



SICS SWEDISH ICT

SICS TECHNICAL REPORT T2015:05

Timetabling based on generalised cost¹

Victoria Svedberg

Martin Aronsson

Martin Joborn

vicsve@sics.se

martin@sics.se

mjoborn@sics.se

SICS Swedish ICT AB; Box 1263; SE-164 29 Kista; Sweden

Abstract

The liberalised railway market of today calls for new methods to plan the annual timetable. A conflict between two railway undertakings operating in the same market is hard to solve using the current method. Common price setting strategies such as auctions, framework agreements and spot markets may be of help. To administer a long term stability for an operator, framework agreements can be entered. These framework agreements should be valid for several years but still not consume too much capacity from other operators. In this report we propose a novel mathematical model calculating the benefit of subsidised traffic, based on the generalised cost of it's timetable and optimisation models are developed. The calculated benefit is weighted against traffic from other operators, to provide a decision support for concluding the content of the framework agreements.

February 18, 2016

¹This research is performed under Trafikverket, grant TRV 2014/77971

1 Introduction

In 1988 Banverket was separated from Statens Järnvägar to take on the role as an infrastructure manager in Sweden. The main responsibilities were to maintain the railway tracks and organising timetables for trains requesting to operate on the railway network. Previously, Statens Järnvägar had been responsible for operating both tracks and trains as a government agency but would in the following years be reorganised into a state-owned company. This separation was the start of the liberalisation of the Swedish railway market. Currently, trains on the Swedish railways are operated by multiple companies such as SJ, MTR, and Green Cargo. An increased interest from different companies to operate trains on the Swedish network along with already heavily congested tracks causes challenges to the current methods used to plan the timetable.

In this paper, a new approach for the timetable planning is suggested. This approach is first suggested by Eliasson and Aronsson [5]. A part of this approach is to enter framework agreements for capacity utilisation. Since framework agreements are entered years in advance, and there are no outlook on the timetabling needs in the future, a method for weighting the cost of the framework agreement against the cost of possible other future traffic is introduced to judge between the different timetabling needs. In this report the weight of a possible framework agreement for non-commercial traffic is considered. This weight is found by calculating the welfare cost of a suggestion to a framework agreement. The welfare cost consists of the generalised cost and the operating cost. A common model describing the generalised cost and the operating cost of a timetable will be described and modelled as an optimisation model and minimised. The generalised cost will be based on the cost of travel time and deviation from desired departure time. The aim is to provide a computational method so that common tools for economical evaluation, such as consumer and producer surplus, can be applied in the timetable planning.

This paper have the following structure. First a description of the current timetable planning method and it's limitation is given in Section 2. A new approach to the timetable planning is given in Section 3 which aims to remedy the deficiencies described in Section 2. A part of the remedy is the welfare cost of a timetable and this is explained in Section 4. The intent is to minimise the welfare cost of a timetable so optimisation models are developed and explained in Section 5. In Section 6 a test case and the results from this test case are described and a conclusion is made in Section 7.

2 Current timetabling method

The process at Trafikverket (the infrastructure manager in Sweden) is briefly described in the flowchart in Figure 1.

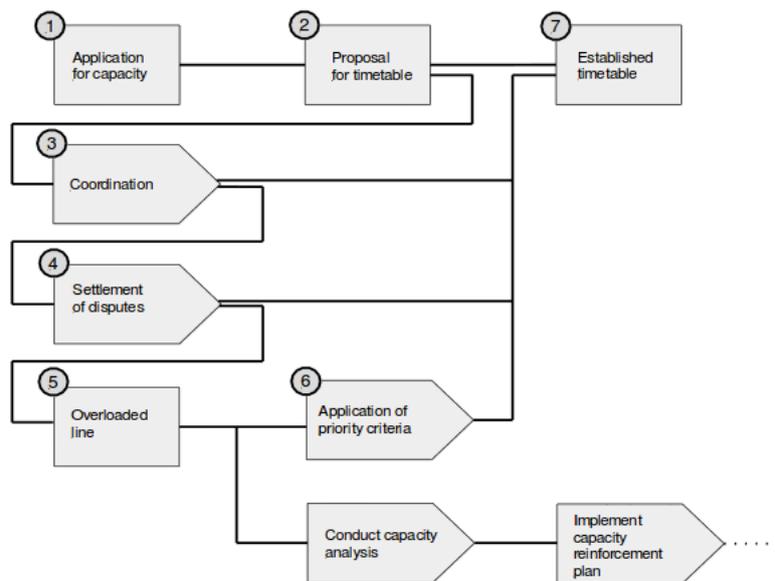


Figure 1: Trafikverket's annual timetabling process. Picture from Trafikverket's Network Statement [17].

The timetabling process starts over a year in advance when the Network Statement is written and published. After the Network Statement is finished and has been published, the railway undertakings and maintenance entrepreneurs apply for their requested train paths and maintenance work. The infrastructure manager investigates whether the requests can be met or not. A train path is a timetable for a specific train. If two or more train paths are causing a resource conflict which cannot be resolved within reasonable adjustments, the corresponding requests cannot be met and the infrastructure manager must confer with the concerned operators to discuss possible changes to their requested train paths in order to fit all of them in the timetable. If no such agreements are found, the track is declared congested and the infrastructure manager calculates the value of the train paths by applying the priority criteria calculus. The name priority criteria might be a bit confusing since it is actually a value of the trainpath and its transport which is calculated. A value of, for instance travel time, stops and passenger exchange is set. This value depends on whether the train is a passengertrain or a freighttrain, the speed of the train, etc. The name "criteria" comes from the fact that there are some criteria which have to be met for being assigned to certain cost parameters. The priority criteria rely on self declared information from the operators. When the values of the train paths are found, the train path with the lowest value loses the dispute and the other train paths are included in the timetable. When all applications are treated and scheduled, the timetable is published, along with a list of the cancelled train paths. This point in time is also the start of the ad-hoc process. Many applications for train paths are submitted after the deadline and cannot participate in the annual process previously described. Some demand for train paths might not yet be apparent during the application period and the ad-hoc process should cover these late requests. The train paths in the ad-hoc process are then dealt with on a first-come-first-served basis and doesn't change any of the already planned and published train paths.

2.1 Limitations to the current timetabling method

The European Union strongly advocates a fair competition on the tracks by the First Railway Directive. One part of the road to a fair competition is to let operators compete for a conflicting train paths on equal and fair conditions. There are some issues regarding the fair competition in the current timetabling process. These issues are thoroughly discussed in Eliasson and Aronsson [5]. The main issue concerns the priority criteria used to obtain a value of the train paths in disputes. The infrastructure manager is completely dependent on correct and honest information from the operators. By design, some properties may be exploited. As an example, a train travelling a longer distance has a higher value than other trains travelling shorter distances. This can be used by an operator to obtain a higher value on the requested train path by prolonging the transport a few kilometers. If two train paths in a dispute are to be operated by trains attracting to similar markets, the outcome of the priority criteria is very random. In this case the criteria cannot provide any good guidance and the consequence is a slightly unfair capacity allocation. To achieve a perfect competition, the conditions need not only be equal, but also be completely transparent to all operators.

Eliasson and Aronsson [5] also address the problem of the different preferences of the companies. Passenger train operators must know their timetable in advance, so they can sell tickets. Freight train operators may not know by the application deadline when or between which stations a transport should be arranged. They must then settle with submitting an application in the ad-hoc process and adjust to the already planned timetables. In the subsequent sections a new approach addressing these issues is described.

3 Suggested timetable planning process

Eliasson and Aronsson [5] propose a new approach for timetable planning consisting of four parts. These parts are

- Framework agreements
- Auctions
- Spot market
- Economical evaluation

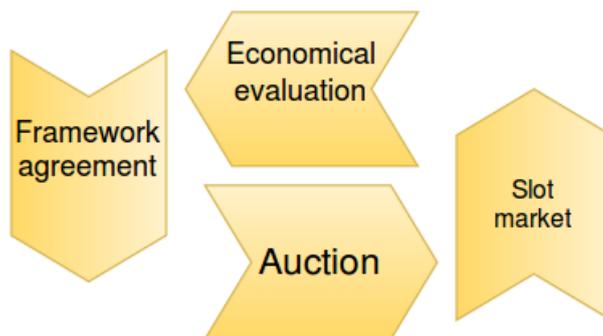


Figure 2: A coarse flowchart of the suggested approach.

