precondition when/2, where the starting time of the activities, based on the global state, are set equal to the activities' first possible starting time), there is just one plan, which is the plan presented above.

9.6.5 Priority Among Activities

It is easy to specify different priorities among the activities to implement different scheduling strategies. A simple example is to extend the top level strategy for the scheduling phase with a call to a priority relation, which specifies some ordering among the activities.

```
priority(akt(1,_,_)).
priority(akt(X,_,_)) :- X \=< 1.

schedule <=
  (ready_sch <- true), % All activities scheduled?
  (\((\text{priority}(\text{Act}) \rightarrow \\
    \text{search}(\text{akt}(\_\_\_),\_I, \\
    \text{d_left}(\_\_,\_I, \\
    a\_\_left(\_\_,\_I, \\
    weak_all(\text{akt}(\_\_\_), \\
    weak_all(\text{performed}(\_\_\_,\_\_\_),\text{schedule_pre})), \\
    a\_\_left(\_\_,\_I, \\
    weak_all(\text{state}(\_\_\_), \\
    weak_all(\text{akt}(\_\_\_),\text{schedule_calc})),

    v\_\_left_all(\text{schedule_post}(\text{A \- C}))))))
  % <- (schedule \rightarrow \text{A \- C}),
  false).
```

Note that the only extension to the original schedule strategy is the call to `priority(Act)`, and that `Act` is passed to the first `d_left/3` inference rule call. Compare the following plan and resource graph with the first ones in section 9.6.2:

We can note that the activity `Work on ground` has got the maximum allocation of resources (10 teams with 2 persons in each), and that the other activities in parallel with `Work on ground` have to manage with the resources that are left. Notably, the activity `Electric installation` has got only one team allocated.
10. Discussion

We have described a system which spans the set of all possible plans given a design. The system has no expert knowledge about what plans are good and what plans are poor, and which method to apply in a given situation. But the system has expert knowledge about what different systematical ways there are to solve a complex task. The next step would be to add expert knowledge about what plans should be preferred over other plans, for example, if there are several methods to choose among, the system should be able to pick one which seems better based on the overall information. As for the described system, this additional information should be editable by the user, to reflect his personal knowledge and preferences. There are different ways to do that, but one promising way is to use decision theory, which is used to calculate a priority when there exist choices, for example resource allocation to competing activities. By this an ordering of the possible plans can be imposed, which in turn guides both the method choosing algorithm and the scheduling algorithm. Whether it is possible to find priority functions for preference of methods and resource allocation, and if the expressiveness of them is enough for experts in the field, is not clear, but to us it seems the most promising approach.

The basis of the planning theory described here is very basic. It should be possible to generalize it to other domains as well. The basic entities of our planning theory are activities which are applied to and change a global state when resources are allocated to it. A scheduler reasons about when and with what resources different activities are allocated, as soon as the activities are known. To generate the activities, methods are used to systematically group activities, that solve a more complex task. The overall goal is expressed as the design of a building, representing a state change. We hope to be able to apply these ideas to some other domain as well, to see what parts are general and what are parts are specific for this domain.

There is one known logical deficiency that ought to be mentioned. The application described here lacks the possibility to add constraints on the global state. For example, assume that there is one activity $a'$ changing the state into a state $s'$, and then an activity $a''$ that changes the state before state $s'$ into a state $s''$, from which $s'$ cannot be reached. This means that $s'$ must contain the information that all states before $s'$ must satisfy some constraints that should not violate it.

\[ a'' \quad a' \]

\[ s \quad s'' \quad s' \]

It is possible to order the activities for the scheduling phase by imposing conditions in the preconditions, that cause such critical activities as $a'$ to be performed after $a''$, when the state $s''$ is known. This is quite natural, since one often delays an activity $a'$ that is known to be dependent on another activity $a''$ until its duration and place in time are calculated.

To be able to manage the search space, the user must have ways to guide the generation of plans to obtain plans he is interested in. Some such guiding possibilities have been sketched in the paper, e.g. partially specifying what plans the choice-of-method phase should produce, interaction with the user to fill in values in the preparation phase between the choice-of-method phase and the scheduling phase, and one could also add user interaction in the scheduling phase when there are several ways to solve the produced
constraints. The user interaction could take place in several ways, not only as events generated by the program as sketched in section 9.6.4, but also give the user possibilities to change the control level, as suggested in section 9.6.5. This gives him the possibility to write priority functions for different kinds of situations, the way he himself prioritizes some solution over another.

Since the programmer has got hold of the object level structure in each deduction step, if he wants to, it is easy to extend and manipulate the search behaviour in GCLA. The programmer is not forced to specify for each step what is to be done, but he has the opportunity. As he gathers more knowledge about what to do and when to do it, only the control level needs to be extended, as it is completely separated from the declarative definition. Since it is easy to build modular search strategies, the programmer can change one particular search strategy without affecting the rest, thus preserving the overall structure. To the programmers help there are also tools for diagnosing the behaviour of an application (see the performance package of GCLA in [Aro91a]).

There is a difference of a factor of 5 to 10 in execution time between this application and a Prolog application of the same system, in favour of the Prolog application. In GCLA's favour it must be said that we feel that we have a clearer understanding of what we are doing in GCLA. It is easier to change parts of the system, or to add some further functionality to the system. Also, we feel that the clear distinction between declarative knowledge, in this case methods and activities, and procedural knowledge, here how to plan using the methods and activities, has increased the ability for us to reason about what is declarative during the development of the application, and what the interesting possibilities there are to use that declarative knowledge. For example, the ability to reason about all objects satisfying a partially instantiated term through a \((V_{ar})\)-terms got its solution when we turned from Prolog to GCLA. In Prolog we did not have the underlying framework to sort things out, while in GCLA the way of thinking about those partially instantiated terms as a partial inductive definition helped us to sort the different problems out. This is not to say that the application cannot be implemented in any language (Prolog in particular since GCLA is implemented on top of Prolog), but it is to say that the higher level of the GCLA language helped us to reason on a higher level, which was what was needed in order to understand the nature and solutions of the high level problems.

Note that apart from arithmetic rules and some simple new rules, the whole control part consists of strategies, and thus is a 'clean' GCLA program, without any obscure things done hidden in new, very specialized rules. We think that one should try to stick to the general GCLA rules as much as possible until one knows what one is doing, i.e. has understood the problem at hand, and then, gain efficiency by introducing more specialized rules. For the application described in this paper, we are convinced that efficiency could be increased by some new rules, which would implement some of the things that are now handled by strategies. For example, all weakening could be replaced by a rule, which performs all weakening in a proviso.

We are planning a continuation of this project, and we will then perform the next step of the development methodology described in [Aro92], which would be to write new rules of the kind described above, followed by more specialized provisos. With these improvements, we would gain efficiency, and still know what we are doing. The system described in this paper stands as a model for the next version.

