Bounds on the Energy Consumption of Routings in Wireless Sensor Networks

Juan Alonso[1], Adam Dunkels[1,2], and Thiemo Voigt[1]

[1] SICS, Swedish Institute of Computer Science, Stockholm Sweden
[2] Dept. of Comp. Sci. and Engineering, Mälardalen University, Sweden
E-mail: first.lastname@sics.se
SICS Technical Report T2003:22
ISSN 1100-3154
ISRN:SICS-T–2003/22-SE

Abstract. Energy is one of the most important resources in wireless sensor networks. We use an idealized mathematical model to study the impact of routing on energy consumption. Our results are very general and, within the assumptions listed in Section 2, apply to arbitrary topologies, routings and radio energy models. We find bounds on the minimal and maximal energy routings will consume, and use them to bound the lifetime of the network. The bounds are sharp, and can be achieved in many situations of interest. We illustrate the theory with some examples.

1 Introduction

Recent technological advances have made the production of small and inexpensive wireless sensor devices possible, prompting a flurry of research and experiment. The starting point for this paper was a statement by Mainwaring et al. [1], one of the initial exciting deployments of this type of sensors. When discussing different routing algorithms, the authors write (in Section 6.2): "Although these methods provide factors of 2 to 3 times longer network operation, our application requires a factor of 100 times longer network operation...". We thought this was intriguing: What factor is reasonable to expect of a routing algorithm?

Typically, communication is the most expensive activity in terms of energy [2]. In this paper we focus on the energy consumed in communication (only the energy required to receive and send data is considered), regardless of the particular routing scheme used, and address questions such as: How much improvement in the lifespan of a network can be expected by changing only the routing algorithm? Which factors (as far as routing is concerned) affect the network's lifespan the most? How good is my favorite routing?

There is a vast literature relating energy consumption to routing, see for instance [4]-[6]. With few exceptions (see e.g. [3]) this previous work has concentrated on the performance of specific routing algorithms. Our main contribution is to provide fundamental limits to the energy consumption of routings, applicable regardless of the topology, routing algorithm, or radio energy model.

2 Assumptions on the sensor network

We assume the nodes in the network are of two types: sensor nodes and base nodes. Sensor nodes (or, simply, nodes) are low-energy and have very limited memory and processing capabilities, whereas base nodes are high-energy and have significantly more processing power and memory capacity than sensor nodes. We make the assumption that there is an underlying hierarchic architecture whereby the base nodes control the sensor nodes deciding, in particular, which routing to use. We use the term *routing* to denote a specific set of paths (or multi-paths) that packets take through a network. A routing is the result of the particular routing algorithm used.

The sensor nodes take readings and send them to the bases using other sensor nodes to reach them. This process is repeated until nodes die, eventually breaking connectivity and making the network non-operational. Another assumption is that during the whole process all nodes transmit at the same, constant power. No data aggregation is done in the network: all data gathered is sent unchanged to the base nodes.

3 Routings and their energy consumption

We model the network by a directed graph $G = (V, E)$. Given a link $e = (v, w)$, we let $\overline{e} = (w, v)$ denote the reverse link. We assume that if $e \in E$ then also $\overline{e} \in E$. We assume given a set $B \subseteq V$ of base nodes with $0 < |B| < |V|$.

The network operates with the following traffic pattern. For each iteration t, $1 \leq t \leq T$, every node sends a packet of a certain length to some base node. Informally, each way to do this is a routing. More formally, a *routing* is a vector

$$y = (y_e^t)_{1 \leq t \leq T,\, e \in E}$$

where y_e^t represents the total number of packets destined to some base node that are sent through e during the t:th iteration. Observe that we can think of the routing y as being a sequence $y = (y^1, \ldots, y^T)$, where y^t is the routing used during the t:th iteration. The only restriction we place on routings is that they should be *effective*, in the sense of not having loops. A *routing has no loops* if for all $1 \leq t \leq T$ the following holds: for every node, the directed path used to send its packet to a base node never visits a node more than once. Let

$$R^T = \{y = (y_e^t)_{1 \leq t \leq T,\, e \in E} \,|\, y \text{ is a routing with no loops}\}$$