Figure 2 illustrates the main outline of the process.

Framework agreement: A framework agreement can be concluded between the infrastructure manager and an operator. The agreement consists of a contract for operating train according to a specific departure pattern during a period (for instance having a departure each 20th minute between 7:00 and 9:00). Deviations from the departure pattern are allowed. The maximum allowed deviation is also stated in the contract. This allows some flexibility for the planner, since there is no possibility to know how other operators want to plan their traipaths. This contract is valid for a period of, say, five years, and is then renegotiated. State-subsidized traffic is only allowed to conclude framework agreements since they are financially stronger than commercial traffic and thus should be excluded from the auctions and spot market described below.

Auctions: The auctions regard contract times. Contract time is a time for a departure from or an arrival to a station. Hence a complete timetable is not negotiated (which is a set time on every consecutive signal and station), only a promise that the planner will plan a timetable with respect to the contract times. This also allows some flexibility for the planner when planning the daily timetable. Gestrelus [7] gives a more thorough explanation of contract times and its properties. The auctions are held one year in advance and only the winners are allowed to state their desired contract times.

Spot market: This is similar to the current ad-hoc process. The idea is that a train operator applies for contract times and the infrastructure manager checks the possibility of fulfilling the request. The infrastructure manager also calculates a price based on the auction price and the demand for conflicting contract times. The operator can then renegotiate the times, accept or decline. The slot market is open until the day of operation.

Economical evaluation: To decide the auction-objects, the conditions for the framework agreement and other parameters, an economical evaluation is performed. This evaluation ties the process from the previous year to the process of next year, ensuring that the complete process is flexible and responsive to the train operating companies' opinions.

The complete methods of this annual process are investigated within the project "Economically efficient track allocation" where research is carried through by SICS Swedish ICT and KTH.

3.1 Related Work

To use framework agreements for determining capacity utilisation have previously been investigated at Trafikverket [16]. The notion of economically maximised timetables for railways is not a new area, but still a quite unexplored one. Previous literature describing how to calculate a generalised cost of a timetable to be used when investigating consumer surplus have not been found although models using a dynamic demand and focusing on passenger welfare are more common. A welfare enhancing timetable will depend on the routes passengers are willing to take. In the meantime passenger routes are dependent on the timetable. Schmidt et al. [15] and Borndörfer et al [3] try to address this problem by exploring the integrated periodic timetabling and passenger routing problem. Another common approach [6, 8, 13, 1] is an iterative procedure where firstly a timetable is found, then the demand is recalculated and the timetabling problem is solved again. The iterations continue until a stopping criteria is met. Espinosa-Aranda et al. [6] introduces a model based on two submodels. The first is a discrete event scheduling model which represents the supply of rail services and the second is a constrained logit-type choice model which takes the user behaviour into account and calculates a demand. The behaviour is

based on timetable fare, travel time and seat availability. Kuo et al. [8] applies a simulation based iterative framework for recalculating the demand from an optimised timetable where the operating cost is minimised for freight trains. A similar iterative approach is treated in the case of line planning in Parbo et al. [13]. Barrena et al. [1] minimise the average passenger waiting time where the model is adjusted to a dynamic demand behaviour using a cumulative function. Niu and Zhou [11] optimised a schedule on a heavily congested urban rail corridor including a condition for train crowding. Robenek et al. [14] investigates a model where timetables are planned after the passengers point of view using their in-vehicle time, waiting time, number of transfers and the scheduled delay. Niu et al. [12] minimise the waiting time for passengers using a time-varying origin-destination passenger demand matrices. Wang et al [18] propose an model based on departure, arrival and passenger arrival rate changes. The routing of passengers at transit station is also included and their behaviour. An auction approach for track allocation is investigated in Nilsson [10, 9], Brännlund et al. [4] and Borndörfer et al. [2]. All the previously described models are considering the passenger welfare, but does not provide a model calculating the generalised and operating cost. This is an important aspect when doing an economical evaluation using the consumer and producer surplus, which are common tools for economical evaluations currently used when investigating investments. This consumer and producer surplus will then be used as an estimated value of the traffic for different suggestions to a framework agreement.

4 The welfare cost of a timetable

The welfare cost is regularly used when investigating the welfare benefits of an investment. Usually the producers' difference in benefit and cost before and after an investment is estimated along with the change in cost for the society. The cost an investment inflicts on society is called the generalised cost. The generalised cost is not only a monetary cost induced by for instance fare which is paid by every affected person, but also an estimated cost for the travel time, waiting time, congestion and other non-monetary cost imposed on a user. This can also be applied to timetables.

To calculate the welfare benefit of traffic from a state-subsidised operator, the generalised cost is weighted against the production cost of a timetable. Using the result a consumer surplus and a producer surplus can be calculated respectively. This is used when comparing and negotiating different possibilities of framework agreements. In this report, only the objective is investigated. The underlying model where the technical aspects of a timetable is considered, such as conflict freeness, is the same as the one used by Gestrelus et al. described in [7].

4.1 Operating cost

To calculate the cost of operating a train the travel time, travel distance and the amount of travelling passengers are considered. A simple linear model for calculating the operating cost for a train r is suggested in Equation 1. Table 1 describes the included parameters and variables. The operating cost can be used for calculating the producer surplus when comparing suggested framework agreements.

Sets:

\mathcal{R} trains

Variables:

t_r total travel time for train r

N_r amount of passengers on train r

Constants:

θ_1 cost/(train · time unit)

θ_2 cost/(amount of passengers · time unit)

θ_3 cost/(traveldistance · time unit)

L_r total traveldistance for train r

Table 1: Explanation of constants and variables for the operating cost in Equation 1.

$$\theta_1 t_r + \theta_2 N_r + \theta_3 L_r, \quad \forall r \in \mathcal{R} \quad (1)$$

4.2 Generalised cost

As previously mentioned, the generalised cost is minimised, which also means that the service should be available to as many as possible. Therefore, there is a distribution of passengers travelling between an OD-pair included in the objective. The cost is minimised when the time between the desired departure and the actual departure is kept at a minimum for a maximum number of people. To describe the cost of a certain departure time $T_{i,k}$ for a passenger who wants to travel at time s , the mathematical model stated in Equation 2 is used. We assume that the cost function is the same for all passengers.

Sets:		Variables:	
\mathcal{I}	stations	$T_{i,k}$	departure time from station i for departure $k \in \mathcal{K}_i$
\mathcal{K}_{ij}	departures between stations $i, j \in \mathcal{I}, i \neq j$	$t_{ij,k}$	travel time between stations i and j for departure $k \in \mathcal{K}_{ij}$. ($= T_{j,k} - T_{i,k}$)
		s	desired travel time
Functions:		Constants:	
$N_{ij}(s)$	distribution of passengers who wants to travel from station i to j	α	cost of travel time
$(x)^+$	$= \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$	β	cost inflicted when travelling later than desired
$\tau_{ij,k}$	breakpoints, i.e. the time when a passenger is indifferent when choosing either an earlier or a later departure.	γ	cost inflicted when travelling earlier than desired

Table 2: Explanation of constants and variables.

$$\alpha t_{ij,k} + \beta(T_{i,k} - s)^+ + \gamma(s - T_{i,k})^+ \quad (2)$$

Table 2 consists of a complete description of the parameters and variables to Equation 2.