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Appendix A:
A Complete Listing of the Example Building

The example building constitutes the following design database

```plaintext
%% Patches to simulate a design database consisting of these elements as well as
%% the supporting parts below.
design([1.0, a()], [1.0, a()], [area(30000)]).
design([3.0, a()], [1.0, a()], [area(3000)]).
design([3.4, a(), [1.2, a()], a(), [area(288)], length(n(24)), width(n(12))].
design([3.6, a()], [1.1, a()], [area(24)], [area(1150)]).
design([3.6, a()], [1.2, a()], [area(24)], [area(1150)]).
design([5.0, a(), [1.5, a()], a(), [area(1150)]).
design([6.0, a(), [1.5, a()], a(), [area(1150)]).%
```

%% Supporting parts
design([3.3, l], [1.1, 0001], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(24)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(14.4)), relative_height(n(1))).
design([3.3, l], [1.1, 0002], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(24)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(14.4)), relative_height(n(1))).
design([3.3, l], [1.1, 0003], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(1))).
design([3.3, l], [1.1, 0004], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(1))).
design([3.3, l], [1.1, 0005], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(1))).
design([3.3, l], [1.1, 0006], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(1))).
design([3.3, l], [1.2, 0007], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(24)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(14.4)), relative_height(n(4))).
design([3.3, l], [1.2, 0008], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(24)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(4))).
design([3.3, l], [1.2, 0009], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(4))).
design([3.3, l], [1.2, 0010], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(4))).
design([3.3, l], [1.2, 0011], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(4))).
design([3.3, l], [1.2, 0012], [e, 3, 3, l, 1],
[grid([11, l]), construction_weight_kg_per_sqm(n(15)), single_layer,
length(n(12)), height(n(3)), thickness(n(0.20)), area(n(12))],
volume(n(7.2)), relative_height(n(4))).
```

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Appendix B:
A Complete Listing of the Example Methods and Section Divisions

The Object Level Code
plan(N,A,X) <= expand(X,Y) -> pl(N,A,Y).

expand(\{\},\).
expand((X,Y),[X1,Y1]) <= expand(X,X1),expand(Y,Y1).
expand(pl(N,A,X),Z) <= plan(N,A,X) -> Z.
expand(X,X) \& X = [\_\_\_] X \& \_ \& X = pl(\_\_\_,\_) <= expand_one(\_\_\_,X) -> X1.

max_resources(\_\_\_,n(30)).
expand_one(1, X) <- X.
expand_one(2, X) <- m(_, _, X).
expand_one(N, X) # (N \= 1, N \= 2) <- e(_, _, X).

member(X, [X1 | _]).
member(X, [X1 | R]) # (f(X, X1) \= f(W, W)) <- member(X, R).

%%% Method DB

m(0, [], activity(0, [a(V1), a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res)) <- plan(main_groups,
activity(0, [a(V1), a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res),
activity(3, [3, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res),
activity(81, [5, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % installations, HVAC
activity(83, [6, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % el. installations
activity(4, [1, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res)).

m(1, [], activity(3, [3, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res)) <- plan(main_groups_of_building,
activity(3, [3, a(V2), a(V3)], [V4, V5, V6], _, Tst, Tst, Res),
activity(2, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % basis
activity(33, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % supporting parts
activity(48, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % outer roof
activity(66, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % outer walls
activity(7, [6, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % rooming in
activity(66, [6, a(V3)], [V4, V5, V6], _, Tst, Tst, Res)).

m(2, [], activity(33, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res)) <- plan(supporting_parts,
activity(33, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res),
activity(3, [3, a(V3)], [V4, V5, V6], _, Tst, Tst, Res), % slab on ground = slab 0
m(3, [], activity(33, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res)) # (V5 = \= 0, V5 = a(_)) <- plan(supporting_parts,
activity(33, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res),
activity(3, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res), % supporting walls
activity(3, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res), % pillars
activity(3, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res), % supporting slabs
activity(3, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res), % supporting parts of stairs
activity(3, [3, a(V3)], [V4, V5, a(V6)], _, Tst, Tst, Res)).

%%% Partition DB

e(1, building_partitioning,
activity(X, [3, V2, V3], [a(V4), V5, a(V6)], Bsab, Tst, Tst, Res)) <- bagof([V2, V3, Bsab2], Hs,
  (design([3, V2, V3], [Hs, V5, a(V6)], Bsab2, a(_)), number(n(Hs))), Temp),
  sort(Temp, List1),
  bagof([Tst, Tst, RES], activity(X, [3, V2, V3], [V4, V5, a(V6)], Bsab2, Tst, Tst, RES),
  member(V4, List1), List).