The energy consumption of a routing y will be measured by the following cost function $f^T : R^T \to \mathbf{R}_+$:

$$f^T(y) = \max\nolimits_{v \in V} \{ \sum_{t=1}^{T} (\sum_{e \in I^v} \rho y_e^t + \sum_{e \in O^v} \tau y_e^t) \} \tag{1}$$

where ρ [resp. τ] is the cost for the reception [resp. transmission] of one packet, I^v is the set of incoming links of v, $I^v = \{(i,j) \in E | j = v\}$, and O^v is the set of outgoing links of v, $O^v = \{(i,j) \in E | i = v\}$. Thus, $f^T(y)$ measures the maximum energy used by nodes when transmitting and receiving according to routing y. When $T = 1$ we write simply $f(y)$.

4 How parsimonious is my favorite routing?

Set $m^T = \min_{y \in R^T} f^T(y)$, and $M^T = \max_{y \in R^T} f^T(y)$. We thus have, for an arbitrary routing $y \in R^T$, $m^T \leq f^T(y) \leq M^T$. When $T = 1$ we write simply m, M. In this section we find bounds on the size of the interval $[m^T, M^T]$. For this purpose, we partition the set of nodes into subsets $S_0, \ldots, S_n$ satisfying $V = S_0 \cup S_1 \cdots S_n$, $S_i \cap S_j = \emptyset$ for all $i \neq j$, and no S_i is empty. The definition of the S_i is as follows: $S_0 = B$, and for $i > 0$, S_i is the set of nodes can be reached in i hops, but not less than i hops, from some node in S_0 (i.e. S_i is the "sphere" of radius i around S_0). Thus,

$$|V| = |S_0| + |S_1| + \cdots + |S_n|$$

and all $|S_i| > 0$. Notice that $n \geq 1$, since $|B| < |V|$. Corresponding to the spheres S_i, there are "balls" of radius i, denoted B_i, and defined by $B_i = S_0 \cup \cdots \cup S_i$. It will be convenient to introduce the following notation: $s_i = |S_i|$, $b_i = |B_i|$, and $N = |V|$. Finally, for $i = 1, \ldots, n$, we set:

$$m_i = \frac{N - b_i}{s_i} \cdot \rho + \frac{N - b_i + s_i}{s_i} \cdot \tau \tag{2}$$

These constants are interesting because of Theorem 1 below. Inequalities (i)-(iii) of the theorem can be seen as providing fundamental limits to the possible amount of improvement in energy consumption that can be derived from changes in the routing algorithm, and benchmarks to compare your favorite routing(s) against. The strength of Theorem 1 derives from its generality, as its results apply to any graph, routing, and radio energy model.

Theorem 1. *With the notation above,*

i) $M^T \leq T \cdot [\rho \cdot (N - s_0 - 1) + \tau \cdot (N - s_0)] = T \cdot [(\rho + \tau) \cdot (N - s_0) - \rho]$
ii) $m^T \geq T \cdot \max\{m_1, \ldots, m_n\}$
iii) $M^T \leq s_1 \cdot m^T + T \cdot \rho \cdot (s_1 - 1)$
iv) $m_n = \tau$.

Proof. Notice first that (iv) follows immediately from the definitions, since $N = b_n$. Next, for arbitrary $v \in V$ and $y \in R^T$, notice that $\sum_{e \in I^v} y_e$ is the total number of packets received by v and, likewise, $\sum_{e \in O^v} y_e$ is the total number of packets transmitted by v. We claim these numbers cannot exceed the total number of packets being sent throughout the network at each iteration, i.e. $N - s_0$ packets transmitted and $N - s_0 - 1$ packets received. This is true because y has no loops and hence v will receive and send at most one packet for every non-base node. (i) follows immediately from this.