During the day, multiple trains depart between an OD-pair. We assume that a person who wants to leave at 10am is more likely to chose the train departing at 10.30am than 12am. Hence, a day may be split into different breakpoints, $\tau_{ij,k}$. A person with a desired departure time before $\tau_{ij,k}$ takes departure k and if a person wants to travel after $\tau_{ij,k}$ she/he takes departure $k + 1$. The τ -value is obtained by first assuming that people always chooses the departure which minimizes their cost. τ is then the time when the cost for taking an earlier departure equals to cost of taking a later departure, i.e. when $\alpha t_{ij,k} + \gamma(\tau_{ij,k} - T_{i,k}) = \alpha t_{ij,k+1} + \beta(T_{i,k+1} - \tau_{ij,k})$. Solving for $\tau_{ij,k}$ the function described in Equation 3 is obtained.

$$\tau_{ij,k} = \frac{\gamma T_{i,k} + \beta T_{i,k+1} + \alpha(t_{ij,k+1} - t_{ij,k})}{\beta + \gamma} \quad (3)$$

To find the cost of a specific departure we utilise the model displayed in Equation 4. The generalised cost is integrated over the distribution of the passengers requested travel time between the two breakpoints $\tau_{ij,k-1}$ and $\tau_{ij,k}$. Hence the expected cost for all passengers travelling from station i to station j with departure k train can be calculated as in Equation 4.

$$\int_{\tau_{ij,k-1}}^{\tau_{ij,k}} (\alpha t_{ij,k} + \beta(T_{i,k} - s)^+ + \gamma(s - T_{i,k})^+) N_{ij}(s) ds \quad (4)$$

This must then be done for all departures and over all OD-pairs in order to get the total cost for the entire timetable. Figure 3 illustrates the main idea of this model.

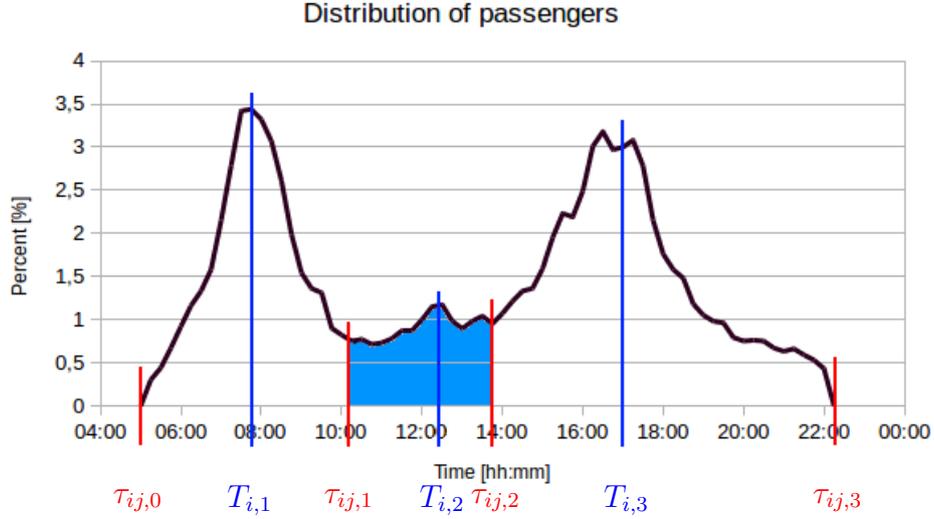


Figure 3: The distribution of the passengers desired departure time. Departure time $T_{i,k}$ and breakpoint $\tau_{ij,k}$ are displayed. The amount of passenger travelling with departure 2 is marked in bright blue.

5 Optimisation models

Different models for solving the optimisation problem have been tested. In this section, a description will be given of seven of the models which have been given the best results and which best describes the general idea of modelling the cost function. Test runs on an example from a part of Sweden are given in Section 6. Six of the models describe the cost function as given. The seventh model is an alternative interpretation trying to imitate the behaviour of the original cost function.

A passenger distribution is generally non-convex, and usually with no chance whatsoever to make it convex. The ambition is to in the end have a linear optimisation problem. This non-convex passenger distribution puts additional spokes in the wheel for our ambition. To remedy this problem the day is split into a set of time intervals, $q = (q_l, q_u) \in \mathcal{Q}_{ij}, \forall i, j \in \mathcal{I}, (i \neq j)$, where the number of passengers in each interval is defined to N_q . Then a Dirac delta-function² is assigned giving an impulse with the amount of passengers desiring to travel in the interval at the mean time $q_m = \frac{q_l + q_u}{2}$ of the interval. The amount of people desiring to travel between time s_1 and s_2 is then

$$\int_{s_1}^{s_2} N(s) ds \sim \int_{s_1}^{s_2} \sum_{q \in \mathcal{Q}_{ij}} N_q \delta_{q_m}(s) ds = \sum_{q | q_m \in (s_1, s_2), q \in \mathcal{Q}_{ij}} N_q. \quad (5)$$

Thus, the integral over a non specified function is made linear. Remember the generalised cost of a timetable. This is again stated in Equation 6. This can in a similar manner be rewritten into the sum in Equation 7. This is also suitable since a preferred departure time might not be exactly defined but rather be in a small interval.

$$\int_{\tau_{ij,k-1}}^{\tau_{ij,k}} \sum_{q \in \mathcal{Q}_{ij}} (\alpha t_{ij,k} + \beta(T_{i,k} - s)^+ + \gamma(s - T_{i,k})^+) N_q \delta_{q_m}(s) ds \quad (6)$$

$$\sum_{q | q_m \in (\tau_{ij,k-1}, \tau_{ij,k}), q \in \mathcal{Q}_{ij}} (\alpha t_{ij,k} + \beta(T_{i,k} - q_m)^+ + \gamma(q_m - T_{i,k})^+) N_q \quad (7)$$

In the following sections a description to the different optimisation models developed to minimise the generalised cost will be described. In each sections new constraints and variables will be defined. Nothing will be passed on between sections. It is also important to note that this is just a model of the objective function. A complete description of the model of the infrastructure is given in Gestrelus et al. [7].

²Dirac delta-function, also called the impulse function, $\delta_a(x) = \begin{cases} 1 & x = a \\ 0 & \text{otherwise} \end{cases}$

5.1 Model 1

Firstly, one must make sure that the restriction of the sum in Equation 7 to only include $q|q_m \in (\tau_{ij,k-1}, \tau_{ij,k})$, $q \in \mathcal{Q}_{ij}$ is followed. This is done by introducing a binary indicator variable $z_{ij,k,q}, \forall k \in \mathcal{K}_{ij}, q \in \mathcal{Q}_{ij}, i, j \in \mathcal{I} (i \neq j)$. Then $z_{ij,k,q}$ can be defined according to Equation 8 by using the constraints defined in Equation 9 and 10.

$$z_{ij,k,q} = \begin{cases} 1 & \tau_{ij,k-1} \leq q_m \leq \tau_{ij,k} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$\tau_{ij,k} - Mz_{ij,k,q} \leq q_m \leq \tau_{ij,k+1} + Mz_{ij,k,q} \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}_{ij}, k \in \mathcal{K}_{ij} \quad (9)$$

$$\sum_{k \in \mathcal{K}_{ij}} z_{ij,k,q} = |\mathcal{K}_{ij}| - 1 \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}_{ij} \quad (10)$$

These constraints allocate one q -interval to one train, and it will always be to the train where the q_m -value is in the interval $(\tau_{ij,k-1}, \tau_{ij,k})$. The constraint in Equation 9 makes sure that if $z_{ij,k,q} = 0$, q_m have to be within $(\tau_{ij,k-1}, \tau_{ij,k})$. The constraint in Equation 10 makes sure that only one of the $z_{ij,k,q} = 0$ for a specific q equals to 0. If one only looks at the generalised cost, Figure 4 gives the main idea. The y-axis illustrates the cost for choosing a specific departure depending on the desired departure time. A person always seeks the option which minimises his cost. Thus, the departure causing the minimum cost for a person is chosen. The value to include in the objective is then always the minimal generalised cost for each train. This minimal generalised cost we want to consider in the objective is the dashed line in Figure 4. The intent of the $z_{ij,k,q}$ -variable is to force the objective to only consider the smallest cost, and remove the impact from the other departures yielding a higher cost.