-> plan(building_partitioning,
activity(X, [3, V2, V3], [a(V4), V5, a(V6)], Bsab2, Tst, Tst, Res, List).

e(2, floor_partitioning,
activity(X, [3, V2, V3], [V4, a(V5), a(V6)],
  Bsab, Tst, Tst, Res)) <- bagof([V2, V3, Bsab], Ven,
  (design([3, V2, V3], [V4, Van, a(V6)], Bsab, a(_)), number(n(Van))), Temp),
  sort(Temp, List1),
  bagof([Tst, Tst, RES], activity(X, [3, V2, V3], [V4, V5, a(V6)], Bsab, Tst, Tst, RES),
  member(V5, List1), List).

-> plan(floor_partitioning,
activity(X, [3, V2, V3], [V4, a(V5), a(V6)], Bsab, Tst, Tst, Res, List).

The Control Code

%%% From rules.rul, added that B should not be false, and if true
%%% then do not execute the sequent (P \= true), succeed at once.
d_right(C, CT) <-
  atom(C),
  clause(C, B),

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B \iff \text{false},
(B \iff \text{true}, (PT \iff (P \iff B)))
\iff (P \iff C).

%%% Choice-of-method phase
\text{a_left2}(A \iff C1), I, PT, PT1, left) \iff
\text{data}(C1) \iff (I@[(A \iff C1) \iff _] \iff _).
\text{a_left2}(A \iff C1), I, PT, PT1, left) \iff
\text{a_left1}(X, I, PT, PT1).
\text{a_left1}(A \iff C1), I, PT, PT1, right) \iff
\text{not(data}(C1)) \iff (I@![A \iff C1] \iff _) \iff _).
\text{a_left2}(A \iff C1), I, PT, PT1, right) \iff
\text{a_left}(X, I, PT, PT1).
\text{a_left1}(A \iff C1), I, PT, PT1, left) \iff
\text{(PT1 \iff (I@[C1][Y] \iff C)),
(PT \iff (I@[Y] \iff A))
\iff (I@[A \iff C1][Y] \iff C).
\text{d_right1}(C, PT) \iff \text{not(function(C,plan, _)), not(function(C, pl, _)) \iff (_ \iff C))}.
\text{d_right1}(C, PT) \iff \text{d_right}(C, PT).
\text{d_left1}(C, I, PT) \iff
\text{(not(function(C, pi, _)),
not(function(C, activity, _)),
not(function(C, state, _))}
\iff (I@[C] \iff _) \iff _).
\text{d_left1}(T, X, PT) \iff
\text{atom}(T),
definiens(T, Dp, N),
N > 0,
(PT \iff (X@[Dp][Y] \iff C))
\iff (X@[T][Y] \iff C).
\text{sort_right} \iff
\text{sort}(X, Y) \iff
(I \iff \text{sort}(X, Y)).
\text{findall_right}(PT) \iff
\text{lift_from_a}(B, B1, [], [],
(i([A], PT"Ass^\text{B1}"(PT \iff (Ass \iff B1))) \iff C)
\iff (Ass \iff \text{findall}(A, B, C)).
\text{bagof_right}(PT) \iff
\text{lift_from_a}(B, B1, [], [], Vars),
\text{append}(EVars, Vars, Vars1),
(i([A], Vars1"PT"(PT \iff (Ass \iff B1))) \iff C)
\iff (Ass \iff \text{bagof}(EVars, A, B, C)).
\text{axiom1}(T, C, I) \iff
\text{data}(T)
\iff (I@[T] \iff C).
\text{axiom1}(T, C, I) \iff
\text{axiom}(T, C, I).
\text{lift_from_a}(V, V1, L, L) \iff \text{var}(V).
\text{lift_from_a}(A, V, L, [V \iff L]) \iff \text{functor}(A, a, l), A = a(V).
\text{lift_from_a}(A, V, L, L) \iff \text{atomic}(A).
\text{lift_from_a}(X, [F1][R1], L, L2) \iff \text{nonvar}(X), X = [F1][R1],
\text{lift_from_a}(F, F1, L, L1),
\text{lift_from_a}(R, R1, L, L2),
\text{lift_from_a}(Str, Str1, L, L1) \iff \text{nonvar}(Str),
Str =..[S][A], S =..', S =.. a, A =.. [],
\text{lift_from_a}(A, A, L, L1),
Str1 =..[S][A1].
\text{data}(X) \iff \text{functor}(X, pl, _).
\text{data}(X) \iff \text{functor}(X, activity, _).
number_right <=
    number(C) ->
    (\_ \= number(n(C))).

right1(PT) <=
    sort_right,
    finall_right(PT),
    begof_right(PT),
    number_right,
    v_right(_,PT,PT),
    a_right(_,PT),
    c_right(_,PT),
    true_right,
    d_right1(_,PT).
left1(PT) <=
    v_left(_,PT),
    a_left2(_,PT,PT,\_),
    c_left(_,PT,PT),
    d_left1(_,PT),
    pi_left(_,PT),
    false_left_.

plan <- axiom1(_,r_),right1(plan),left1(plan).

Appendix C:
A Complete Listing of the Intermediate Step

The Object Level Code

flatten_act([pl(N,activity[A,B,C,D,E,F,Res],L]),[{E,F}]) <=
    cons(akt(-1,activity[A,B,C,D,E,F,Res],times(E,F,List)),
          flatten_act(L,List)).
flatten_act([pl(N,activity[A,B,C,D,E,G,Res],L),F[R],[{E,G|R1}]) <=
    cons(akt(-1,activity[A,B,C,D,E,G,Res],times(E,G,List)),
          append(flatten_act(L,List),flatten_act([F[R],R1]))).
flatten_act([activity(A,B,C,D,E,G,Res),F[R],[{E,G|R1}]) <=
    cons(akt(A,activity(A,B,C,D,E,G,Res),N),flatten_act([F|R],R1)).
flatten_act([activity(A,B,C,D,E,G,Res),[]]) <=
    cons(akt(A,activity(A,B,C,D,E,G,Res),[]),[]).

append([],L) <= L.
append([F[R],L] <= cons(F,append(R,L)).
append(X,Y) #{X \= [\_],X \= [\_\_]} <=
    {{X \= Z},Z\= [\_];Z\= [\_\_]} -> append(Z,Y).

The Control Level

%%%% Intermediate step, convert from tree structure to flat structure.
%%%% Also, keep track of and collect time slots of each activity,
%%%% so that superactivities have a list of their subactivities
%%%% time slots.

flattened <-
    axiom_flatten(_,r_,r_).
    left_flatten(flattened),
    right_flatten(flattened).

axiom_flatten(T,C,I) <=
    {functor(T,akt,3) ;
     functor(T,[\_],0) ;
     functor(T,'+',2)}
    -> (\#[T] \= - C).
axiom_flatten(T,C,I) <= axiom(T,C,I).

d_right_flatten(C,PT) <= functor(C,\_2) -> (\_ \= C).
    d_right_flatten(C,PT) <= d_right(C,PT).
right_flatten(PT) <=
    true_right,
    d_right_flatten(_, PT),
    o_right(_, PT),
    a_right(_, PT),
    v_right(_, PT, PT).

left_flatten(PT) <=
    d_left_flatten(C, I, PT),
    a_left(_, _, PT, PT).

d_left_flatten(A, I, PT) <=
    (functor(A, flatten_act, 2) ;
     functor(A, append, 2) ;
     functor(A, cons, 2))
    -> (I@[A_] \-= \_).

d_left_flatten(A, B, PT) <= d_left(A, B, PT).

Appendix D:
A Complete Listing of the Example Activities

