on the benefits associated with higher alpha values. The authors give an increase in alpha value of 0.29% per degree. A further development of the control strategy for a multi-unit power plant was performed in [20].

In [14], the operating area of CHP plants in terms of heat versus electricity production is modelled by adding a third dimension: return temperature. This allows to consider the variations of the operating area due to return temperature variations. Return temperature is modelled as a variable that depends on the outdoor temperature.

In [8], the temperature dependence of alpha values is modelled by discretizing the supply temperature range in a number of segments and assuming that alpha values are constant in each segment.

In [25], the temperature dependency of COP is linearized by introducing a set of temperature intervals and introducing binary variables corresponding to each interval. A set of constraints ensure that only one temperature interval is active and linearizes the product of the binary variables and the electric power consumption that appear in the COP constraint.

## 4.3 Consideration of temperature quality into a MILP

The formulation given in Section 3 is nonlinear because of the products of mass flows and temperatures in Equations 8, 9 and 16 to 19. Several works in the literature have looked at linearising these equations.

In [11], it is assumed that the district heating network is operated according to the so-called constant-flow variable-temperature mode, as is common in China. This assumption allows to fix all flows and only temperatures are taken as variable. It is similar to the assumption of a two-level controlled pump in [22] reviewed below. This assumption may not hold for systems with thermal storage because the charging and discharging processes entail a variable flow out of the storage.

In [4], an iterative approach using a detailed model and a MILP model is proposed to optimize pumping powers and delivery temperatures. This requires the use of detailed model and the iterative approach implies a higher computational burden than direct linearisation of the equations in the MILP problem.

In [13], a temperature-centric formulation using batches of water at different predetermined temperatures is proposed. An exclusion constraint is used to ensure that water only has one temperature. This allows for linearising the mass flow - temperature equations directly in the MILP problem. The drawback is that it introduces new binary variables, associated with the choice of one temperature batch.

In [21] and [2], the TES is modelled as several variable masses at fixed temperatures. In [21], effort is made to reduce the number of binary variables in the problem formulation (see [22] below for an analysis of the additional computation burden), for example by allowing several masses of water to be partially active at the same time instead of using binary variables to ensure that only one of them is active.

In [3], the nonlinearity are addressed by formulating a relaxed MILP using McCormick relaxations. However, mass flows are noticed to be largely underestimated by such an approach. The authors propose heuristics to adjust the mass flows out of the relaxed MILP. These fixed mass flows are then set in the original MINLP formulation, thus leading to a reduced MILP formulation where only temperatures are optimisation variables. The drawback is that it introduces heuristics and requires the need to solve two optimisation problems instead of one.

In [27], an iterative process is used to tackle the nonlinearities induced by the products of mass flows and temperatures. As for [4], the iterative approach implies a higher computational burden than direct linearisation of the equations in the MILP problem.

In [23], it is assumed that the storage always charges and discharges with the supply temperature, thus avoiding to keep the storage temperature as an optimisation variable. This approach cannot consider the cases where the temperature stored in the storage is lower than the supply temperature.

In [9], an iterative process is developed to linearize products of mass flows and temperatures. First, supply temperatures are set according to a control curve as a function of outdoor temperature. Second, return temperatures and mass flows are computed based on supply temperatures and forecasted heat demand. Third, a MILP optimisation problem is solved to minimize costs using heat production and variable supply temperatures. Steps 2 and 3 are repeated until convergence of supply temperatures. The drawback of this approach is the iterative nature of the process which would make it too time consuming for long-term simulations.

In [12], Taylor's first-order expansions are used to linearize nonlinearities to formulate an MPC controller. This approach may work if deviations between different time steps are small, for example if the MPC controller if called often enough. It would not be appropriate in a day-ahead scheduling problem.

In [15], temperatures are discretized and constraints are formulated to ensure that an arbitrary temperature is approximated by appropriate masses of at most two different temperatures. The approach is similar to [25] where it was applied to linearizing the temperature dependency of COP. This results in a MILP problem for scheduling, so-called water-flow MILP model. It is compared to an energy-only MILP model and to a MINLP model for different numbers of temperature intervals and on different system designs. The results show the importance of including temperature to avoid overestimating the contribution of lower temperature sources (as in the solution of the energy-only model) which would result in inappropriate supply temperatures and underestimating costs. The results show that the water-flow MILP model approximates reality (as modelled by the MINLP model) as the number of temperature intervals increase. It is also shown that the inclusing of thermal storage in the system design increases the complexity of the water-flow MILP substantially by introducing time-coupling constraints. The same discretization of temperatures is also used in [7].