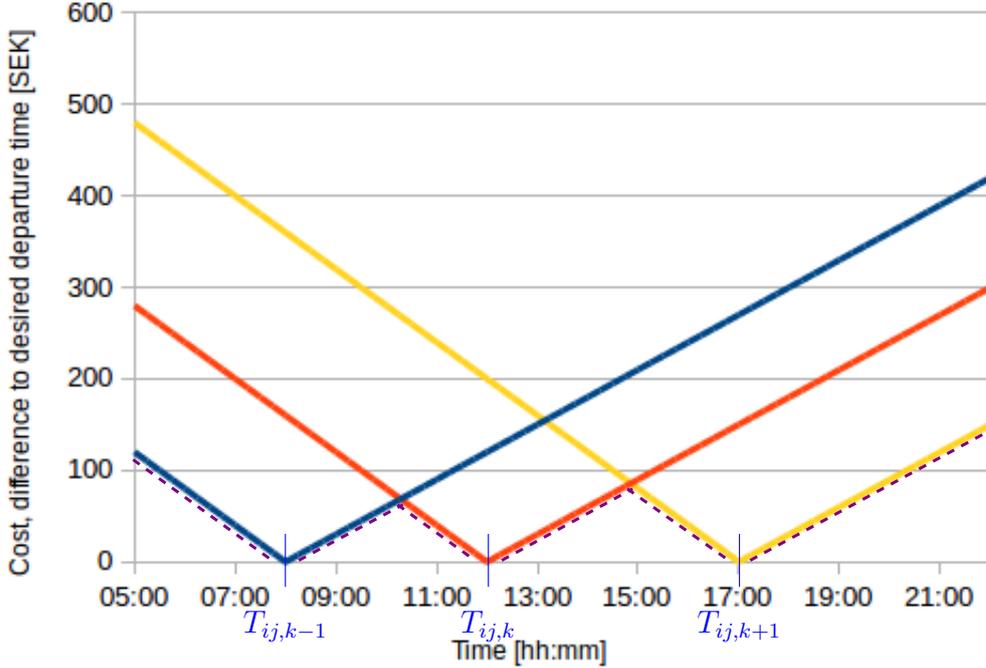


Figure 4: The cost for taking a departure distributed over time. The dashed line is the minimum cost.

To speed up the calculations the time for a departure is only allowed to vary a specified number of time from the initial value, $T_{init,i,k}$. Call this time-interval (l_k, u_k) for each departure k . $z_{ij,k,q}$ must then equal 0 outside (l_{k-1}, u_{k+1}) . For every $q \in \mathcal{Q}_{ij}$ a variable $o_{ij,q}$ is introduced. The generalised cost can then be defined using the constraints in Equation 11 and 12.

$$o_{ij,q} \geq \alpha t_{ij,k} + \beta(T_{i,k} - q_m) - Mz_{ij,k,q}, \forall q \in \mathcal{Q}_{ij} (q_m \in (l_{k-1}, u_k)), i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (11)$$

$$o_{ij,q} \geq \alpha t_{ij,k} + \gamma(q_m(1 - z_{ij,k,q}) - T_{i,k}), \forall q \in \mathcal{Q}_{ij} (q_m \in (l_k, u_{k+1})), i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (12)$$

Using big M -constraints ensure that the train yielding the smallest generalised cost will be considered in the objective. The constraint in Equation 11 refers to the generalised cost of travelling with a later train and the constraint in Equation 12 refers to the cost of travelling with an earlier train. If a q -interval should be allocated to a departure k , then $z_{ij,k,q} = 0$ and the impact of the large M -constraints disappear. The objective function is then

$$\min \sum_{ij \in \mathcal{I} | i \neq j} \sum_{q \in \mathcal{Q}_{ij}} N_q o_{ij,q} \quad (13)$$

5.2 Model 2

Model 1 yields a large amount of binary variables and constraints. The structure of the constraints defining the $z_{ij,k,q}$ -variables opens up for many simplifications in the preprocessing, but still doesn't solve the problem fast enough. To decrease the amount of variables one may look at only two consecutive departures. By the definition of the objective, it can be split into two parts. One for the objective when a person needs to commence travelling at a later time than desired, $\alpha t_{ij,k} + \beta(T_{i,k} - q_m)$, and one for the case when a person needs to take an earlier departure than desired $\alpha t_{ij,k-1} + \gamma(q_m - T_{i,k-1})$. Call this the β - and γ -part respectively. Then one may only look at the γ -part for the earlier train and the β -part for the later train. Figure 5 shows the cost-situation for this arrangement. Note that this is exactly the same situation as in Figure 4, but only considering the $k-1$:th and the k :th departure. As previously stated the departure yielding the minimum cost will be considered in the objective, which is the dashed line in Figure 5. To remove the impact from the higher cost a binary variable $z_{ij,k,q}$ is introduced. $z_{ij,k,q}$ should be defined in line with Equation 14. Since the departure time $T_{ij,k}$ is as earlier only allowed to vary within a time interval (l_k, u_k) , $z_{ij,k,q}$ -variables only need to be defined for all q in (l_k, u_{k+1}) , call this set $\mathcal{Q}_{ij,k}$. Let q_l^{min} and q_u^{min} be the lowest time interval such that q_m still contained in (l_k, u_{k+1}) , and δ be the length of each interval (they are assumed to have equal length). Then the constraints on $z_{ij,k,q}$ can be defined as in Equation 15 and 16. Notice that this is not the same $z_{ij,k,q}$ as described earlier, nor is it the same as the $z_{ij,k,q}$ -variable which will be used in later sections.

$$z_{ij,k,q} = \begin{cases} 1 & q_l \leq \tau_{ij,k} \leq q_u \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

$$q_l^{min} + \sum_{q \in \mathcal{Q}_{ij,k}} p \cdot \Delta \cdot z_{ij,k,q} \leq \tau_{ij,k} \leq q_u^{min} + \sum_{q \in \mathcal{Q}_{ij,k}} p \cdot \Delta \cdot z_{ij,k,q} \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}_{ij,k}, k \in \mathcal{K}_{ij} \quad (15)$$

$$\sum_{q \in \mathcal{Q}_{ij,k}} z_{ij,k,q} = 1 \quad \forall i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (16)$$

The constraint in Equation 15 finds the q -interval in which $\tau_{ij,k}$ are located. The constraint in Equation 16 makes sure that only one q -interval is found. Now if $\tau_{ij,k}$ is within a certain q -interval, the corresponding $z_{ij,k,q}$ -variable is equal to one, and all later γ -cost parts will not have any effect on the resulting cost using a large M constraint. Vice versa for all earlier β -cost parts. In the end the cost variable $o_{ij,q}$ can be defined using the constraints in Equation 17 (cost for choosing a later train) and 18 (cost for choosing an earlier).

$$o_{ij,q} \geq \alpha t_{ij,k} + \beta(T_{i,k+1} - q_m) - M \sum_{q \in \mathcal{Q}_{ij,k}} z_{ij,k,q}, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (17)$$

$$o_{ij,q} \geq \alpha t_{ij,k} + \gamma(q_m - T_{i,k+1}) - M \sum_{q \in \mathcal{Q}_{ij,k}} z_{ij,k,q}, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (18)$$

Now the objective is the same as in Model 1, i.e. Equation 13.