The Object Level Code

akt(0, activity(0, [a(V1), a(V2), a(V3)], [V4, V5, V6], B, Time, End, NT), project) <=
  when(started(openning_start, _), Time) ->
    (area([3, 4], [a(1), a(1), a(1)]) -> n(TotArea)),
    (area([3, 4], [a(1), 0, a(1)]) -> n(BasArea)),
    (defun((time_formula -> n(9) * n(TotArea))),
     (defun((building_area -> n(Area))),
      (defun((team_size(general_workers) -> n(2))),
       (defun((place_coeff -> n(1.0))),
        (density(general_workers) -> (n(BasArea) / n(25))))
        -> (get_manpower(team_size(general_workers), density(general_workers), NT)
         -> n(Res))),
        (consume(T, n(Res), time_formula, End,
         activity(0, [a(V1), a(V2), a(V3)], [V4, V5, V6], B, Time, End, NT)
         -> quote(Act))
         -> (change(started([a(V1), a(V2), a(V3)], [V4, V5, V6], B), End, Act))).

akt(1, activity(1, [1, a(V2), a(V3)], [V4, V5, V6], B, Time, End, NT), work_on_ground) <=
  when(started([3, a(V2), a(V3)], [V4, V5, V6], a(_), Time) ->
    (area([1, a(V2), a(V3)], [a(1), 0, a(1)]) -> n(Area)),
    (defun((time_formula -> ground_area * n(0.2))),
     (defun((building_area -> n(Area))),
      (defun((place_coeff -> n(1.0))),
       (density(gardenworkers) -> (ground_area / n(300))))
       -> (get_manpower(team_size(gardenworkers), density(gardenworkers), NT) -> n(Res)),
       (consume(Time, n(Res), time_formula, End,
         activity(1, [1, a(V2), a(V3)], [V4, V5, V6], B, Time, End, NT) -> quote(Act))
         -> (change(started([1, a(V2), a(V3)], [V4, V5, V6], B), End, Act)).

akt(3, activity(3, [3, a(V2), a(V3)], [V4, a(V5), a(V6)], B, Time, End, NT), actual_building) <=
  when(started(openning_start, _), Time) ->
    (area([3, 4], [V4, 0, a(V6)]) -> n(BasArea)),
    (area([3, 4], [V4, a(V5), a(V6)]) -> n(TotArea)),
    (defun((time_formula -> n(6) * n(TotArea))),
     (defun((building_area -> n(Area))),
      (defun((place_coeff -> n(2))),
       (density(gardeners) -> n(Res))),
       (consume(Time, n(Res), time_formula, End,
         activity(3, [3, a(V2), a(V3)], [V4, a(V5), a(V6)], B, Time, End, NT)
         -> quote(Act))))

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akt(3, activity(3, [a(V2), a(V3)], [V4, 0, a(V6)], B, Time, End, NT),
    actual_building_palace) <=
when(started(official_start, _), Time) ->
    (area(area([3, 3, 4], [V4, 0, a(V6)])) -> n(Area)),
    (defn(timeformula -> n(6) * building_area),
     defn(bldg_area -> n(Area)),
     defn(teamsize(general_workers) -> n(2)),
     defn(density(general_workers) -> (building_area / n(25))),
     get_manpower(teamsize(general_workers), density(general_workers), NT)
     -> n(Res)),
    (consume(Time, n(Res), timeformula, End, activity(3, [a(V2), a(V3)], [V4, 0, a(V6)], B, Time, End, NT)))
    -> quote(Act))
-> (change(started([3, a(V2), a(V3)], [V4, 0, a(V6)], B), End, Act)).
akt(3, activity(3, [a(V2), a(V3)], [V4, V5, a(V6)], B, Time, End, NT),
    building_floors)
#(V5 \ a(_) <=
  (n(V5) = n(1) -> n(V5p)),
when(started([3, a(V2), a(V3)], [V4, V5p, a(V6)], a(_), Time)) ->
    (area(area([3, 3, 4], [V4, V5, a(V6)])) -> n(Area)),
    (defn(timeformula -> n(6) * building_area),
     defn(bldg_area -> n(Area)),
     defn(teamsize(general_workers) -> n(2)),
     defn(density(general_workers) -> (building_area / n(25))),
     get_manpower(teamsize(general_workers), density(general_workers), NT)
     -> n(Res)),
    (consume(Time, n(Res), timeformula, End, activity(3, [a(V2), a(V3)], [V4, V5, a(V6)], B, Time, End, NT)))
    -> quote(Act))
-> (change(started([3, a(V2), a(V3)], [V4, V5, a(V6)], B), End, Act)).
akt(81, activity(81, [5, a(V2), a(V3)], [V4, V5, V6], _), Time, End, NT),
    'HVAC installations') <=
when(started([3, a(V3)], [V4, V5, V6], a(_), Time) ->
    (area(area([3, 3, 4], [V4, V5, V6])) -> n(Area)),
    (defn(timeformula -> area * n(3.0))),
    defn(area -> n(Area)),
    defn(teamsize(plumber) -> n(2)),
    defn(density(plumber) -> (area / n(100))),
    defn(place_coeff -> n(1)))
    -> get_manpower(teamsize(plumber), density(plumber), NT) -> n(Res)),
    (consume(Time, n(Res), timeformula, End, activity(81, [5, a(V2), a(V3)], [V4, V5, V6], _, Time, End, NT)))
    -> quote(Act))
-> (change(started([5, a(V2), a(V3)], [V4, V5, V6], _, End), Act)).
akt(83, activity(83, [6, a(V2), a(V3)], [V4, V5, V6], _), Time, End, NT), install_electric) <=
when(started([3, a(V3)], [V4, V5, V6], a(_), Time) ->
    (area(area([3, 3, 4], [a(_, a_), a(____)])) -> n(Area)),
    (defn(timeformula -> area * n(1.0))),
    defn(area -> n(Area)),
    defn(teamsize(electrician) -> n(1)),
    defn(density(electrician) -> (area / n(100))),
    defn(place_coeff -> n(1.0))
    -> get_manpower(teamsize(electrician), density(electrician), NT) -> n(Res)),
    (consume(Time, n(Res), timeformula, End, activity(83, [6, a(V2), a(V3)], [V4, V5, V6], _, Time, End, NT)))
    -> quote(Act))
-> (change(started([6, a(V2), a(V3)], [V4, V5, V6], _, End), Act)).
(get_manpower(teamsize(moulders), density(moulders), NT) -> n(Res)),
(consume(Time, n(Res), timeformula, End, activity[2, [3, 2, a(V3)], [V4, V0, V6], _Time, End, NT)) -> quote(Act))
- (change(started([3, 2, a(V3)], [V4, V0, V6], _, End, Act)).

akt(33, activity[33, [3, 3, a(V3)], [V4, V5, V6], _Time, End, NT),