To prove (ii) it suffices to prove that $m^T \geq T \cdot m_i$, for all $1 \leq i \leq n$. The idea of the proof is to consider S_i as a bottleneck for nodes outside B_i trying to reach S_0. More formally, notice that in every routing, packets in $V \setminus B_{i-1}$ can only reach S_0 by either going through S_i (i.e. these packets originate outside of B_i and, hence, are both received and transmitted by some element of S_i) or by being transmitted by some node in S_i (i.e. these packets originate at S_i). Thus, the nodes in S_i must receive $N - b_i$ packets, and they must transmit $N - b_i + s_i$ packets . For every $y \in R$ we have:

$$\begin{aligned} f^T(y) &\geq \max\nolimits_{v \in S_i} \{ \sum_{t=1}^{T} (\sum_{e \in I^v} \rho \cdot y_e^t + \sum_{e \in O^v} \tau \cdot y_e^t) \} \\ &\geq \rho \cdot T \cdot \frac{N - b_i}{s_i} + \tau \cdot T \cdot \frac{N - b_i + s_i}{s_i} = T \cdot m_i \end{aligned} \tag{3}$$

Inequality (3) follows from Lemma 1 below. We show this for reception, the case for transmission being similar. By the discussion above, $\sum_{v \in S_i} (\sum_t \sum_e y_e^t) = T \cdot (N - b_i)$, so Lemma 1 gives $\max\{\sum_t \sum_e y_e^t | v \in S_i\} \geq T \cdot (N - b_i)/|S_i|$, as desired. Since $f^T(y) \geq T \cdot m_i$ holds for all $y \in R$, we obtain $m^T \geq T \cdot m_i$, as desired. Finally, using (i) and $T \cdot m_1 \leq m^T$, we get

$$\begin{aligned} M^T &\leq T \cdot \rho \cdot (N - s_0 - 1) + T \cdot \tau \cdot (N - s_0) \\ &= T \cdot \rho \cdot (N - b_1) + T \cdot \rho \cdot (s_1 - 1) + T \cdot \tau \cdot (N - s_0) \\ &= T \cdot m_1 \cdot s_1 + T \cdot \rho \cdot (s_1 - 1) \end{aligned} \tag{4}$$

This completes the proof of (iii) and of the theorem.

Lemma 1. *Let I denote a finite set. If $\sum_{i \in I} A_i = a$, then*

$$\max\{A_i | i \in I\} \geq \frac{a}{|I|}$$

Proof. Suppose, for contradiction, that the conclusion of the Lemma is false. Then $A_i < a/|I|$, for all $i \in I$. But then $\sum_I A_i < \sum_I a/|I| = a$, contradicting the hypothesis. This proves the lemma.

It is meaningful to distinguish two cases in Theorem 1, according to whether or not $n = 1$. We consider first the rather trivial case when $n = 1$, i.e. when all nodes are one hop away from a base node. Inequality (ii) in Theorem 1 reduces to $m^T \geq T \cdot \tau$, i.e. the minimal energy use after T iterations is the transmission cost times T. It is easy to find an optimal routing, i.e. a routing achieving this minimum: for each node, select a unique base node one hop away, and transmit the node's unique packet to the chosen base node; repeat T times. In this case, the upper bound for M^T, $T \cdot [(\rho+\tau) \cdot s_1 - \rho]$, can be achieved if, for instance, the non-base nodes can use each other to transmit their packets to a specified non-base node that receives all the packets minus its own, and transmits all s_1 packets to a base node. Summarizing:

Corollary 1. *In the special case when every node is only one hop away from a base node, we have:*

i) $M^T \leq T \cdot [(\rho+\tau) \cdot s_1 - \rho]$
ii) $m^T \geq T \cdot \tau$

Moreover, (ii) *is a sharp bound, i.e. there is a routing $y \in R^T$ with $f^T(y) = m^T$.*

One can obtain a nicer form for the coefficient in Theorem 1(iii), by moving from the above case, when S_0 is "thick", to the opposite case, when S_1 is "thin" in the sense that $s_1 \leq s_2 + \cdots + s_n$ or, equivalently, when $N - b_1 \geq s_1$. In this case Theorem 1 takes the neater form expressed in Corollary 2. The corollary says that, in terms of f^T-value, no routing is worse than $2 \cdot s_1 - 1$ times the best possible routing. This gives an answer to a question asked in Section 1: What factor is reasonable to expect of a routing algorithm?