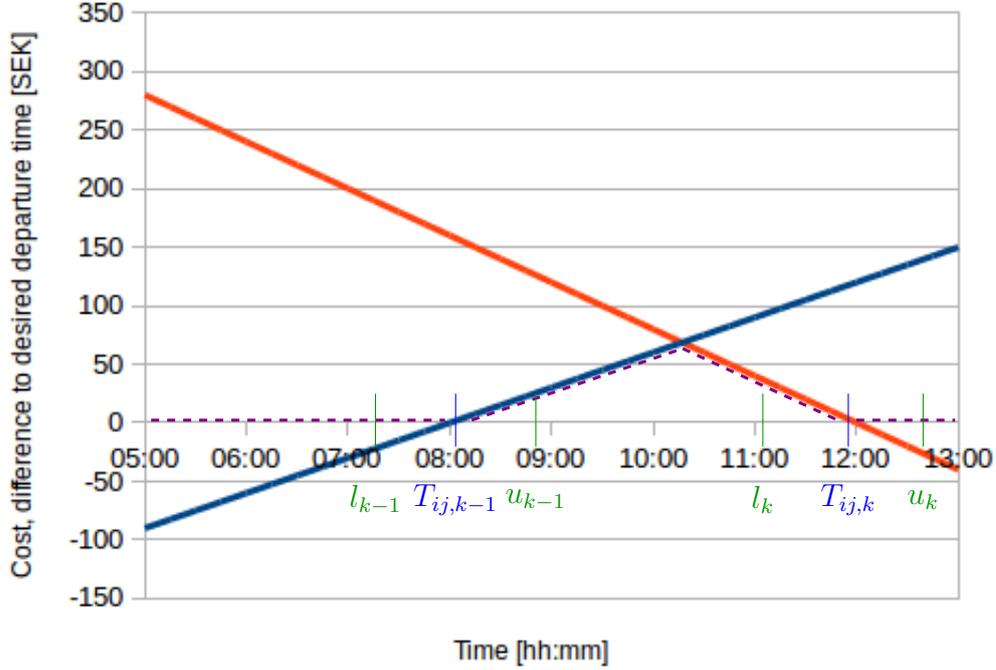


Figure 5: The cost situation between two consecutive departure. The blue line is the generalised cost for earlier departure than desired and the red is the cost of a later departure than desired. The dashed line is the cost included in the objective.

5.3 Model 3

Another option of restricting the binary indicator variables, while only looking at consecutive departures as in Model 2, is to use the constraint defined in Equation 19 and 20. These constraints indicate that either q_m is larger than $\tau_{ij,k}$ (constraint in Equation 19) or lower than $\tau_{ij,k}$. In this case, the amount of variables is the same as in Model 2.

$$\tau_{ij,k} - s_m \leq M z_{ij,k,q} \quad \forall i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (19)$$

$$s_m - \tau_{ij,k} \leq M(1 - z_{ij,k,q}) \quad \forall i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (20)$$

The constraints on $o_{ij,q}$ should then be matched with the $z_{ij,k,q}$ -variables such that the if q_m is lower than $\tau_{ij,k}$, the β -part should be used in the objective and vice versa. Then the constraints are:

$$o_{ij,q} \geq \alpha t_{ij,k} + \beta(T_{i,k+1} - q_m) - M(1 - z_{ij,k,q}), \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (21)$$

$$o_{ij,q} \geq \alpha t_{ij,k} + \gamma(q_m - T_{i,k+1}) - M z_{ij,k,q}, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (22)$$

The objective is the same as in Equation 13.

5.4 Model 4

Since we are restricting how much the departure time may vary (previously defined to (l_k, u_k) for departure k) we can find the minimum possible generalised cost of the γ -part for first train and the β -part for second train. The minimum generalised cost for each part is:

$$c_{\min}(T_{ij,k}) = \alpha t_{ij,k} + \gamma(l_{i,k} - T_{i,k}) \quad (23)$$

$$c_{\min}(T_{ij,k+1}) = \alpha t_{ij,k+1} + \beta(T_{i,k+1} - u_{i,k+1}) \quad (24)$$

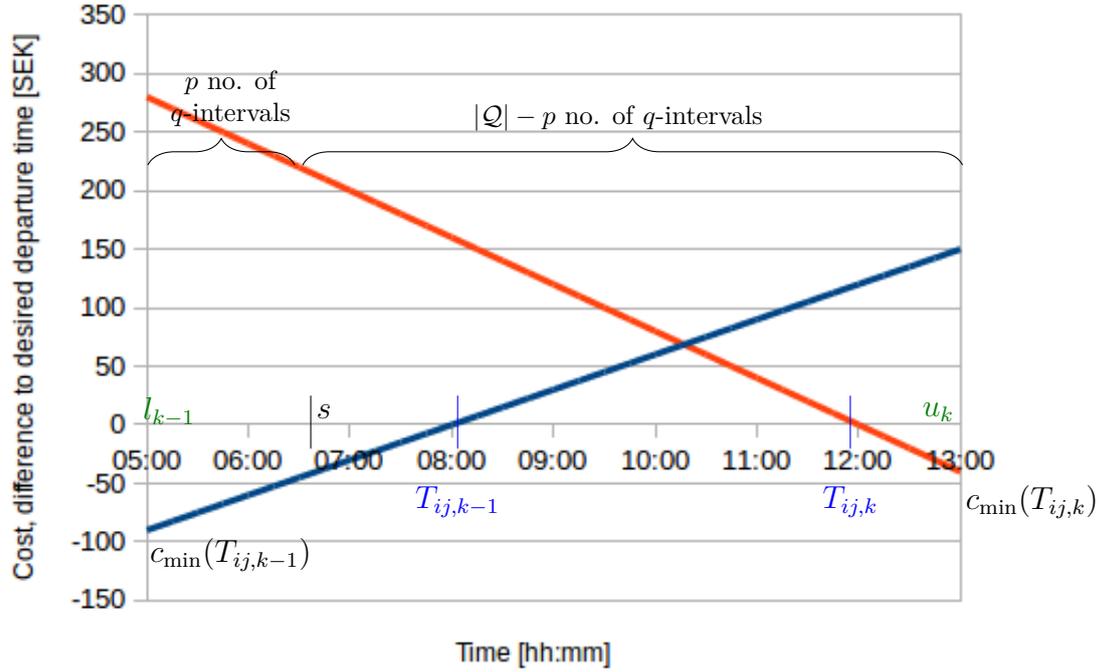


Figure 6: $c_{\min}(T_{ij,k-1})$ is the generalised cost at time l_{k-1} and $c_{\min}(T_{ij,k})$ is the value at time u_k .

Between two subsequent departures, there are only two options of which generalised cost there may be on an interval q . Either the cost from the earlier departure or the later departure. Regard point s in Figure 6. Since the length of an interval, Δ , is fix and if point s belongs to the p :th interval in $\mathcal{Q}_{ij,k}$ ($\mathcal{Q}_{ij,k}$ is the time intervals used to describe the passenger distribution), then we know the two possible options for the generalised cost. The generalised cost for all people desiring to start travelling in the point s is

$$\text{either } c_{\min}(T_{i,k-1}) + \gamma \cdot p \cdot \Delta \text{ or } c_{\min}(T_{i,k}) + \beta \cdot (|Q| - p) \cdot \Delta \quad (25)$$

This is modelled using the constraints in Equation 26 and 27, for all q -intervals with a $q_m \in (l_k, u_{k+1})$.

$$o_{ij,q} \geq c_{\min}(T_{i,k-1}) + \gamma \cdot i \cdot d \cdot z_{ij,k-1,q} \quad (26)$$

$$o_{ij,q} \geq c_{\min}(T_{i,k}) + \beta \cdot (Q - i) \cdot d \cdot (1 - z_{ij,k-1,q}) \quad (27)$$

No further restrictions on $z_{ij,k,q}$ are needed since the optimisation always will chose a solution minimising the travellers cost. Again, the objective is as in Equation 13.