'Constructing supporting parts of building') <=
when(started([3, 2, a(V3)], [V4, V5, V6], a(_)), Time) ->
(area design([3, 3, 4], [V4, V5, V6]) -> n/Area),
(defun(timeformula -> floor_area * n(2.5))),
(defun(floor_area -> n/Area)),
(defun(answer_unit -> hours)),
(defun(teamsize(moulders) -> n(4))),
(defun(density(moulders) -> (floor_area / n(250))),
(defun(driftcoefficient -> n(1.0))) ->
(get_manpower(teamsize(moulders), density(moulders), NT) -> n(Res)),
(consume(Time, n(Res), timeformula, End, activity[33, [3, 3, a(V3)], [V4, V5, V6], _Time, End, NT)) -> quote(Act))
- (change(started([3, 3, a(V3)], [V4, V5, V6], _, End, Act)).

akt('4_B', activity('4_B', [3, 3, a(V3)], [V4, V5, V6], _Time, End, NT), work_on_outer_roof) <=
when(started([3, 3, a(V3)], [V4, V5, V6], a(_)), Time) ->
(area design([3, 4, a(V3)], [V4, V5, V6]) -> n/Area),
(defun(timeformula -> 3.5 * roof_area)),
(defun(roof_area -> n/Area)),
(defun(answer_unit -> hours)),
(defun(teamsize(carpenters) -> n(10))),
(defun(density(carpenters) -> (roof_area / n(500))),
(defun(driftcoefficient -> n(1))) ->
(get_manpower(teamsize(carpenters), density(carpenters), NT) -> n(Res)),
(consume(Time, n(Res), timeformula, End, activity[4_B, [3, 4, a(V3)], [V4, V5, V6], _Time, End, NT]) -> quote(Act))
- (change(started([3, 4, a(V3)], [V4, V5, V6], _, End, Act)).

akt(6, activity(6, [3, 5, a(_)], [V4, V5, V6], _Time, End, NT), work_on_outer_walls) <=
when(started([3, 3, a(V3)], [V4, V5, V6], a(_)), Time) ->
(area design([3, 5, a(V3)], [V4, V5, V6]) -> n/Area),
(defun(timeformula -> n(2.0) * area_outer_walls)),
(defun(area_outer_walls -> n/Area)),
(defun(answer_unit -> hours)),
(defun(teamsize(brick_layer) -> n(6))),
(defun(density(brick_layer) -> (area_outer_walls / n(100))),
(defun(place_coeff -> n(1))) ->
(get_manpower(teamsize(brick_layer), density(brick_layer), NT) -> n(Res)),
(consume(Time, n(Res), timeformula, End, activity[6, [3, 5, a(_)], [V4, V5, V6], _Time, End, NT]) -> quote(Act))
- (change(started([3, 5, a(V3)], [V4, V5, V6], _, End, Act)).

akt(7, activity(7, [3, 6, a(V3)], [V4, V5, V6], _Time, End, NT), stomkompplettering) <=
when(started([3, 3, a(V3)], [V4, V5, V6], a(_)), Time) ->
(area design([3, 3, 4], [V4, V5, V6]) -> n/Area),
(defun(timeformula -> floor_area * n(2.0))),
(defun(floor_area -> n/Area)),
(defun(answer_unit -> hours)),
(defun(teamsize(carpenters) -> n(5))),
(defun(density(carpenters) -> (floor_area / n(200))),
(defun(place_coeff -> n(1))) ->
(get_manpower(teamsize(carpenters), density(carpenters), NT) -> n(Res)),
(consume(Time, n(Res), timeformula, End, activity[7, [3, 6, a(V3)], [V4, V5, V6], _Time, End, NT]) -> quote(Act))
- (change(started([3, 6, a(V3)], [V4, V5, V6], _, End, Act)).

akt(66, activity(66, [3, 7, a(V3)], [V4, V5, V6], _Time, End, NT), inner_surfaces) <=
when(started([3, 6, a(V3)], [V4, V5, V6], a(_)), Time) ->
(area design([3, 3, a(V3)], a(_), a(_), a(_)) -> n/Area),
(defun(timeformula -> area * n(0.1))),
(defun(area -> n/Area)),
(defun(answer_unit -> hours)),
(defun(teamsize(painters) -> n(1)),
defn(density(painters) -> (area / n(250))),
defn(driftcoefficient -> n(1))) ->
(get_manpower(teamsize(painters),density(painters),NT) -> n(Res)),
(consume(Time,n(Res),timeformula,End,
   activity(66, [3,7,a(V3)], [V4,VS,VS,_.Time,End,NT]) -> quote(Act)))
   -> (change(started([[3,7,a(V3)], [V4,VS,VS,_.End,Act]]).)
akt(-1,Act,times(Min,Max,List)) <=
   true
   -> (find_max_min(List,Min,Max)
      -> performed(Act,Min,Max,List)).

%%%%%%%%%%%%%%%%%%%%%
find_max_min([],Tst,Tsl) <=
   find_max_min([],[],[],Tst,Tsl).

find_max_min([],Min,Max,n(Tst),n(Tsl)) <=
   constr(Tst = min(Min)),
   constr(Tsl = max(Max)).

find_max_min([(n(Tst),n(Tsl)) | R],Min,Max,ST,SL) <=
   find_max_min([R, Tst|Min], [Tsl|Max], ST,SL).

get_manpower(TeamSize, TeamFormula, n(NoOfTeams)) <=
   max_resources(mn, MaxSize),
   (int(MaxSize / TeamSize) -> n(MaxSize1)),
   (int(TeamFormula) -> n(TeamNum)),
   (min([TeamNum,MaxSize1]) -> n(Top)),
   (gen(n(1),n(Top)) -> n(NoOfTeams)) ->
   TeamSize = n(NoOfTeams).

cons(X,Y) <= (
   (X -> X1), (Y -> Y1) -> [X1|Y1]).

max(A) <= (constr((X -> max(A))) -> n(X)).

min(A) <= (constr((X -> min(A))) -> n(X)).

gen(A,B) <= ((int(B) -> n(B1)) -> gen1(A,n(B1),_)).

gen1(n(_),n(X),1) <= n(X).

gen1(n(Y),n(X),Z | (Z \= 1) <= n(Y) < n(X) -> gen1(n(Y),n(X) - n(1),_).

gen1(X,Y,_) | (Y \= 2) <= (Y -> 2) -> gen1(X,2,)._.

when((A,B),Time) <=
   when(A,Time1),
   when(B,Time2),
   constr((Time = max([Time1,Time2]))).
when(started([B1,B2,B3],n(Time)) <=
   %started means when some state
   forall([B1,B2,B3,Prop,T1,T2],
   (design([B1,B2,B3,Prop] -> %started to exist
      state(bsab(B1,B2,B3),T1,T2))),
   findall(Start,state(bsab(B1,B2,B3),n(Start),_),List),
   constr((Time = max(List))),
   findall(End,state(bsab(B1,B2,B3),n(End)),List1),
   constr((Time = min(List1))).
when(finished([B1,B2,B3],n(Time)) <=
   %finished means when some state
   forall([B1,B2,B3,Prop,T1,End],
   (design([B1,B2,B3,Prop] -> %finished to exist
      state(bsab(B1,B2,B3),T1,n(End))))),
   findall(End,state(bsab(B1,B2,B3),n(End)),List1),
   (max(List) -> n(Time))).

change(started([B1,B2,B3],n(Time)) <=
   forall([B1,B2,B3,Prop,End],
   (design([B1,B2,B3,Prop] ->
      state(bsab(B1,B2,B3),n(Time),n(End))))).