Corollary 2. *Suppose the network contains many nodes at least two hops away from all base nodes, i.e. that $N - b_1 \geq s_1$. Then for all $T \geq 1$:*

$$M^T \leq (2 \cdot s_1 - 1) \cdot m^T$$

Proof. The condition $N - b_1 \geq s_1$ implies that m_1 in (2) satisfies $m_1 \geq \rho$. Together with equation (4) this gives $M^T \leq T \cdot m_1 \cdot (2 \cdot s_1 - 1) \leq (2 \cdot s_1 - 1) \cdot m^T$, since $T \cdot m_1 \leq m^T$ by Theorem 1(ii). This completes the proof.

5 Bounds on the lifetime of a sensor network

Suppose each node has the exact same amount EE of energy and we use a routing y in a traffic pattern consisting of T iterations. The network will be operational as long as $f^T(y) \leq EE$ and, to compute the break point[1], we set $f^T(y) = EE$, and let $T_{\max}$ denote the corresponding value of T. The next theorem bounds the life of the network in terms of $T_{\max}$.

[1] $T_{\max}$ is time to first node failure. When (iii) of Thm.1 is sharp, i.e. when, say, $m^T = T \cdot \max\{m_1, \ldots, m_n\} = T \cdot m_i$, all nodes of the sphere S_i will fail *at the same time*, breaking connectivity. That $m^T > T \cdot \max\{m_1, \ldots, m_n\}$ indicates that it is not possible to balance traffic evenly. This lends support to the conjecture that the portion(s) of the network depending on the corresponding dead node(s) to reach B, will be disconnected.

Theorem 2. *The maximum number $T_{\max}$ of readings a sensor network can take under the given assumptions is bounded as follows:*

$$\frac{EE}{(\rho+\tau)\cdot(N-s_0)-\rho} \leq T_{\max} \leq \frac{EE}{\max\{m_1,\ldots,m_n\}}$$

Proof. It follows from $f^{T_{\max}}(y) = EE$, that $m^{T_{\max}} \leq EE \leq M^{T_{\max}}$. Applying Theorem 1 to these inequalities, $T_{\max}\cdot\max\{m_1,\ldots,m_n\} \leq EE \leq T_{\max}\cdot[(\rho+\tau)\cdot(N-s_0)-\rho]$. Theorem 2 follows immediately from this.

6 Examples

The results of Sections 4 and 5 highlight the role of the spheres S_i in the longevity of a sensor network. Notice that the m_i (see equation (1)) decrease as s_i increases, suggesting that the larger the s_i are, the more one stands to gain by devising and implementing smart routing algorithms, i.e. those that exactly, or nearly so, achieve the minimum value m^T. This brings us to the question as to how sharp is bound (ii) of Theorem 1, which is also related to the question of whether increasing the s_i will always result in an increased lifespan for the network. Ways to increase s_i are, e.g. to place more sensor nodes in the vicinity of the given ones, and/or to increase the transmission range.

Theorems 1 and 2 show that $max\{m_1,\ldots,m_n\}$ is the theoretical minimum for the energy consumption of routings. Examples 1 and 5 below illustrate two different ways in which the theoretical value can fail to be achieved, i.e. $m \neq max\{m_i\}$. In both cases it is impossible to balance the load evenly among the nodes of S_1. Example 2 shows that $max\{m_i\}$ need not equal m_1. Example 3 is a simple case to illustrate that using the same routing at each iteration can be far from optimal. Example 4 is characteristic for rectangular networks with "judicious" choice of base nodes, while Example 5 shows that size and placement of the base are important parameters in order to obtain the most of the network.