5.5 Model 5

The maximum cost between two consecutive departures is always $\beta(T_{i,k+1} - \tau) = \gamma(\tau - T_{i,k})$. The main idea of this model is illustrated in Figure 7. If Δ is the q -interval length, then the cost in one interval closer to the k :th departure is then approximately $\gamma\Delta$ less than the maximal cost. One extra interval closer yields $2\gamma\Delta$ less than maximal cost. The same is true for a third interval closer to the k :th departure. On the other hand one interval closer to the $k + 1$:th departure is, using the same reasoning, $\beta\Delta$ less than the maximum. Introduce an indicator variable $z_{ij,k,q}$, similarly as in Equation 8, i.e. $z_{ij,k,q}$ is one if the q -interval contains τ . This is done by using the constraints defined in Equation 28 and 29. Constraint 29 allows only one $z_{ij,k,q}$ -variable to be one, and Constraint 28 forces $z_{ij,k,q}$ to one when its corresponding q -interval is containing $\tau_{ij,k}$.

$$q_i^{\min} + \sum_{q \in \mathcal{Q}_{ij,k}} p \cdot \Delta \cdot z_{ij,k,q} \leq \tau_{ij,k} \leq q_u^{\min} + \sum_{q \in \mathcal{Q}_{ij,k}} p \cdot \Delta \cdot z_{ij,k,q} \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}_{ij,k}, k \in \mathcal{K}_{ij} \quad (28)$$

$$\sum_{q \in \mathcal{Q}_{ij,k}} z_{ij,k,q} = 1 \quad \forall i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (29)$$

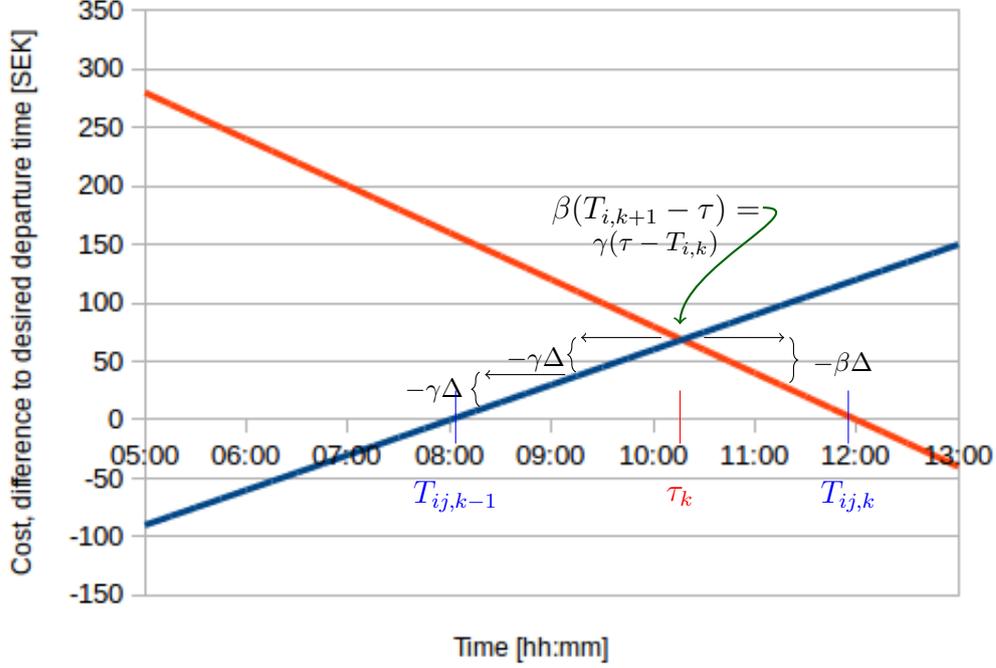


Figure 7:

If we know in which q -interval $\tau_{i,k}$ is contained, it is easy to calculate the cost in all intervals between the departures using the matrix below in Equation 30.

$$\mathbf{A} = \begin{pmatrix} 0 & -\beta\Delta & -2\beta\Delta & \dots & -(|\mathcal{Q}| - 1)\beta\Delta \\ -\gamma\Delta & 0 & -\beta\Delta & \dots & -(|\mathcal{Q}| - 2)\beta\Delta \\ -2\gamma\Delta & -\gamma\Delta & 0 & \dots & -(|\mathcal{Q}| - 3)\beta\Delta \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(|\mathcal{Q}| - 1)\gamma\Delta & -(|\mathcal{Q}| - 2)\gamma\Delta & -(|\mathcal{Q}| - 3)\gamma\Delta & \dots & 0 \end{pmatrix} \quad (30)$$

The rows indicate in which q -interval $\tau_{ij,k}$ is included and the columns indicate the impact on the maximum cost on the corresponding interval. If $\tau_{ij,k}$ is included in the third interval the lowest has the cost $\beta(T_{i,k+1} - \tau) - 2 * \gamma\Delta$, second lowest has cost $\beta(T_{i,k+1} - \tau) - \gamma\Delta$, third has cost $\beta(T_{i,k+1} - \tau)$, fourth has cost $\beta(T_{i,k+1} - \tau) - \beta\Delta$ etc. Constraints on $o_{ij,q}$ can then be defined as in Equation 31. The objective function is the same as Equation 13.

$$o_{ij,q} \geq \alpha t_{ij,k} + \beta(T_{i,k+1} - \tau_{ij,k}) - \sum_{q' \in \mathcal{Q}_{ij,k}} \mathbf{A}_{q,q'} z_{ij,k,q'}, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (31)$$

5.6 Model 6

Define again

$$c_{\min}(T_{ij,k}) = \alpha t_{ij,k} + \gamma(l_{i,k} - T_{i,k}) \quad (32)$$

$$c_{\min}(T_{ij,k+1}) = \alpha t_{ij,k+1} + \beta(T_{i,k+1} - u_{i,k+1}) \quad (33)$$

Every interval in the passenger distribution is Δ minutes long. Assume that $o_{ij,q}$ is the p :th interval in the timeline between l_{k-1} to u_k . The constraints on the $o_{ij,q}$ -variables can be redefined into

$$o_{ij,q} \geq c_{\min}(T_1) + \gamma \cdot p \cdot \Delta \cdot \sum \mathbf{Z}_{ij,k}, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (34)$$

$$o_{ij,q} \geq c_{\min}(T_2) + \beta \cdot (Q - p) \cdot \Delta \cdot (1 - \sum \mathbf{Z}_{ij,k}), \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (35)$$

where $\mathbf{Z}_{ij,k}$ is an upper diagonal matrix, containing the binary variables $z_{ij,k,q}$ stated in Equation 36.

$$\mathbf{Z}_{ij,k} = \begin{pmatrix} z_{ij,k,1} & z_{ij,k,2} & z_{ij,k,3} & \cdots & z_{ij,k,|\mathcal{Q}_{ij,k}|} \\ z_{ij,k,1} & z_{ij,k,2} & z_{ij,k,3} & \cdots & 0 \\ z_{ij,k,1} & z_{ij,k,2} & z_{ij,k,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{ij,k,1} & z_{ij,k,2} & 0 & \cdots & 0 \\ z_{ij,k,1} & 0 & 0 & \cdots & 0 \end{pmatrix} \quad \forall i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (36)$$

Also here, one $z_{ij,k,q}$ -variable have to be one.

$$\sum_{q \in \mathcal{Q}_{ij,k}} z_{ij,k,q} = 1, \quad \forall q \in \mathcal{Q}_{ij,k}, i, j \in \mathcal{I} (i \neq j), k \in \mathcal{K}_{ij} \quad (37)$$

This is a modification of Model 4. In Model 4 the vector $(z_{ij,k,1}, z_{ij,k,2}, z_{ij,k,3}, \dots, z_{ij,k,|\mathcal{Q}_{ij,k}|})$ will end up on the form $(1, 1, 1, \dots, 1, 0, \dots, 0)$. By the formulation of this problem, the vector structure will follow. This is a possible way to speed up the calculations. Again, the objective is as in Equation 13.

5.7 Passenger windows

In the case of a coarse departure pattern, a person might rather want to explore a different travel option than to take the train due to a long waiting time. Here the notion of *passenger windows* is introduced. The main idea is that there is a passenger window around a departure time where people choose the take the train. All other passengers are assumed to chose another transportation mode or decide to stay at home. The amount of these passengers is penalised in the objective. This is different to the previous models. The models in the previous sections tried to minimise the generalised cost for every passenger by minimising the deviation between desired travel times and the actual departure while minimising the travel time cost for each passenger. This model is trying to imitate the main idea of the previous models but using less computationally difficult constraints. The main goal of this model is to cover as many people as possible by inducing a cost for the passengers who chose not to take the train.