change(finished([B1,B2,B3],n(Time)) <=
   forall([B1,B2,B3,Prop,St],
   (design([B1,B2,B3,Prop],
      state(bsab(B1,B2,B3),St,n(Time)) -> true)).
change((A,B),Time) <= change(A,Time),change(B,Time).

consume(n(Time),n(0),timeformula,n(End),Act) <=

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call(constr(End = Time)) ->
  quote(performed(Act,n(Time),n(Time),n(0))).
consume(n(Time),Resources,TimeFormula,n(End),Act) \{Resources \= n(0) <= ((Int((TimeFormula / Resources) / n(B)) -> n(Duration)),
call(constr(End = Time + Duration)) ->
overload_check(Act,n(Time),n(Duration),Resources)).

overload check(Act,Xst,Dur,Res) <=
  max_resources(max,Xst,MR),
  (Xst + Dur -> Xsl),
  findall(n(R),\{Xst1,Xst1,n(R)\},Xst1=Xst1,Xsl1>Xst),Lstart),
  (sum(Lstart) -> n(NStart)),
  (n(NStart) + Res -> StartTotal),
  StartTotal =< MR, % checking start
  findall(Xst1-n(R),\{Xst1\},Xst<Xst1,Xst1>Xst1),
  Lst_in),
  findall(Xsl1=R1,\{Xsl1\},
  Xst<Xsl1,Xsl1>Xsl1,\{n(O)-n(R)=R\}),Lsl1_in),
  append(Lst_in,Lsl1_in,L),
  keysort(L,L1),
  check_all_res_changes(L1,StartTotal,MR)
-> quote(performed(Act,Xst,Xsl,Res)).

append([],L,L).
append([F|R],L,[F|R]) <= append(R,L,R).

check_all_res_changes([],__).
check_all_res_changes([N-X],So far,MaxRes) <=
  (So far + X -> So far1),
  So far1 =< MaxRes.
check_all_res_changes([N-X,N-Y]|R),So far,MaxResource) <=
  (X + Y -> Z),
  check_all_res_changes([N-Z]|R),So far,MaxResource).
check_all_res_changes([N-X,N-Y]|R),So far,MaxResource) \{f(N,N)} \{f(W,W)} <=
  (So far + X -> So far1),
  So far1 =< MaxResource.
  check_all_res_changes([N-Y]|R),So far,MaxResource).

X = X.

%%% Some general routines for arithmetics
area(design(A,B)) <=
call(bagof([C,D],X,\{design(A,B,D,C),member(area(X,C)),L\}) -> sum(L).
volume(design(A,B)) <=
call(bagof([C,D],X,\{design(A,B,D,C),member(volume(X,C)),L\}) -> sum(L).
length(design(A,B)) <=
call(bagof([C,D],X,\{design(A,B,D,C),member(length(X,C)),L\}) -> sum(L).
reinforcement_weight(design(A,B)) <=
call(bagof([C,D,X,Y],Z,
  \{design(A,B,D,C),
  member(construction_weight_kg_per_sqm(X,C)),
  member(area(Y,C),(X * Y -> Z)),L\})
-> sum(L).
call(X) <= X.

%%% Used when all activities are scheduled, and the plans should be plotted
plot <= findall(Y\{X,Y,Z,W\},performed(X,Y,Z,W),L),
present_plans(L).

---

The Control Level

%%% Scheduling phase, new rules

%%% For all is a kind of pi-declaration.
%%% Special generalized index function used:
%%% i(Constructor, Base-constant, Template, Call), where Constructor is the functor
used for building the answer structure, and Base-constant is the empty structure
if there are no solutions to Call.
forall_l_r(forall(X, (A -> C)), PT) <=
    lift_from_a(I, (A -> C), (A1 -> C1), [], Vars),
    i(1, true, C1, X"Vars"*"definias"(A1, true, Y) -> Struct),
    PT -> (Ass \- Struct))
    -> (Ass \- forall(X, (A -> C))).
schedule_forall <=
    right_sch forall(schedule_forall),
    axiom_sch(_r_r_).
right_sch forall(PT) <=
    v_right(_r_r_),
    true_right,
    d_right_sch(_r_r_).

In principle, this can be done by first doing a lot of contractions,
then eliminate the forall quantification with the elements produced by
the bagof.
forall_l_generate_all(forall(X, (A -> G)), I, PT, PTbag, 2) <=
    lift_from_a(I, (A -> G), (A1 -> C1), [], Vars),
    i(1, (G1), X"Vars"*"bagof"(PTbag) -> ([] \- A1)) -> Goals),
    PT -> (I0Goals@R \- C)
    -> (I0(forall(X, (A -> G)) |R \- C).

Strategy for eliminate all forall-conditions
all forall_l_generate_all(PT) <=
    {forall_l_generate_all(_, _, (A \- C), ra_sch, 2) <=
    {all forall_l_generate_all(PT) -> (A \- C)}, PT).
ra_sch <=
    (right(ra_sch) <- true), axiom(_r_r_).

General strategies for eliminating terms through weakening
weak all(T, PT) <=
    copy_term(T, T1) ->
    (search1(_, _i, weak all(T1, I, (A \- C))) <-
    weak all(T, PT) -> (A \- C)), PT).
v left all(PT) <=
    (v left (_, _, I, (A \- C)) <- (v left all(PT) -> (A \- C))), PT.

General strategy for searching the first applicable assumption
search1(N, I, PT) <=
    length(Ass, N) -> (Ass \- _).
search2(N, I, PT) <=
    (prev(I, I1, I2, PT) -> (I1 \- PT) <- true), search1([I1], I2, N, PT).

top level strategy, for the scheduling phase
incorporate(F|R), I, PT) <=
    {PT -> (I0(F|R)@R \- C)} ->
    (I0(F|R)|R1 \- C).
schedule <=
    (ready_sch <- true),
    (search1(_, I),
    d left(akt(_r_), I),
    a left(_, I),
    weak all(akt(_r_), weak all(formed(_, _, _)), schedule pre)),
    a left(_, I),
    weak all(state(_, _, _), weak all(akt(_, _), schedule calc)),
    v left all(schedule post(A \- C))))
    -> (schedule -> (A \- C)),
    false).
    % No act possible
ready_sch <- check_assumptions(Ass) -> (Ass \_ \_).
ready_sch <- (plot <- true), axiom(_,_,_).

check_assumptions(\{}\{}).
check_assumptions([A|:R]) :-
  functor(A,P,_,_),
  (P = state ;
   r = performed),
  check_assumptions(R).

schedule_pre <=
  (a_right(_,math) <- true),
  true_right,
  d_right(_,schedule_pre),
  v_right(_,schedule_pre,schedule_pre),
  forall_r(_,schedule_forall),
  findall_right(schedule_findall),
  constr_right(_.).

schedule_calc <=
  (mathing <- true),
  (right_sch(schedule_calc),
   axiom_sch(_,_,_),
   left_sch(schedule_calc)).

schedule_post(PT) <=
  (d_left(change(_,_,_,_,_,(A \_ C)) <- (schedule_post(PT) -> (A \_ C))),