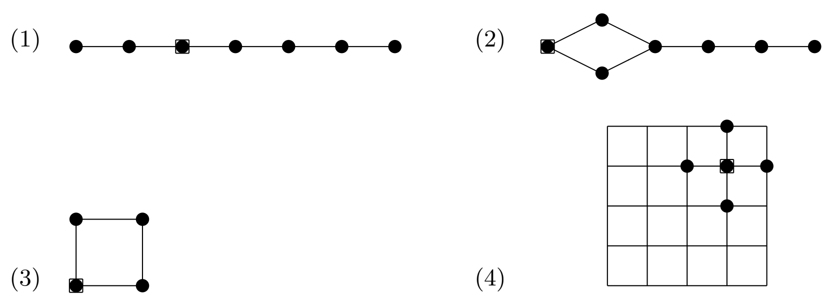

Fig. 1. Networks (the square node is the base node)

1. Consider network (1) of Fig.1, where B consists of the square node. The network consists of two trees rooted at the base node. In this case $m_1 = 2\rho + 3\tau$, $m_2 = m_3 = \rho + 2\tau$, $m_4 = \tau$, and $max\{m_i\} = m_1$. However, $m = f(y) = 3\rho + 4\tau > m_1$, where y is the only routing without loops on each of the rooted trees.
2. The network in Fig.1(2) illustrates a case where $max\{m_i\} \neq m_1$. In this case, according to equation (2), $m_1 = 2\rho + 3\tau = m_3$, $m_2 = 3\rho + 4\tau$, $m_4 = \rho + 2\tau$, and $m_5 = \tau$. It is easy to see that $m = m_2 = max\{m_i\}$.
3. For the graph in Fig.1(3), $m_1 = (1/2)\rho + (3/2)\tau$ and $m_2 = \tau$. In this case $m_1 = max\{m_i\}$, but $m^T \neq T{\cdot}m_1$. However, if we let $y = (y_1, y_2, y_1, y_2, \ldots)$, then $f^T(y) = m^T$ "asymptotically", in the sense that this equality is true for all even values of T.
4. The graph in Fig.1(4) consists of 25 nodes, one for each intersection. The figure emphasizes only B_1. For this graph, $m_1 = 5\rho + 6\tau$, $m_2 = (7/3)\rho + (10/3)\tau$, $m_3 = (4/3)\rho + (7/3)\tau$, $m_4 = (3/5)\rho + (8/5)\tau$,$m_5 = (1/2)\rho + (3/2)\tau$, and $m_6 = \tau$. In this case $m = m_1 = max\{m_i\}$.
5. Consider again the graph in Fig.1(4), but this time with five base nodes consisting of the whole fourth row (from, say, top to bottom). The sphere S_1 consists then of ten nodes, namely rows three and five. In this case $max\{m_i\} < m$ since one cannot take advantage of all ten nodes to balance the traffic load.

7 Conclusions

In this paper, using an idealized mathematical model, we have quantified the fundamental role played by the spheres of different radii in determining the energy consumption of routings in networks satisfying the assumptions of Section 2. We have computed the theoretical optimal value, and applied this to bound the lifetime of sensor networks. Finally, we have given some examples to illustrate the theory we developed.

References

[1] A. Mainwaring, J. Polastre, R. Szewczyk, D. Culler and J. Anderson "Wireless sensor networks for habitat monitoring" in ACM International Workshop on Wireless Sensor Networks and Applications (WSNA'02) 2002.

[2] V. Raghunathan, C. Schurgers and M. Shrivastava "Energy aware wireless microsensor networks" IEEE Signal Processing Magazine, Vol. 19 pp. 40-50 2002.

[3] M. Bhardwaj and A. Chandrakasan "Bounding the Lifetime of Sensor Networks via Optimal Role Assignment" Proceedings of INFOCOM 2002 pp. 1587-1596 2002.

[4] J.H. Chang, L. Tassiulas "Energy conserving routing in wireless adhoc networks" IEEE INFOCOM 2000 2000.

[5] Y. Xu, J. Heidemann, and D. Estrin "Adaptive Energy-Conserving Routing for Multihop Ad Hoc Networks" Research Report 527, USC/Information Sciences Institute 2000.

[6] W.R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor" HICSS'00. 2000.