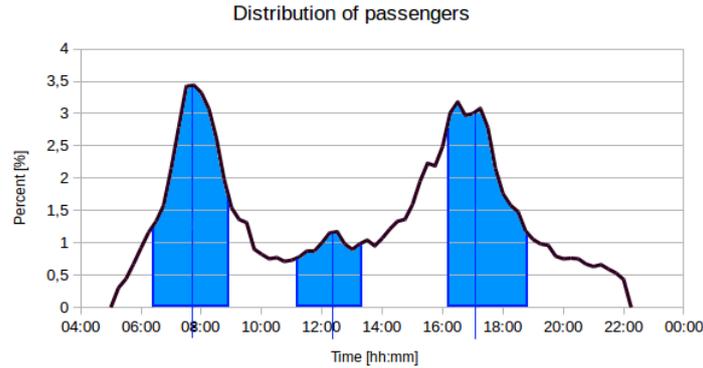


Figure 8: An illustration of the departure times and the passenger windows in bright blue.

The binary variable $z_{ij,k,q}$ is introduced. Let p_l and p_u be the amount of time people are willing to leave early or wait for the train, that is the lower and upper limit of the passenger window respectively. By the constraint in 38 and 39, $z_{ij,k,q}$ is 1 outside the passenger window.

$$T_k + p_u - q_m + Mz_{ij,k,q} \geq 0, \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}, k \in \mathcal{K}_{ij} \quad (38)$$

$$q_m - (T_k - p_l) + Mz_{ij,k,q} \geq 0, \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q}, k \in \mathcal{K}_{ij} \quad (39)$$

Let $x_{ij,q}$ be equal to 1 if the q -interval is not covered by a passenger window and $y_{ij,q}$ be the amount of passenger windows overlapping the time interval q . These variables can be defined by the constraint 40.

$$\sum_{k \in \mathcal{K}_{ij}} z_{ij,k,q} - x_{ij,q} + y_{ij,q} = 0, \quad \forall i, j \in \mathcal{I} (i \neq j), q \in \mathcal{Q} \quad (40)$$

The objective is now

$$\min \sum_{ij \in \mathcal{I} | i \neq j} \sum_{q \in \mathcal{Q}} (N_q x_{ij,q} + y_{ij,q}) \quad (41)$$

5.8 Summary of the models

The objective of Model 1-6 models is:

$$\min \sum_{ij \in \mathcal{I} | i \neq j} \sum_{q \in \mathcal{Q}_{ij}} N_q o_{ij,q} \quad (42)$$

The objective for the passenger window is:

$$\min \sum_{ij \in \mathcal{I} | i \neq j} \sum_{q \in \mathcal{Q}} (N_q x_{ij,q} + y_{ij,q}) \quad (43)$$

A binary decision variable $z_{ij,k,q}$ is introduced in every model. In model 1 and in the model describing the timewindows (PW-model) $z_{ij,k,q}$'s are defined for the entire timeline for all departures. In the other models (Model 2-6) $z_{ij,k,q}$'s are only defined for the timeline between the lowest allowed value for the earlier departure (l_k) and the largest allowed time for the later departure (u_{k+1}), between two subsequent departures.

The constraint equations for the models are specified in Table 3 below.

Model	Constrains
1	9-12
2	15-18
3	19-22
4	26-27
5	28-29, 31
6	35-34, 37
PW	38-40

Table 3: Summary of the constraints to the models

5.9 Iterative approach

All models yield a large set of variables. To further increase the speed without loosing too much accuracy some iterative approaches have been introduced to solve the problem. Previously, the allowed interval for the departure time to vary have been defined to (l_k, u_k) for departure k . Using large intervals also mean more possible conflicts on the tracks. Conflicts will give rise to a large amount of variables which introduce complexity and very large execution time. To decrease this amount of variables, an iterative approach can be made. This approach is described below.

1. Set up model and solve with small time windows.
2. Use the resulting value from 1 as initial values. Set new domains around the new solution and iterate.
3. Does the solution change? If yes, go to 2. Otherwise stop.

6 Test case

We have chosen to study the railway line between Hallsberg, Boxholm and Linköping displayed in Figure 9. This line consists of single and double track, stations and meeting stations. The traffic data are from 2014 and consist of a mixture of freight and passenger trains using these tracks and a total of 190 trains (94 regional). The data for the distribution of passengers provided by Östgötatrafiken show the amount of people registering their card at a station each hour. No information about destination is given. Hence there is no information about the amount of people changing trains. Therefore, OD-pairs where the passengers have to change train are not considered in the model.

The OD-pairs considered are between the stations on the following lines.

- Motala - Skänninge - Mjölby
- Boxholm - Mjölby - Mantorp - Vikingstad - Linköping

For each of these OD-pairs, a model for the generalised cost is set up for the regional trains. The other trains (freight and long-distance passenger trains) are solved using the priority criteria from the current timetable planning process at Trafikverket described in Section 2.

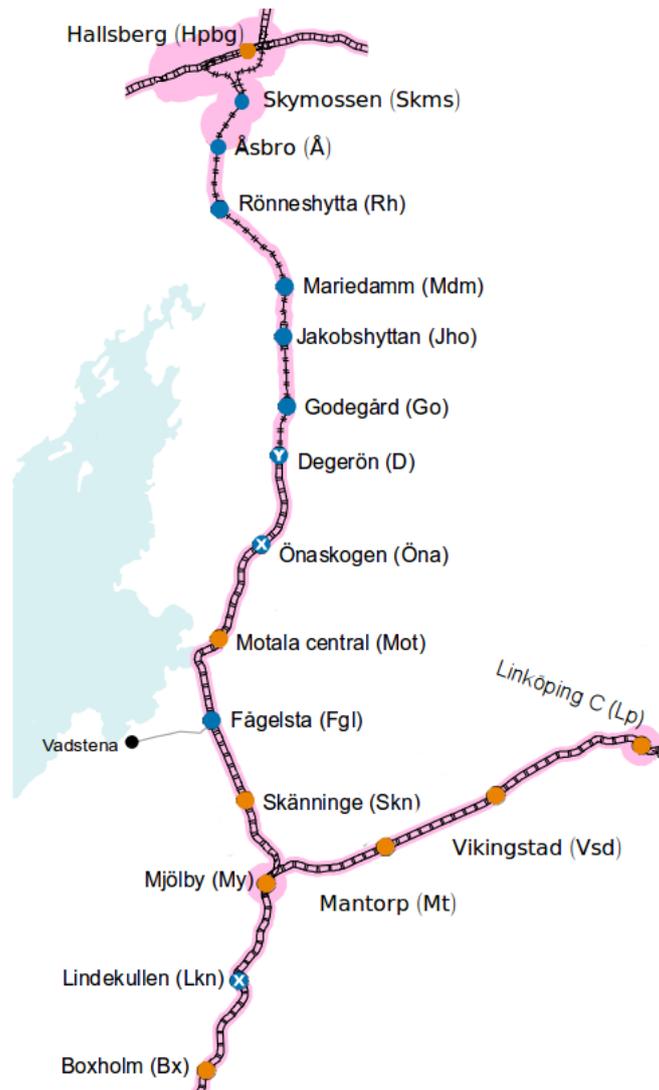


Figure 9: The line considered in the test case. The orange circles are the stations where a train may stop.

The passenger distributions between the larger stations are displayed in Figure 10. The passenger

distributions from or to the smaller stations are very small and not plotted, but are still considered in the optimisation.