  (forall_1_generate_all(_,_,_,(A \_ C),r,2)
   <- (schedule_post(PT) -> (A \_ C))),
  PT).

keysort_right(keysort(L,L1)) <=
keysort(L,L1) -> (_ \_ keysort(L,L1)).

% % % Strategies used by schedule_calc
right_sch(PT) <=
  (not(functor(C,state,_,_)) -> (_ \_ C)).
right_sch(PT) <=
  relations,
  findall_right(schedule_findall),
  bagof_right(PT),
  forall_r(C,schedule_forall),
  constr_right(C),
  v_right(_,PT,PT),
  a_right(_,v_left_all(PT)),
  o_right(_,o_left(PT)),
  true_right,
  d_right_sch(_,PT),
  keysort_right(_).

left_sch(PT) <=
  forall_1_generate_all(C,I,PT,r,2),
  v_left(_,_,PT),
  a_left(_,_,PT,v_left_all(PT)),
  o_left(_,_,PT,PT),
  d_left_sch(_,_,PT),
  pi_left(_,_,PT).

axiom_sch(T,C,I) <= axiom(T,C,I).

d_left_sch(C,I,PT) <=
  (not(functor(C,performed,4)) -> (I\_\_C \_ \_)),
  d_left_sch(C,I,PT) <=
  (not(functor(C,state,3)) -> (I\_\_C \_ \_)),
  d_left_sch(C,I,PT) <= d_left(C,I,PT).

d_right_sch(C,PT) <=
  (not(functor(C,plot,_,_)) -> (_ \_ C),
  d_right_sch(C,PT) <= d_right(C,PT).

schedule_findall <=
  (a_right(_,A \_ C) <- (math -> (A \_ C)),
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right_sch_forall(schedule_findall),
axiom_sch(\_,\_,\_),
relations.

%%% -------------------------------
%%% Some general strategies
mathing <= (var(C) ; functor(C,n,1) ; functor(C,quote,1)) -> (_, \_ , C).
mathing <= math.

constr_right(constr(\_)) <=
  constraint(Constr) -> (_, \_ , constr(Constr)).

%%% Overwrites the definition in math.rul.
if_statement <= schedule_calc.

%%%---------------------------
%%% Strategy for plotting a plan.
plot <= (_, \_ , plot).
plot <= d_right(_, v_right(_, findall_right(axiom(_,\_,\_)), presenting)).

presenting <=
  plot_plan(L)
  -> (_, \_ , presentPlans(L)).

%%% PROVISOS
constructor(findall,3).
constructor(bagof,4).
constructor(forall,2).
constructor(constr,1).
constructor(keysort,2).

append([],L,L).
append([F|R],L,[F|R|L]) :- append(R,L,R).

%%%---------------------------
%%% CONSTRAINTS
constraint((X =< Y)) :- prolog:when([ground(X),ground(Y)], X =< Y).
constraint((X >= Y)) :- prolog:when([ground(X),ground(Y)], X >= Y).
constraint((X = Y)) :- prolog:when([ground(X),ground(Y)], X = Y).
constraint((X =< Y)) :- prolog:when([ground(X),ground(Y)], X =< Y).
constraint((X =< Y)) :- prolog:when([ground(X),ground(Y)], X =< Y).
constraint((X = max(L))) :- user:max(L,X). % max(X,L): X is max of L's elements
constraint((X = min(L))) :- user:min(L,X). % min(X,L): X is min of L's elements

Appendix E:
The Mathematical Rules

%%% Rewritten rules
%%% For evaluating user-defined functions
d_left_funs(T,1,PT) <=
  atom(T),
  not(num(T)),
  not(functor(T,'..',2)),
  definiens(T,\_,Dp,N),
  N > 0,
  (PT -> (\[\$[Dp|Y] \_ C\])
  -> (\[\$[T|Y] \_ C\)).

%%% Overwrites the pi_left rule in rules.rul
pi_left((pi X \ C),1,PT) <=
  inst(X,C,C2),
  (PT -> (\[\$|C2|R] \_ C1))
  -> (\[\$|C\$[pi X \ C]|R] \_ C1).

%%%---------------------------
%%% New rules
numax(T,C) <=
  (num(T) ; functor(T,'.',2) ; T == [] ; functor(T,quote,1)),
  unify(C,T)
-> (I@[T[1] \ C).

integer_left(Int(X),I,PT) <=
  (PT -> (I@[X(R) \ n(X)])),
  Y is integer(X1),
  (PT -> (I@[n(Y) \ R \ C]))
-> (I@[int(X) \ R \ C].

mod_left(mod(A,B),I,PT) <=
  (PT -> (I@[A(R) \ n(A)])),
  (PT -> (I@[B(R) \ n(B)])),
  X is A1 mod B1,
  (PT -> (I@[n(X) \ R \ C]))
-> (I@[A \ B \ R \ C].

add_left(+,(A,B),I,PT) <=
  (PT -> (I@[A(R) \ n(A)])),
  (PT -> (I@[B(R) \ n(B)])),
  X is A1 + B1,
  (PT -> (I@[n(X) \ R \ C]))
-> (I@[A + B \ R \ C].

mul_left(*)((A,B),I,PT) <=
  (PT -> (I@[A(R) \ n(A)])),
  (PT -> (I@[B(R) \ n(B)])),
  X is A1 * B1,
  (PT -> (I@[n(X) \ R \ C]))
-> (I@[A \ B \ R \ C].

div_left(/,(A,B),I,PT) <=
  (PT -> (I@[A(R) \ n(A)])),
  (PT -> (I@[B(R) \ n(B)])),
  X is A1 / B1,
  (PT -> (I@[n(X) \ R \ C]))
-> (I@[A / B \ R \ C].

minus_left(-,(A,B),I,PT) <=
  (PT -> (I@[A(R) \ n(A)])),
  (PT -> (I@[B(R) \ n(B)])),
  X is A1 - B1,
  (PT -> (I@[n(X) \ R \ C]))
-> (I@[A - B \ R \ C].

sum_left(sum(\,I,PT,PT1) <=
  (functor(L,'\',2),strip_n(L,L1)) ;
  not(functor(L,'\',2)),
  (I([X],PT1 -> (I@[L(R) \ n(X)])) -> L1)),
  add_list(L1,S),
  (PT -> (I@[n(S) \ R \ C]))
-> (I@[sum(L) \ R \ C].

strip_n([],[]).
strip_n([n(X)|R],[X|R1]) :-
  strip_n(R,R1).
strip_n([F|R],[F|R1]) ;
  not(functor(F,n,1)),
  strip_n(R,R1).

add_list(L,N) :-
  add_list(L,0,N).
add_list([],N,Answer) :-
  N1 is N + F,
  add_list(R,N1,Answer).

%%% For closures
defun_left(defun(X),I,PT) <=
  (PT -> (I@[X(Y) \ C})
-> (I@[defun(X) \ Y \ C].

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less_than <=
  X < Y -> \_ \_ n(X) < n(Y)).

greater_than <=
  X > Y -> \_ \_ n(X) > n(Y)).

greater_or_equal_than <=
  X >= Y -> \_ \_ n(X) >= n(Y)).
equal_or_less_than <=
  X <= Y -> \_ \_ n(X) <= n(Y)).

% % New strategies
% %-----------------------------------------------
math <= math(math).

relations <=
  less_than,
greater_than,
greater_or_equal_than,
equal_or_less_than.

math(PT) <=
  (system_defined(math(PT)) <- true),
eager(math(PT)).

system_defined(PT) <=
  numax(_,_),
  integer_left(_,_,PT),
  mod_left(_,_,PT),
  add_left(_,_,PT),
  mul_left(_,_,PT),
  div_left(_,_,PT),
  minus_left(_,_,PT),
  sum_left(_,_,PT,sumstrat).

% % Default summing strategy. Can be replaced.