A similar linearization is performed in [16]. However, instead of splitting a temperature interval by equally spaced temperatures, the authors use a so-called binary expansion to approximate temperatures. Assuming a temperature interval of $[T_{\text{min}}, T_{\text{max}}]$ and using $K$ binary variables $z_i$, $i = 1, \ldots, K$, set $\Delta T = (T_{\text{max}} - T_{\text{min}})/K$. A temperature $T$ can then be approximated by $T = T_{\text{min}} + \Delta T \sum_{k=1}^{K} 2^{k-1} z_k$ by choosing the binary variables $z_k$ in an adequate manner. In this way, temperatures are approximated from below with a precision of $1/2^K$. This gives a better precision than using $K$-equally spaced temperatures in the interval $[T_{\text{min}}, T_{\text{max}}]$, see Table 12

**Table 12:** Comparison of the precision of binary expansion and equally-spaced intervals

| Temperature interval | Number of points | Precision with binary expansion | Precision with equally spaced |
|---|---|---|---|
| 30 | 1 | 15 | 15 |
| 30 | 2 | 7.5 | 10 |
| 30 | 3 | 3.75 | 7.5 |
| 40 | 1 | 20 | 20 |
| 40 | 2 | 10 | 13.3 |
| 40 | 3 | 5 | 10 |
| 50 | 1 | 25 | 25 |
| 50 | 2 | 12.5 | 16.7 |
| 50 | 3 | 6.25 | 12.5 |

Discretization is also applied in [26] but this time on the mass flow rates. The authors point out that, although this results in a MILP problem, it is still complex because of the number of binary variables. They propose a two-stage solution approach to speed up the process. First, all binary variables for choosing mass flow rate levels are relaxed to being continuous except for one. This gives a lower bound for the optimal solution (because it is a relaxation). Second, the mass flow rates are fixed as being equal to the solution in the first step and the temperatures let free. This gives an upper bound for the optimal solution (since mass flow rates are constrained to fixed values instead of being free). The results show small difference between the optimal costs in the first and second step and the authors use the solution from the second step as an approximation of the solution to the original problem.

Five different linearisations techniques of the product of mass flows and temperatures are compared in [22] to model thermal energy storage: energy-only model (no consideration of temperature and mass flows), fixed temperature difference between supply and return, mass flows fixed to nominal mass flow (two-level controlled pump), Taylor's first order expansion (as in [12] above) and binary expansion of the mass flows (as in [16] but for mass flows instead of temperatures). Results show that energy-only model is not able to meet comfort criterion. The Taylor's first order expansion leads to physically impossible solutions (too low mass flow) due to the large error in excluding higher order terms. The binary reformulation with 3 binary variables is very complex and required large computation times. The other two approaches, constant temperature differences and two-level control circulation pump achieves good trade-off between physical accuracy and computation times.

## 5 Conclusion

In this report, the importance of considering temperature quality in district heating networks when scheduling district heating units is presented. Temperature quality refers to controlling mass flows and supply temperatures to ensure acceptable confort at the end-consumers. Traditional scheduling models use an energy-only formulation where energy is related to the product of temperaturate and mass flows. They do not consider these two quantities separately and, therefore, are unable to capture temperature quality aspects. On the other hand, these traditional scheduling models are less complex than a full representation of both temperatures and mass flows. Traditional scheduling models based on an energy-only formulation can be expressed as MILP optimization problems, whereas considering both temperatures and mass flows lead to MINLP optimization problems which are must harder to solve.

Several simplications and reformulations have been proposed in this report to make the full MINLP problem less complex. These simplifications reduce the number of non-linear equations and, for one of them, even leads to a MILP formulation. In addition, a literature review about exisiting linear reformulations of the full MINLP problem and the importance of considering temperature quality is performed.

This report gives a ground to further develop scheduling models that make tradeoffs between model complexity and accurate representation of temperature quality.

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