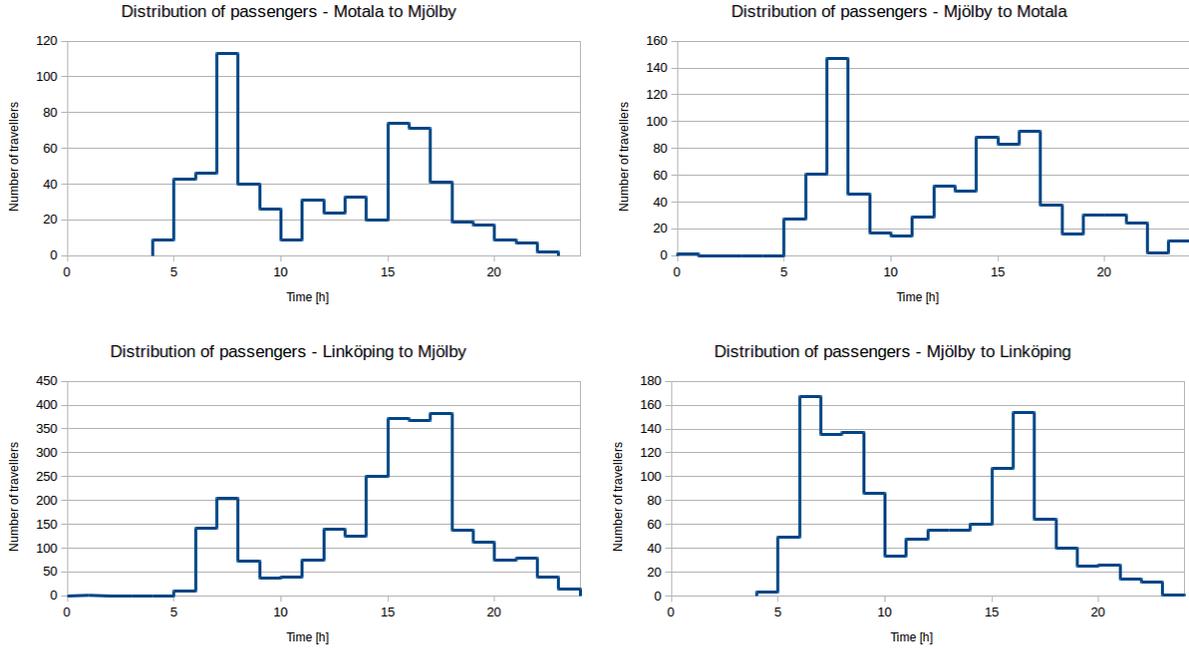


Figure 10: The passengerdistribution for the OD-pairs considered in the problem.

6.1 Details

The departure time is only allowed to vary ± 15 minutes from it's initial value. First an optimal value for only the regional trains is found. This is because we use this timetable as an initial value to the complete problem. Nine iterations are made for all models in the iterative approach described in Section 5.9 and only regional trains are considered when adjusting the timetable domains. The execution time limit in each iteration is 15 minutes. In the last optimisation the result from the previous iteration is used for the regional trains, and the timetable is used as an initial value for all other trains. The execution time for the last optimisation is very long, thus we only allow it to optimise for 45 minutes. The duality gap is then noted, as well as the time where the duality gap decrease under 1%.

6.2 Results

The results of the different models are stated in Table 4. The amount of variables generated for other traffic than regional (based on the priority criteria) is 1 731 in all cases. This is also the amount of constraints from the cost based on the priority criteria in all cases. The variables and constraints generated by the infrastructure constraints are included in the table. These should be the same in all cases, but may vary depending on the outcome of the first iterative part where the optimal for the regional trains is found. It was only model 5 which could not be solved for this test instance. The total preprocessing time is 2h 15min for all models. This dominates the execution time but the result does not vary between the models.

	Model	# variables	# constraints	Duality gap [%]	Execution time [min] (to 1% duality gap)
1	Infrastructure	14 405	32 870	0.56	41
	Generalised cost	17 983	50 528		
2	Infrastructure	14 604	33 383	0.14	2
	Generalised cost	1 483	1 980		
3	Infrastructure	14 415	32 877	0.15	0.1
	Generalised cost	1 114	9 143		
4	Infrastructure	14 742	33 734	0.51	41
	Generalised cost	1 170	1 637		
5	Infrastructure	-	-	-	-
	Generalised cost	-	-		
6	Infrastructure	14 586	33 453	0.29	40
	Generalised cost	1 340	1 462		
PW	Infrastructure	14 765	33 748	0.06	0.3
	Generalised cost	18 975	50 268		

Table 4: Resulting execution time for the different models described in Section 5.

7 Conclusion and Future work

To plan a timetable by minimising a generalised and operator cost is possible. A continuous work will be made on improving the execution time. This will however not be the only focus. A focus will also be put on how this model can be used to conclude framework agreements. Another issue not regarded in this article when planning welfare enhancing traffic is the crowding of trains. This aspect can be important on some lines and could provide a deeper insight to how the timetable should be planned.

7.1 Acknowledgement

This paper has been carried through within the research project "Economically effective track allocation", which is funded by Trafikverket (the Swedish Transport Administration) and affiliated to the research program KAJT.

References

- [1] Eva Barrena, David Canca, Leandro C. Coelho, and Gilbert Laporte. Single-line rail rapid transit timetabling under dynamic passenger demand. *Transportation Research Part B: Methodological*, 70:134 – 150, 2014.
- [2] Ralf Borndörfer, Martin Grötschel, Sascha Lukac, Kay Mitusch, Thomas Schlechte, Sören Schultz, and Andreas Tanner. An auctioning approach to railway slot allocation. Technical Report 05-45, ZIB, Takustr.7, 14195 Berlin, 2005.
- [3] Ralf Borndörfer, Heide Hoppmann, and Marika Karbstein. Timetabling and passenger routing in public transport. In *CASPT*, 2015.
- [4] U. Brännlund, P. O. Lindberg, A. Nõu, and J.-E. Nilsson. Railway timetabling using lagrangian relaxation. *Transportation Science*, 32(4):358–369, 1998.
- [5] Jonas Eliasson and Martin Aronsson. Samhällsekonomiskt effektiv tilldelning av järnvägskapacitet: Några synpunkter på trafikverkets nuvarande process. *CTS working paper*, 14, 2014.
- [6] José Luis Espinosa-Aranda, Ricardo García-Ródenas, María del Carmen Ramírez-Flores, María Luz López-García, and Eusebio Angulo. High-speed railway scheduling based on user preferences. *European Journal of Operational Research*, 246(3):772 – 786, 2015.
- [7] Sara Gestrelus, Martin Aronsson, and Markus Bolin. On the uniqueness of operation days and delivery commitment generation for train timetables. In *RailTokyo*, 2015.

- [8] April Kuo, Elise Miller-Hooks, and Hani S. Mahmassani. Freight train scheduling with elastic demand. *Transportation Research Part E: Logistics and Transportation Review*, 46(6):1057 – 1070, 2010.
- [9] Jan-Eric Nilsson. Allocation of track capacity: Experimental evidence on the use of priority auctioning in the railway industry. *International Journal of Industrial Organization*, 17(8):1139 – 1162, 1999.
- [10] Jan-Eric Nilsson. Towards a welfare enhancing process to manage railway infrastructure access. *Transportation Research Part A: Policy and Practice*, 36(5):419 – 436, 2002.
- [11] Huimin Niu and Xuesong Zhou. Optimizing urban rail timetable under time-dependent demand and oversaturated conditions. *Transportation Research Part C: Emerging Technologies*, 36:212 – 230, 2013.
- [12] Huimin Niu, Xuesong Zhou, and Ruhui Gao. Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints. *Transportation Research Part B: Methodological*, 76:117 – 135, 2015.
- [13] Jens Parbo, Otto Anker Nielsen, and Carlo Giacomo Prato. Improving passenger-oriented line planning for high frequent railway networks. In *CASPT*, 2015.
- [14] Tomáš Robenek, Shadi Sharif Azadeh, and Michel Bierlaire. Evaluation the quality of railway timetables from passenger point of view. In *CASPT*, 2015.
- [15] Marie Schmidt and Anita Schöbel. Timetabling with passenger routing. *OR Spectrum*, 37:75–97, 2015.
- [16] Anders Svensson. Rapport - införande av ramavtal för nyttjande av infrastrukturkapacitet på järnväg., 2012.
- [17] Trafikverket. Network statement, 2015.
- [18] Yihui Wang, Tao Tang, Bin Ning, Ton J.J. van den Boom, and Bart De Schutter. Passenger-demands-oriented train scheduling for an urban rail transit network. *Transportation Research Part C: Emerging Technologies*, 60:1 – 23, 2015.