sumstrat <= math.

% %-----------------------------
% % To handle user-defined functions.
eager(PT) <=
  (closure(_,PT) <- true),
  user_defined(C,I,PT).

user_defined(C,I,PT) <=
  d_left_fun(C,I,handle(PT)).

handle(PT) <=
  pl_left(_,_,handle(PT)),
  a_left(_,_,if_or_eval(PT),PT),
  and_l(_,_,eager1(PT)),
% if-statement, or evaluation
% case-statement
  % PT.

if_or_eval(PT) <=
  v_right(_,if_or_eval(PT),if_or_eval(PT)),
  a_right(_,PT),
  relations,
  d_right(_,if_statement).
% if-statement
% if-statement

if_statement <= gcla.
% default strategy

% %-----------------------------
% % To handle a closure
closure(I,PT) <= (I[defun(_)]R \_ \_ _).
closure(I,PT) <=
  contr_l(_,I,defun_left(defun(_,),I,closure1(I,PT))).

closure1(I,PT) <=
  pl_left(_,_,closure1(I,PT)),
  a_left(_,I,axiom(_,C,II),weak_l(C,_,handle(PT))),
  and_l(_,I,closure1(I,PT)).
Provisos
constructor(1,1).
constructor(2,2).
constructor(<=,2).
constructor(>=,2).
constructor(>,2).
constructor(<,2).
constructor(\*,2).
constructor(\^,2).
constructor(\^\*,2).
constructor(\^\^,2).
constructor(\-\-,2).
constructor(\-,2).
constructor(n,1).
constructor(quote,1).
construction([],0).
construction([],0).
construction([],0).
construction(defun,1).

num(X) := functor(X,n,1).

Appendix F:

The General Library Rules

:- multifile(constructor/2).

true_right <= (_ \- true).
false_left(I) <= functor(C,\false,0) => (I@[C\_] \- _).

axiom(T,C,I) <=
term(T),
term(C),
unify(T,C)
-> (I@[T\_] \- C).
axiom(T,I) <=
term(T),
term(C),
unify(T,C)
-> (I@[T\_] \- C).

d_right(C,PT) <=
atom(C),
clause(C,B),
(PT -> (P \- B))
-> (P \- C).

d_left(T,I,PT) <=
atom(T),
definiens(T,Dp,N),
(PT -> (I@[Dp\Y] \- C))
-> (I@[T\Y] \- C).

a_left((A -> C1),I,PT,PT1) <=
(PT -> (I@[Y \- A))),
(PT1 -> (I@[C1\Y] \- C))
-> (I@[A -> C1\Y] \- C).

a_right((A -> C),PT) <=
(PT -> ((A|P) \- C))
-> (P \- (A -> C)).

o_right(I,[C1 ; C2],PT) <=
(PT -> (Ass \- C1))
-> (Ass \- (C1 ; C2)).
o_right(2, (C1 ; C2), PT) <=
   (PT -> (Ass \ C2))
   -> (Ass \ (C1 ; C2)).

o_left((A1 ; A2), I, PT1, PT2) <=
   (PT1 -> (I[A1|R \ C]),
   (PT2 -> (I[A2|R \ C]) -> (I[\A1 ; A2|R \ C]).

v_right((C1, C2), PT1, PT2) <=
   (PT1 -> (A \ C1)),
   (PT2 -> (A \ C2))
   -> (A \ (C1, C2)).

t_left((C1, C2), I, PT) <=
   (PT -> (I(C1,C2|Y) \ C))
   -> (I(C1,C2|Y) \ C).

pi_left((pi X \ C), I, PT) <=
   inst(X, C, C1),
   (PT -> (I[\C|R \ Concl])
   -> (I[\pi X \ C]|R \ Concl).

%%% Additional rules that can be incorporated if the
%%% programmer wishes to do so.

sigma_right((X*C), PT) <=
   inst(X, C, C1),
   (PT -> (A \ C1))
   -> (A \ (X*C)).

%%% Principal way to implement comma-left without contraction as
%%% a strategy on top of the ordinary GCLA v_left-rule.
%and_left_1((A,B), I, II, PT) <=
%   append(I,[A,I]) -> (I@[A,B]_1 \ _).
%and_left_1((A,B), I, II, PT) <=
%   v_left((A,B), I, weak_left((B,II,PT)),
%   v_left((A,B), I, weak_left((A,I,PT)).

%%% More efficient version as a new rule, better to use.
and_left_1((A,B), I, PT) <=
   (PT -> (I[A,R] \ C));
   (PT -> (I[B|R] \ C))
   -> (I[A,B|R] \ C).

weak_left_1(T, I, PT) <=
   (PT -> (I[T,R] \ C))
   -> (I[T|R] \ C).

contr_left_1(T, I, PT) <=
   (PT -> (I[T,R] \ C))
   -> (I[T|R] \ C).

add_left_1(X,Y, I, PT) <=
   add(X),
   (PT -> (I[Y]|C) -> (I[add_def(X,Y)|R] \ C).

rem_left_1(X,Y, I, PT) <=
   rem(X),
   (PT -> (I[Y]|C) -> (I[rem_def(X,Y)|R] \ C).

add_right_1(X,Y, PT) <=
   add(X),
   (PT -> (A \ Y)) ->
   (A \ add_def(X,Y)).

rem_right_1(X,Y, PT) <=
   rem(X),
   (PT -> (A \ Y)) ->
   (A \ rem_def(X,Y)).

%%%---------------------------------------------
%% Provisos

constructor(';', 2).
constructor(',->', 2).
constructor(true, 0).
constructor(false, 0).
constructor('?', 2).
constructor(pl, 1).
constructor(contr, 1).
constructor(add_def, 2).
constructor(rem_def, 2).

%% Strategies

gcla <= ari.

ari <= axiom(_, _,_), right(ari), left(ari).
    alr <= axiom(_, _,_), left(alr), right(alr).
    lra <= left(lra), right(lra), axiom(_, _, _).

    ar <= axiom(_, _,_), right(ar).
    al <= axiom(_, _,_), left(al).

    ra <= right(ra), axiom(_, _, _).
    la <= left(la), axiom(_, _, _).

    rl <= right(rl), left(rl).
    lr <= left(lr), right(lr).

    r <= right(r).
    l <= left(l).

right(C, PT) <=
    user_add_right(C, PT),
    v_right(C, PT, PT),
    a_right(C, PT),
    o_right(_, C, PT),
    true_right,
    d_right(C, PT).
	right(PT) <=
    user_add_right(_, PT),
    v_right(_, PT, PT),
    a_right(_, PT),
    o_right(_, PT),
    true_right,
    d_right(_, PT).

c_right(PT) <=
    v_right(_, PT, PT),
    a_right(_, PT),
    o_right(_, PT),
    true_right.

c_right(C, PT) <=
    v_right(C, PT, PT),
    a_right(C, PT),
    o_right(_, C, PT),
    true_right.

left(PT) <=
    user_add_left(_, _, PT),
    false_left(_),
    v_left(_, _, PT),
    a_left(_, _, PT, PT),
    o_left(_, _, PT, PT),
    d_left(_, _, PT),
    pl_left(_, _, PT).

left(C, I, PT) <=
    user_add_left(C, I, PT),
    false_left(I),
    v_left(C, I, PT),
a_left(C,I,PT,PT),
o_left(C,I,PT,PT),
d_left(C,I,PT),
pi_left(C,I,PT).

c_left(PT) <-
false_left(_),
v_left(_,_,PT),
a_left(_,_,PT,PT),
o_left(_,_,PT,PT),
d_left(_,_,PT),
pi_left(_,_,PT).

c_left(C,I,PT) <-
false_left(I),
v_left(C,I,PT),
a_left(C,I,PT,PT),
o_left(C,I,PT,PT),
d_left(C,I,PT),
pi_left(C,I,PT).